Unit 2: Geometry

Learning Goals: We are learning to…

1. Convert units of measurement between the metric and imperial system and vice versa
2. Determine the perimeter and area of simple 2D figures and apply this to real-world problems
3. Apply perimeter and area to composite figures by breaking the figure down into simpler shapes.
4. Determine the volume and surface area of a basic figure
5. Determine the volume and surface area of composite objects by breaking them down into simpler objects
6. Investigate the properties of 2D optimization and determine the dimensions of a shape that would produce the maximum area or minimum perimeter
7. Investigate the properties of 3D optimization and determine the dimensions of an object that would produce the maximum volume and minimum surface area
8. Apply geometry and measurement skills to solve real-world application based problems

Day	Topic	Expectations, Learning Goals	Practice/Homework
1	2.1 Conversions, Perimeter and Area	LG 1, 2 C1.1, 1,2	Worksheet
2	2.2 Area Applications (Composite Figures)	LG 3 C1.3	p.71 #4-6, 9, 10, 12
3	2.3 Volume and Surface Area **Composite Figures Assignment (2.1-2.2)**	LG 4, 8 C1.3	Worksheet
4	2.4 Working with Composite Objects	LG 5, 8 C1.3	p.81 #1-6, 11, 12, 15 Extra Worksheet on GC
5	2.5 Investigating Optimization **Quiz (2.1-2.4)**	LG 8 C2.1	p.94 #1-5, 7
6	2.6 2D Optimization	LG 6, 8 C2.1, 2.2	p.94 #10-13, 16-18
7	2.7 3D Optimization	LG 7, 8 C2.1, 2.3	p.110 #1-5, 8, 9 12-16
8	Review		See review package
9	**Unit 2 Test** **Date:_______________________________**		

Subject to change based on school activities and class needs

Metric and Imperial Conversions

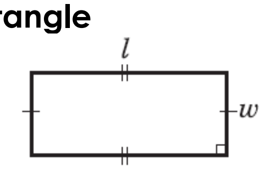
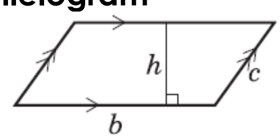
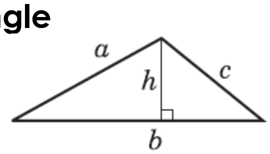
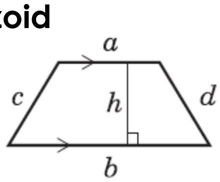
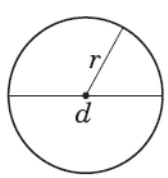
Length

Imperial to Metric	Metric to Imperial
1 inch = 2.54 cm	1 cm $\doteq$ 0.3937 inch
1 foot = 30.48 cm	1 m $\doteq$ 39.37 inches
1 foot = 0.3048 m	1 m $\doteq$ 3.2808 feet
1 mile $\doteq$ 1.609 km	1 km $\doteq$ 0.6214 mile

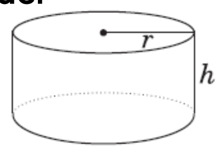
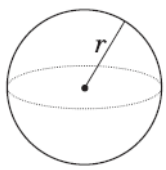
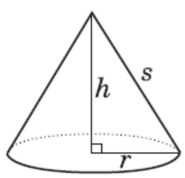
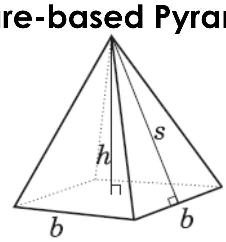
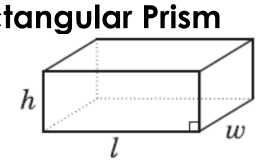
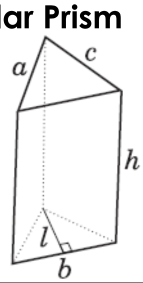
Volume

Imperial to Metric	Metric to Imperial
1 fl. ounce $\doteq$ 28.413 mL	1 mL $\doteq$ 0.0352 fl. ounce
1 pint $\doteq$ 0.568 L	1 L $\doteq$ 1.7598 pints
1 quart $\doteq$ 1.1365 L	1 L $\doteq$ 0.8799 quart
1 gallon $\doteq$ 4.546 L	1 L $\doteq$ 0.22 gallon

Formula Sheet
2-Dimensional Shapes

Geometric Figure	Perimeter	Area
Rectangle 	$P = l + l + w + w$ *or* $P = 2(l + w)$	$A = lw$
Parallelogram 	$P = b + b + c + c$ *or* $P = 2(b + c)$	$A = bh$
Triangle 	$P = a + b + c$	$A = \frac{bh}{2}$ *or* $A = \frac{1}{2}bh$
Trapezoid 	$P = a + b + c + d$	$A = \frac{(a+b)h}{2}$ *or* $A = \frac{1}{2}(a+b)h$
Circle 	$C = \pi d$ *or* $C = 2\pi r$	$A = \pi r^2$

Formula Sheet
3-Dimesional Shapes

Geometric Figure	Surface Area	Volume
Cylinder 	$A_{base} = \pi r^2$ $A_{lateral\ area} = 2\pi rh$ $A_{total} = 2A_{base} + A_{lateral\ area}$ $= 2\pi r^2 + 2\pi rh$	$V = \pi r^2 h$
Sphere 	$A = 4\pi r^2$	$V = \frac{4\pi r^3}{3}$ *or* $V = \frac{4}{3}\pi r^3$
Cone 	$A_{base} = \pi r^2$ $A_{lateral\ area} = \pi rs$ $A_{total} = A_{base} + A_{lateral\ area}$ $= \pi r^2 + \pi rs$	$V = \frac{\pi r^2 h}{3}$ *or* $V = \frac{1}{3}\pi r^2 h$
Square-based Pyramid 	$A_{base} = b^2$ $A_{triangle} = \frac{bs}{2}$ $A_{total} = A_{base} + 4A_{triangle}$ $= b^2 + 2bs$	$V = \frac{b^2 h}{3}$ *or* $V = \frac{1}{3}b^2 h$
Rectangular Prism 	$A = wh + wh + lw + lw + lh + lh$ *or* $A = 2(wh + lw + lh)$	$V = lwh$
Triangular Prism 	$A_{base} = \frac{bl}{2}$ $A_{rectangles} = ah + bh + ch$ $A_{total} = 2A_{base} + A_{rectangles}$ $= bl + ah + bh + ch$	$V = \frac{blh}{2}$ *or* $V = \frac{1}{2}blh$

2.1 Conversions, Perimeter and Area

Learning Goals: I am learning to…
- ☐ Convert units of measurement between the metric and imperial system and vice versa.
- ☐ Determine the perimeter and area of simple 2D figures and apply this to real world application problems.

Recall Key Terms:

Conversion Factors	
Perimeter	
Area	
Volume	
Surface Area	

Part A: Conversions

Example 1: Convert each unit of measure. Round to 2 decimal places.

a) $5\frac{1}{4}$ inches → centimetres (cm)

b) 7.3 km^2 → square miles ($miles^2$)

c) 8.0 gallons → Litres (L)

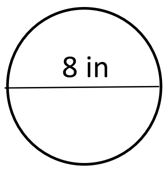
d) 170 mL → fluid ounces (fl. Oz.)

e) In 2005, Canadians consumed on average 94.7 L of milk per person. Americans consumed on average 21.2 US galls per person. Each US gallon is equivalent to 3.785 L. Which country had the greater milk consumption per person? Justify your answer.

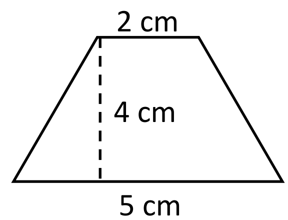
<u>Part B: Perimeter and Area</u>

Example 2: Determine the measure indicated for each geometric figure/object

a) Circumference (perimeter)



b) Find area

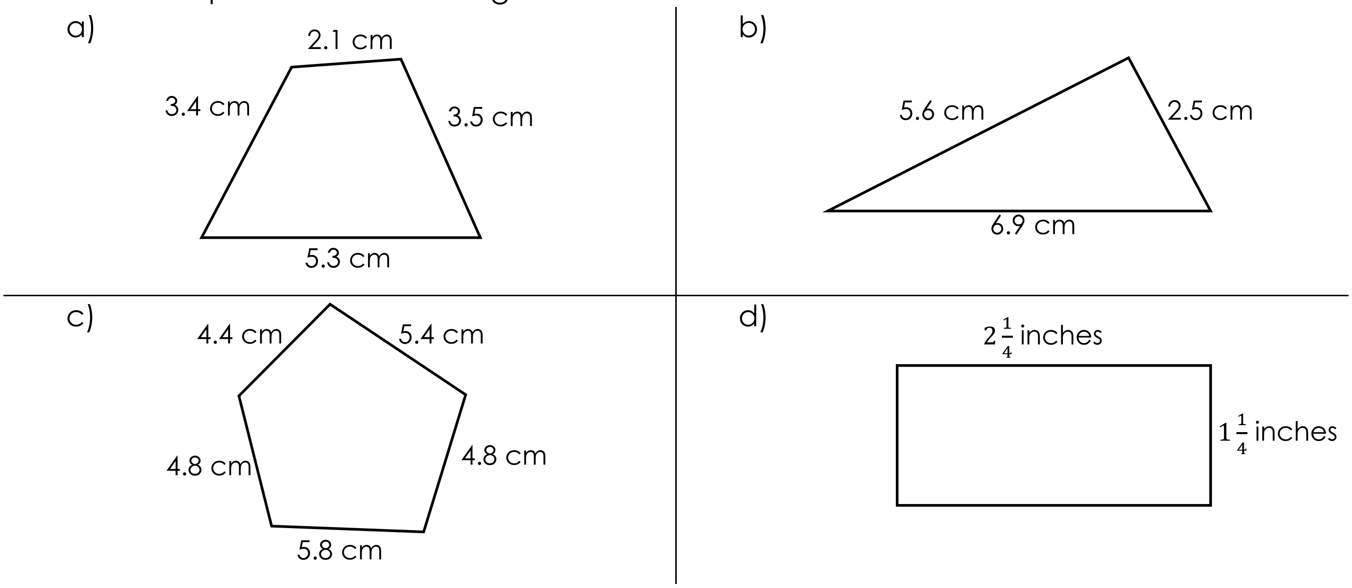


c) Determine the diameter of a circle with an area of 36 m^2.

d) Find the cost of installing tiles on a floor that is 8.4 m long and 5.1 m wide if the tiles cost $34.95/$m^2$.

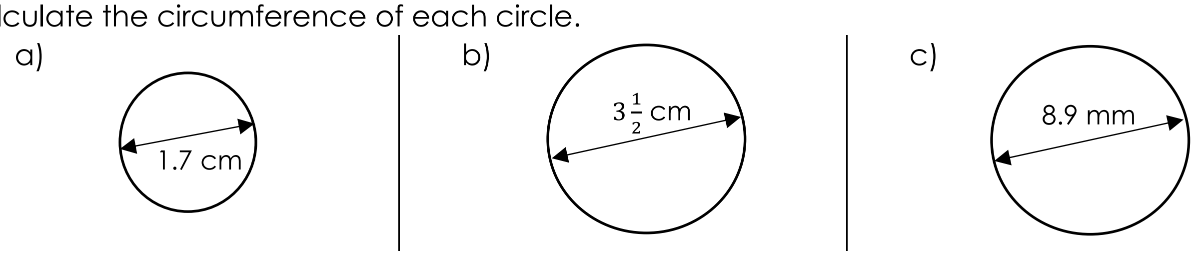
Perimeter Exercises

1. Calculate the perimeter of each figure.



2. Calculate the perimeter of each figure:
   a) A rectangle with sides of 12 feet and 7 feet.
   b) A triangle with side 32.4 m, 18.6 m and 21.5 m.
   c) A regular hexagon (all 6 sides are equal in length) with sides 6.5 inches.
   d) A trapezoid with sides 7.5 cm, 5.1 cm, 4.4 cm and 4.8 cm.

3. Calculate the circumference of each circle.



4. Calculate the circumference of each circle.

a) Radius 25 mm	b) Diameter 7"	c) Radius 15 cm
d) Diameter 3.5 m	e) Diameter 6.5"	f) Radius 12 yards

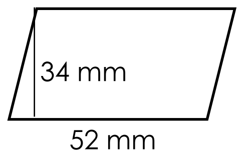
5. Giles has a rectangular swimming pool with dimension 16 feet by 32 feet. A 6-foot wide concrete deck surrounds the pool. What length of fencing is needed to enclose the pool and deck, inclusive?

Answers: 1) a) 14.6 cm b) 15 cm c) 7 inches 2) a) 38 feet b) 72.5 m c) 39 inches d) 21.8 cm 3) a) 3.77 m b) 11 cm c) 27.96 mm 4) a) 157.08 mm b) 21.99 inches c) 94.25 cm d) 11 m e) 20.42 inches f) 75.40 yards 5) 144 feet

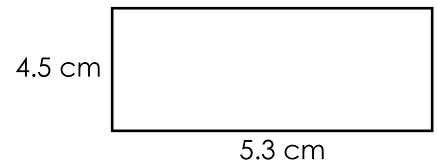
Area Exercises

1. Determine the area of each figure. Round your answers to the nearest tenth where appropriate.

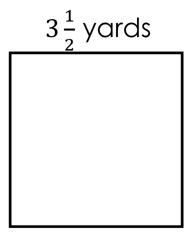
a)



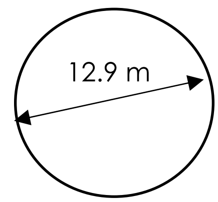
b)



c)



d)



2. Determine the area of each figure. Round your answer to the nearest tenth where appropriate.
   a) A circle with radius 6 feet is painted around the free throw line on a basketball court.
   b) A wall is 7 feet high and 10 ½ feet wide.
   c) A trapezoid shaped lot has parallel sides of length 7 m and 9 m and a depth of 19 m.
   d) A parallelogram-shaped advertising sign has a base 6 ½ feet and height 5 feet.
   e) A decorative circular window has diameter 17 cm.

3. The hypotenuse of a right triangle is 10.8 cm. One leg is 6.1 cm. Calculate the area of the triangle to the nearest tenth.

4. Two rectangular properties have these dimensions.

   Property A: 60 feet by 95 feet
   Property B: 27 feet by 200 feet

   Which property has the greater area?

5. Carol's backyard is the shape of a trapezoid with the parallel sides 40 feet and 100 feet, and a depth of 120 feet. Carol is preparing to fertilize the lawn. A bag of fertilizer will cover 3000 square feet. How many bags of fertilizer must Carol buy?

6. A room is 12 feet by 14 feet. Jorge will buy carpet and underpad for the room from a discount supplier. He will contract a local installer to lay the carpet and underpad. Underpad is $7.97 per square yard and carpet is $49.85 per square yard. The installation costs $0.95 per square foot. Determine the total cost for the materials and installation.

Answers: 1) a) 176.8 mm^2 b) 28.3 cm^2 c) 12.2 yd^2 d) 130.4 m^2 2) a) 113.1 ft^2 b) 73.5 ft^2 c) 152 m^2 d) 32.5 ft^2 e) 227 cm^2 3) 27.1 cm^2 4) Property A 5) 3 bags 6) $1232.73

2.2 Area Applications

Learning Goals: I am learning to…

- ☐ Apply perimeter and area to composite figures by breaking the figure down into simpler shapes.

Sometimes figures are not always simple shapes where the area and perimeter can be determined using one formula. These type of figures can be broken down into smaller figures made up of a combination of small simple figures. These are known as **composite figures.**

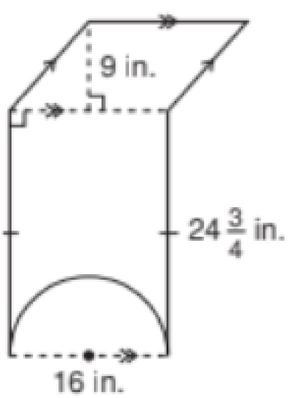
Example: The composite figure below can be broken up into several simple figures. Is there more than one method of breaking this figure down?

Steps to Finding the Area of a Composite figure:

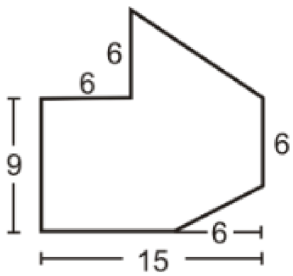
1. Break the composite figure into **simpler figures** which you know how to calculate the area for.
   Note: There is often more than one way to break the figure down!
2. Determine the area of each simpler figure separately.
3. Combine all the areas of each simpler figure by adding.
4. Subtract any areas of any parts that have been removed in the figure.

Note: When working with pi (π), always use the pi button on your calculator! Never round to 3.14 in your solution!

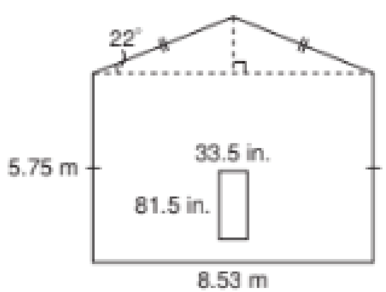
Example 1: Determine the **area** of the following composite figure. *Round to one decimal place*.



Example 2: Determine the **area** and **perimeter** for the following composite figure. *Round to one decimal place.*



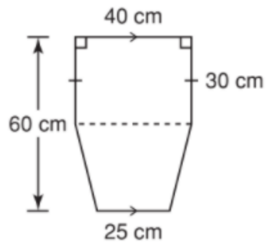
Example 3: Carpenters have constructed the frame for a house and will nail pressboard over the frame. Determine the area of the pressboard they need for the back wall of the house. *Round to* ***one decimal place.*** Hint: You may need to recall trigonometry!



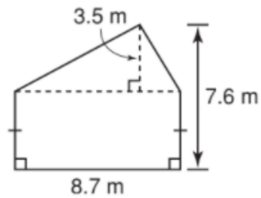
2.2 Area Applications (Composite Figures) Homework

3. Sketch the figures that make up each composite figure. Include measurements in your sketches. All curves are circles or semicircles.

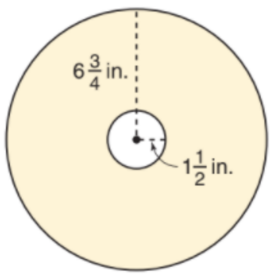
a)



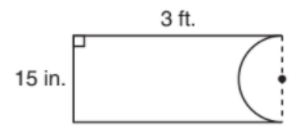
b)



c)

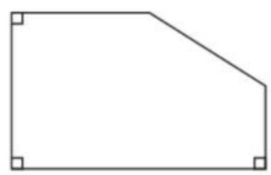


d)



4. Determine the area of each composite figure in question 3.

5. Two students are calculating the area of this figure.



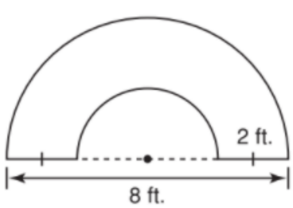
> Tasmin's method
> This is a rectangle with a trapezoid on top. I will add the area of the rectangle to the area of the trapezoid.

> Jeffrey's method
> This is a rectangle with a triangle removed from the corner. I will subtract the area of the triangle from the area of the rectangle.

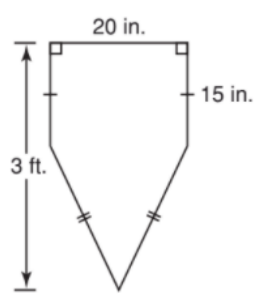
Who is correct? Justify your answer.
Include diagrams in your explanation.

6. Describe the figures that make up each composite figure.
Determine the area of each composite figure. All curves are semicircles.

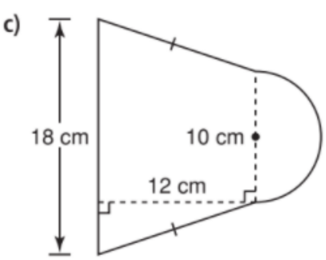
a)



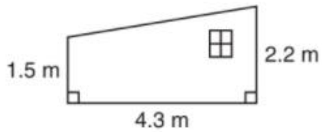
b)



c)



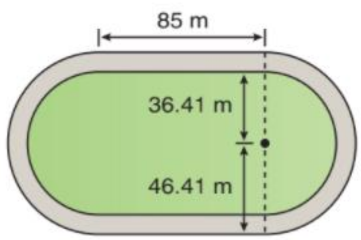
9. A decorator is painting this wall of an attic room. The window measures 0.6 m by 0.5 m.



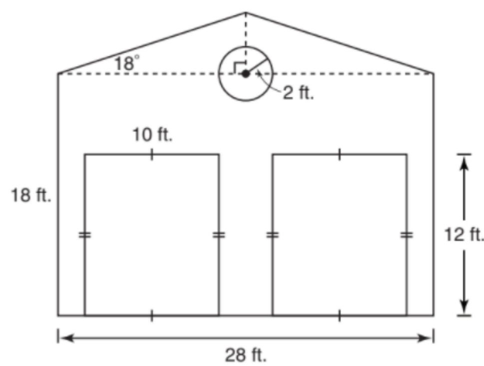
a) What is the area of the wall in square metres and square feet?

b) The paint is sold in 1-pint containers. Each container should cover between 50 square feet and 60 square feet. How many cans of paint should the decorator buy?

10. Assessment Focus The running track in this diagram consists of two parallel sections with semicircular sections at each end. Determine the area of the track.



12. An outdoor garage is being built on a farm to house vehicles and equipment. The front has two congruent garage door entrances and a round window at the top.



a) The front wall will be covered in sheet metal. Determine the amount of sheet metal needed, to the nearest square foot.

b) Suppose the sheet metal is priced by the square metre. How many square metres will be needed for this project?

Answers:

4. a) $2175\ cm^2$ b) About $50.9\ m^2$
c) About 136.07 sq. in. d) About 451.6 sq. in.
5. Both of them are correct.
6. a) A semicircle with a smaller semicircle cut out; $A \doteq 18.85$ sq. ft.
b) A rectangle on top of a triangle; $A = 510$ sq. in.
c) A semicircle on top of a trapezoid; $A \doteq 207.2699\ cm^2$
9. a) $A \doteq 7.955\ m^2$; $A \doteq 82.4$ sq. ft.
b) 2 cans
10. $A \doteq 4301.87\ m^2$
12. a) About 315 sq. ft.
b) About $29\ m^2$

2.3 Volume and Surface Area

Learning Goals: I am learning to…

- ☐ Determine the volume and surface area of a simple object
- ☐ Apply volume and surface area to a real-world scenario

Reminder:

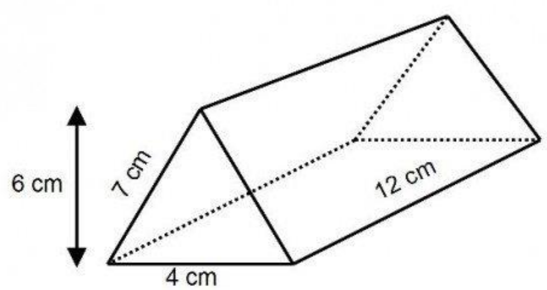
- **Volume** is…
- **Surface Area** is…

Part A: Volume and Surface Area of a Prism

Example 1: Given the following prism.

a) Determine the amount of chocolate that can fit inside this Toblerone package. *Round to one decimal place.*

b) Determine the surface area of the wrapper needed to cover the chocolate. *Round to one decimal place.*



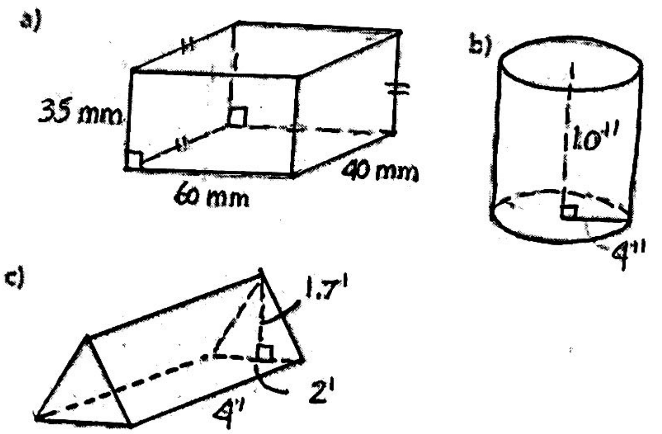
Part B: Volume and Surface Area of a Cylinder

Example 2: Cineplex has just redesigned their jumbo popcorn containers. The container is 2 ft high and holds 1.5 m^3 of popcorn.

a) What is the diameter of the container to the nearest centimetre?

b) What is the surface area of the popcorn bucket? *Round to one decimal place.*

1. Calculate the volume and surface area of each figure, correct to one decimal place.



Volume
a) 8400 mm^2
b) 502.4 in^3
c) 6.5 ft^3

S.A.
a) 11800 mm^2
b) 351.68 in^2
c) 27.4 ft^2

2. Calculate the volume of each prism:
   a) rectangular prism 4' by 3' by 8' {96 ft^3}
   b) a cylinder with diameter 9.8 cm and height 6.3 cm {474.96 cm^3}
   c) a triangular prism 3 m long with triangular height 1.2 m and triangular base 1.8 m {3.24 m^3}

3. A rectangular swimming pool is 18' by 32' by 4' deep. The pool is to be filled to a depth 6" from the top. How many cubic feet of water are needed to fill the pool? {2016 ft^3}

4. A room is 3.8 m by 4.7 m by 2.4 m high. All surfaces except the floor are to be painted. Ignoring doors and windows, how many square metres of surface are to be painted? {58.66 m^2}

5. A cylindrical tank with diameter 1.0 m and height 1.5 m is to be filled with water. How many litres of water are needed to fill the tank? (1 000 cm^3 = 1L) {1177.5 L}

6. A tent in the shape of a triangular prism is being made. The same material will be used for the walls and the floors. The floor is 6 feet by 12 feet. The tent height is 4 feet. Determine the minimum number of square yards of material required to make the tent. {20.6 yd^2}

7. Bob is a stonemason and is preparing to build a rectangular walk with fieldstone. Before laying the stone he must have a base of screenings. This base is to be 6 inches deep. The walk measures 2 feet by 44 feet. How many cubic feet of screenings must Bob order so that he can prepare the walk for the stone? {44 ft^3}

2.4 Working with Composite Objects

Learning Goals: I am learning to…

- ☐ Determine the volume and surface area of composite objects by breaking them down into simpler objects

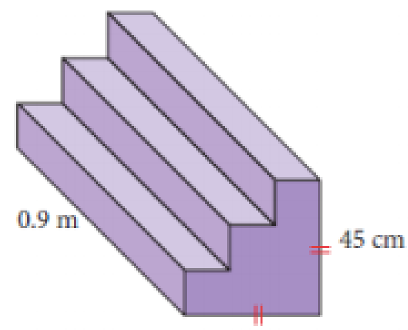
In lesson 2.2 we defined a **composite figure** to be a figure made up of two or more simple geometric shapes. This can also be applied to 3D objects, where the object can be broken up into two or more simpler objects. These are known as **composite objects.**

Steps to Determine the Volume of a Composite Figure

1. Break the composite object into simpler composite objects which you already know how to determine the volume for.
   Note: There is often more than one way to break the object down!
2. Calculate the volume of each simpler composite object separately.
3. Combine all the volumes by adding.
4. Subtract any volumes that have been removed from the total composite object.

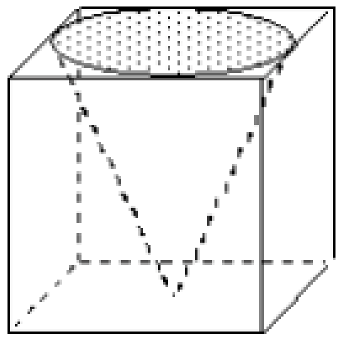
Example 1: You want to construct a concrete staircase with the dimensions shown.

a) Determine the amount of concrete needed to construct the staircase. *Round to* ***two decimal places.***



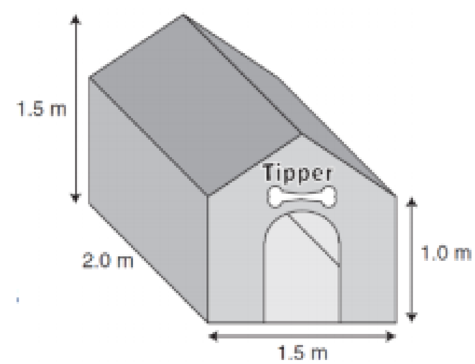
b) If concrete costs \$0.02/ cm^3, how much will it cost to build the stairs.

Example 2: A machinist drilled a conical cone into a cube, as shown in the diagram below. If the cube has sides of 8 cm, what is the volume of the metal **after** the hole is drilled? *Round to* ***one decimal place.***



Steps to Determine the Surface Area of a Composite Object

1. Determine the surface area of each "face" that makes up the composite object.
2. Combine the areas together by adding.
3. Subtract any surface areas that have been removed from the composite object.



Example 3: James is making a doghouse for his dog, Jeff.

a) What is the surface area of the exterior of the doghouse **before the doorway is cut**? Include the floor in your calculations. *Round to* ***one decimal place.***

Face	Shape	Qty	Formula	Area
Roof	Rectangle			
Triangular Panels	Triangle			
Front/Back	Rectangle			
Sides	Rectangle			
Floor	Rectangle			

b) The exterior walls and roof of the doghouse are to be painted a different colour. A 40 cm wide doorway has been cut to allow Jeff to enter and exit his house. The doorway is 60 cm at its highest point. What **area is to be painted** on the doghouse? *Round to* ***one decimal place.***

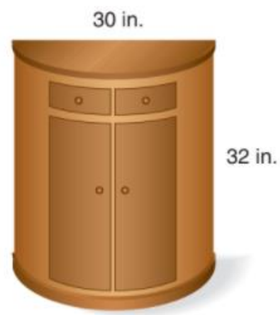
2.4 Composite Objects Homework

14. Olivia owns a furniture store. She has sold a half-cylinder console table like the one shown here. Olivia needs to build a crate to ship the table to the customer.

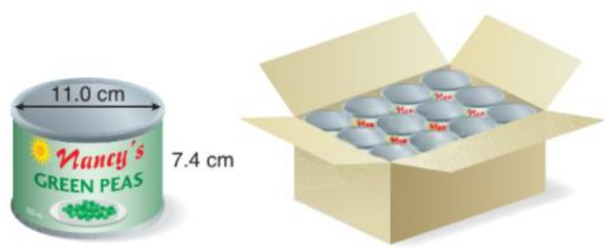
a) What dimensions would you recommend for the shipping crate? Justify your answer.

b) What will be the volume of the shipping crate?

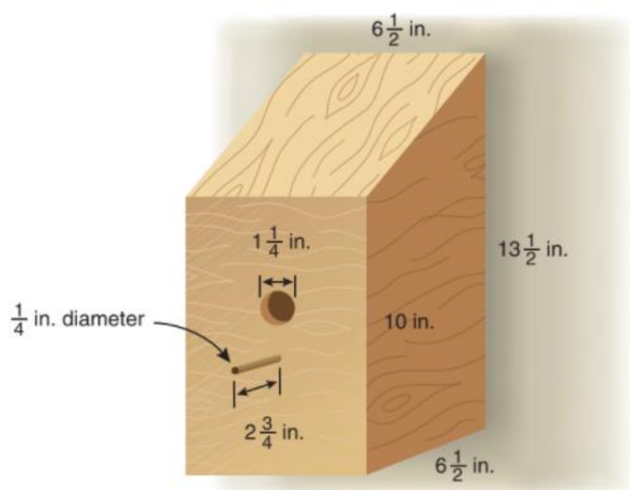
c) How much empty space will there be around the table for protective packing material?



15. A can of peas has a diameter of 11.0 cm and a height of 7.4 cm. The cans are packed for shipping in a box. They are arranged in 2 layers of 3 rows by 4. The box is constructed to fit the cans snugly. Determine the amount of empty space in the box.



16. Amutha has built a birdhouse, and decides to paint it. Determine the surface area that requires painting.



Answers:

14. a) The dimensions for the shipping crate should be slightly greater than the dimensions of the console.
The smallest shipping crate is 31 in. by 16 in. by 33 in.

b) 16 368 cu. in.

c) 5058.27 cu. in.

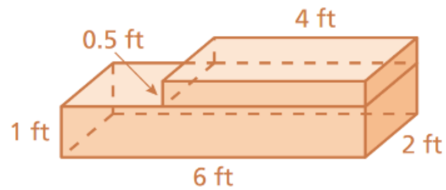
15. About 4611.7 cm^3

16. About 396.72 sq. in.

Working with Composite Objects Practice

<u>Determine the Volume of each Composite Object</u> - *Round all volumes to* ***one decimal place.***

1.



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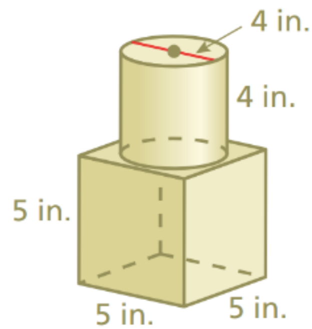

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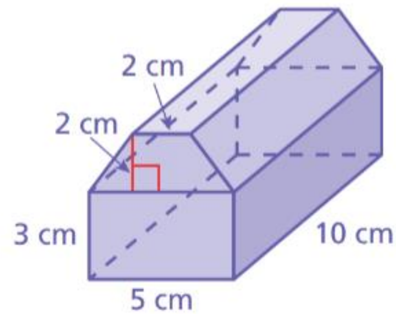

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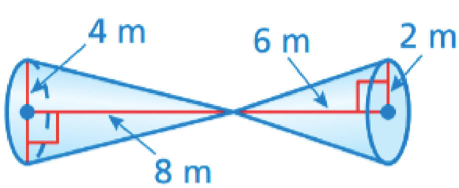

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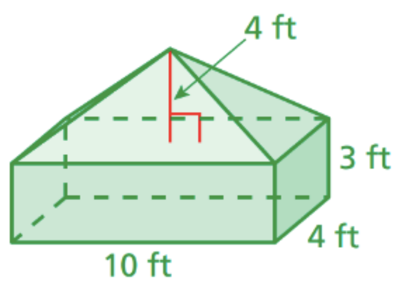

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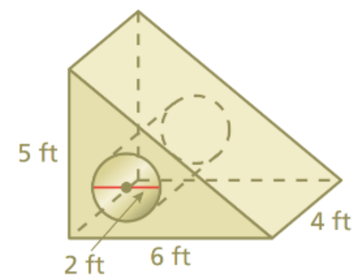

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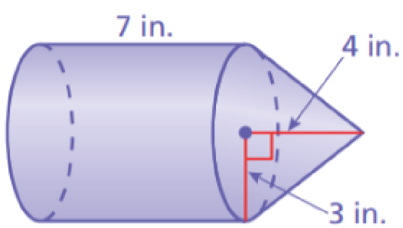

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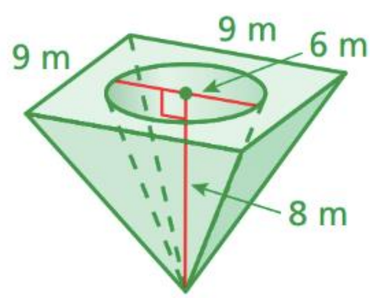

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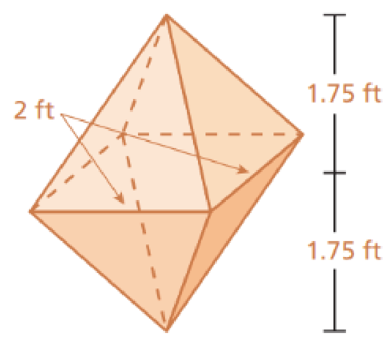

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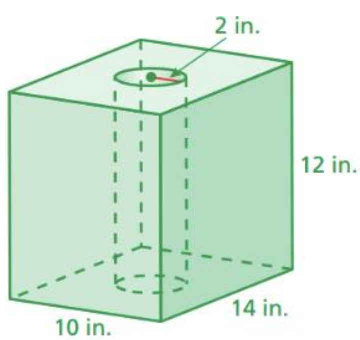

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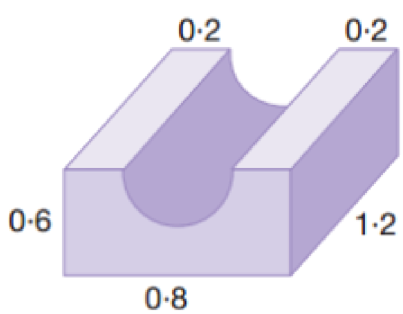

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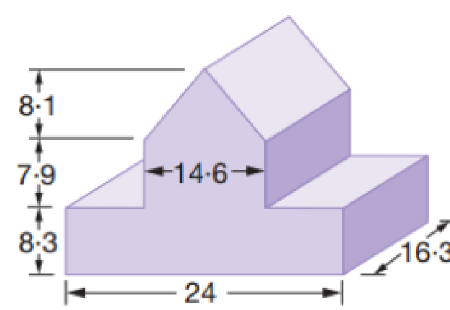
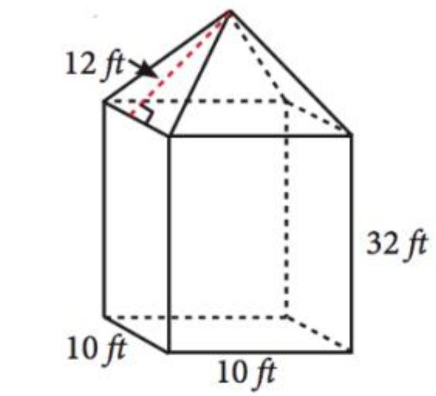


<u>Determine the Surface Area of each Composite Object</u> – *Round all Surface Areas to* ***one decimal place.***

1.



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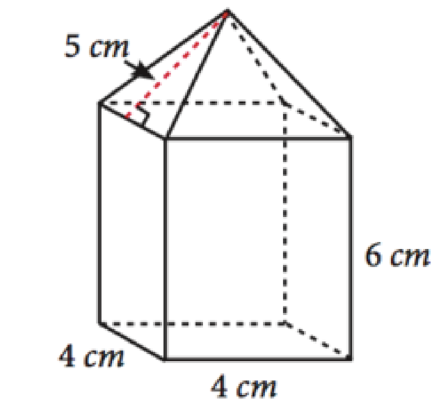

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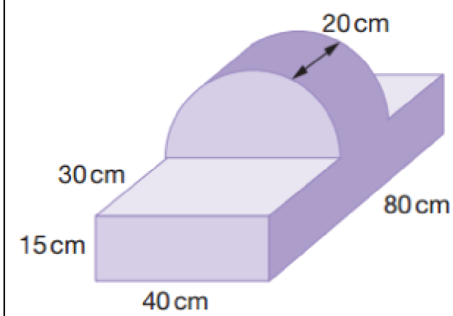

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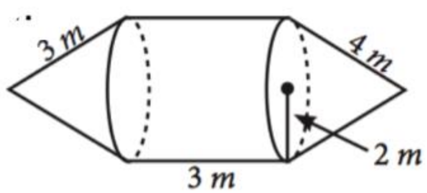

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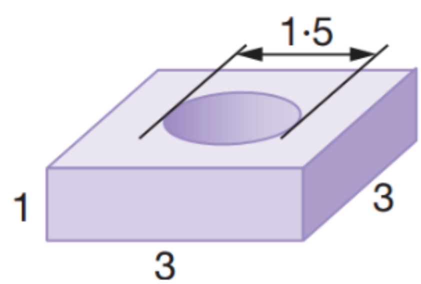

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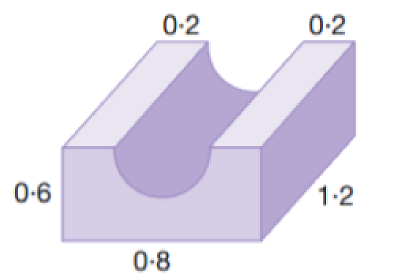

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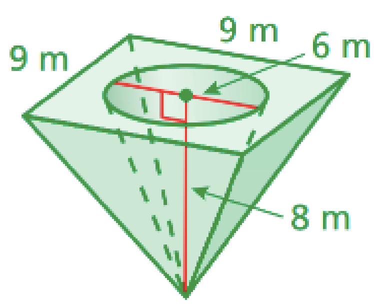

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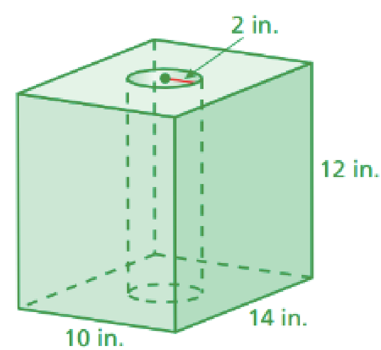

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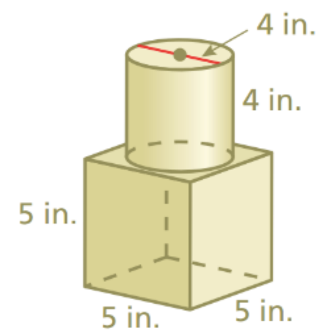

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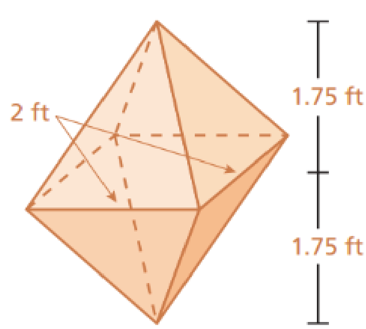

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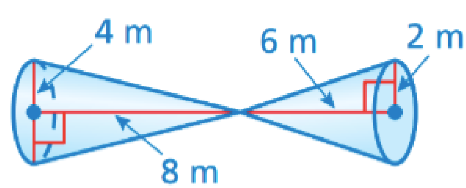

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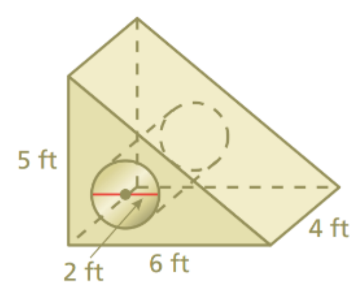


2.5 Investigating Optimization

Learning Goals: I am learning to…
- ☐ Investigate the properties of optimization specific to optimizing area and perimeter.

Part A: Optimizing Area (Maximizing Area)

You are looking to build a rectangular enclosed pen for your pet. You have been given a set amount of fencing to create an enclosure to optimize the area. Why would you want to optimize the area of the enclosure?

-
-

Scenario 1: You have 12 m of fencing: Draw three **rectangular** enclosures that you could build with 12 m of fence. Determine which has the largest area.

Enclosure 1	Enclosure 2	Enclosure 3
Perimeter = 12 m **Dimensions =** **Area =**	**Perimeter = 12 m** **Dimensions =** **Area =**	**Perimeter = 12 m** **Dimensions =** **Area =**

Given a perimeter of 12 m, the enclosure with dimensions _________ x _________ gives an optimal area of ________ m^2.

Scenario 2: You have 16 m of fencing: Draw four **rectangular** enclosures that you could build with 16 m of fence. Determine which has the largest area.

Enclosure 1	Enclosure 2	Enclosure 3	Enclosure 4
Perimeter = 16 m **Dimensions =** **Area =**	**Perimeter = 16 m** **Dimensions =** **Area =**	**Perimeter = 16 m** **Dimensions =** **Area =**	**Perimeter = 16 m** **Dimensions =** **Area =**

Given a perimeter of 16 m, the enclosure with dimensions _________ x _________ gives an optimal area of ________ m^2.

Scenario 3: You have 20 m of fencing

Draw five **rectangular** enclosures that you could build with 20 m of fence. Determine which has the largest area.

Enclosure 1	Enclosure 2	Enclosure 3	Enclosure 4	Enclosure 5
Perimeter = 20m **Dimensions =** **Area =**	**Perimeter = 20m** **Dimensions =** **Area =**	**Perimeter = 20m** **Dimensions =** **Area =**	**Perimeter = 20m** **Dimensions =** **Area =**	**Perimeter = 20m** **Dimensions =** **Area =**

Given a perimeter of 20 m, the enclosure with dimensions _________ x _________ gives an optimal area of ________ m^2.

Part B: Optimizing Perimeter (Minimizing Perimeter)

You are still making an enclosure for your pet, but this time you have a limited area, which must be **36m^2.** You need to design the enclosure in order to optimize the perimeter of the enclosure.

Why would you want to optimize the perimeter of the enclosure?

-
-

Scenario: You have 36 m^2 to fence: Draw five **rectangular** enclosures that you could build for a 36 m^2 enclosure. Determine which has the largest perimeter.

Enclosure 1	Enclosure 2	Enclosure 3	Enclosure 4	Enclosure 5
Perimeter = **Dimensions =** **Area = 36 m^2**	**Perimeter =** **Dimensions =** **Area = 36 m^2**	**Perimeter =** **Dimensions =** **Area = 36 m^2**	**Perimeter =** **Dimensions =** **Area = 36 m^2**	**Perimeter =** **Dimensions =** **Area = 36 m^2**

Given an area of 36 m^2, the enclosure with dimensions _________ x _________ gives an optimal perimeter of ________ m.

Conclusions

1. What can you conclude about optimizing area given a set perimeter?

-
-
-

2. What can you conclude about optimizing perimeter given a set area?

-
-
-

Example: Without drawing the rectangular enclosures, use what you have learned above to determine the optimal area/perimeter for the following situations.

1. Optimize the area, given the following perimeters.

a) P = 100 cm	b) P = 360 m	c) P= 562 m

2. Optimize the perimeter, given the following areas.

a) A = 49 m^2	b) A = 112 m^2	c) A = 570 m^2

2.5 Investigating Optimization Homework

1. For each perimeter, what are the dimensions of the rectangle with the maximum area? What is the area?

a) 40 cm **b)** 110 feet **c)** 25 m **d)** 87 inches

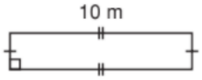
2. For each area, what are the dimensions of the rectangle with the minimum perimeter? What is the perimeter?

a) 25 square feet **b)** 81 m^2 **c)** 144 cm^2 **d)** 169 square inches

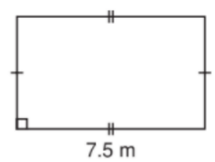
3. For each area, what are the dimensions of the rectangle with the minimum perimeter? What is the perimeter? Round your answers to one decimal place.

a) 30 square feet **b)** 65 m^2 **c)** 124 cm^2 **d)** 250 square inches

4. A gardener uses 24 m of fencing to enclose a rectangular vegetable garden. Some possible rectangles are shown. Determine the missing dimension for each diagram.

a)


b)


c)


5. Calculate the area of each garden shown in question 4.

7. At an outdoor festival, 2-m sections of fencing are used to enclose an area for food sales. There are 100 sections of fencing available.

a) How many metres of fencing are available altogether?

b) Determine the maximum rectangular area that could be enclosed. How does the fact that the fencing is in sections affect your answer?

Answers:

1. The rectangle with the maximum area is a square.
   - **a)** Side length: 10 cm; area: 100 cm^2
   - **b)** Side length: 27.5 ft.; area: 756.25 sq. ft.
   - **c)** Side length: 6.25 m; area: 39.0625 m^2
   - **d)** Side length: 21.75 in.; area: 473.0625 sq. in.
2. The rectangle with the minimum perimeter is a square.
   - **a)** Side length: 5 ft.; perimeter: 20 ft.
   - **b)** Side length: 9 m; perimeter: 36 m
   - **c)** Side length: 12 cm; perimeter: 48 cm
   - **d)** Side length: 13 in.; perimeter: 52 in.
3. The rectangle with the minimum perimeter is a square.
   - **a)** Side length: about 5.5 ft.; perimeter: about 21.9 ft.
   - **b)** Side length: about 8.1 m; perimeter: about 32.3 m
   - **c)** Side length: about 11.1 cm; perimeter: about 44.5 cm
   - **d)** Side length: about 15.8 in.; perimeter: about 63.2 in.
4.
   - **a)** 2 m
   - **b)** 8 m
   - **c)** 4.5 m
5.
   - **a)** $A = 20\ m^2$
   - **b)** $A = 32\ m^2$
   - **c)** $A = 33.75\ m^2$
7.
   - **a)** 200 m
   - **b)** Maximum area: 2500 m^2

2.6 2D Optimization

Learning Goals: I am learning to…

- ☐ Investigate the properties of 2D optimization and determine the dimensions of a shape that would produce the maximum area or minimum perimeter

In lesson 2.5 we investigated how we could optimize the area and then perimeter of an animal pen. We defined **optimization** as the process of finding the most efficient use of materials for a given situation.

Recall:

1. To **optimize area,** given a specific perimeter, a ______________ has the maximum area.
2. To **optimize perimeter,** given a specific area, a ______________ has the minimum perimeter.

Example 1: What dimensions produce an optimal area of a rectangle, with a perimeter of 60 cm? What is the area?

Example 2: What dimensions produce an optimal perimeter of a rectangle, with an area of 81 cm^2? What is the perimeter?

2D Optimization with Restrictions

Sometimes when trying to optimize you may come across a situation where it may not be possible to form a square because of certain restrictions, such as:

- The length and width of the rectangle need to be whole numbers
- One or more sides are enclosed by natural boundaries (i.e. wall, fence, house etc.)

Example 3: A rectangular garden is to be fenced using the wall of a house as one on the sides of the garden. The garden must have an area of 40 m^2. Determine the minimum perimeter and dimensions of the garden if:

a) The dimensions must be whole numbers

b) The dimensions can be decimals

When there are restrictions such as the above situation, a square cannot be formed to form the optimal shape. In this situation,

Total Length = Total Width
2L = W

Example 4: Carlo has 28 m of fencing to build a pen for his dog. He plans to build the pen along one wall of his house to save on materials. What dimensions of the pen will give the greatest area?

Example 5: A farmer wants to create a fenced exercise yard for her horses. She has 900 m of flexible fencing and wants to create the maximum area possible for yard. She can't decide if she should make a rectangular fence or a circular shaped fence. Determine which enclosure will produce the greatest area, the rectangle or the circle.

2.6 2D Optimization Homework

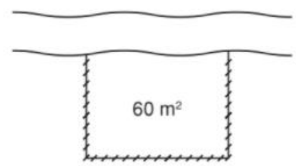
10. A rectangular patio is to be constructed from 80 congruent square tiles.

a) What arrangement of tiles would give the minimum perimeter? Justify your answer.

b) Suppose each tile has side length 50 cm. What would be the minimum perimeter? What would be the area of the patio?

11. A farmer has 650 feet of fencing. Does she have enough fencing to enclose a rectangular area of half an acre? Justify your answer.

12. A rectangular section of a field is to be fenced. Because one side of the field is bordered by a creek, only 3 sides need to be fenced. The fenced section should have an area of 60 m^2. Determine the minimum perimeter and dimensions of the fenced area in each case:



a) The dimensions must be whole numbers of metres.

b) The dimensions can be decimal lengths.

13. A lifeguard is roping off a rectangular swimming area using the beach as one side. She has 200 m of rope.

a) Determine the greatest area she can rope off and its dimensions.

b) Is the area in part a greater or less than 50 000 square feet? Justify your answer.

16. **Assessment Focus** The Tengs are adding a sunroom to their house. The perimeter of the sunroom will be 45 feet, not including the wall that is part of the house.

a) One design is for a rectangular sunroom. Determine the maximum possible area of the room and the dimensions that give this area.

b) Another design is in the shape of a semicircle, where the straight edge is attached to the house. Determine the diameter and area of the room.

c) Which design has the greater area? How much greater is it?

17. Most of the heat loss for outdoor swimming pools is due to surface evaporation. So, the greater the area of the surface of the pool, the greater the heat loss. For a given perimeter, which surface shape would be more efficient at retaining heat: a circle or a rectangle? Justify your answer.

18. A farmer has 1800 m of fencing. He needs to create two congruent rectangular fields, as shown. Determine the maximum possible area of each field.

Answers:

10. a) Arrange the tiles to form a 8-tile by 10-tile rectangle.

b) Minimum perimeter: 1800 cm; $A = 200\ 000\ cm^2$

11. Yes

12. a) The minimum perimeter is 22 m. The dimensions are 10 m by 6 m or 12 m by 5 m.

b) The minimum perimeter is about 21.91 m. The dimensions are about 5.45 m by 11 m.

13. a) Greatest area: 5000 m^2 The dimensions are 100 m by 50 m.

b) Greater

16. a) Maximum area: 253 sq. ft. The dimensions are 22 ft. by 11.5 ft. or 23 ft. by 11 ft.

b) $d \doteq 28.65$ ft.; $A \doteq 322.29$ sq. ft.

c) The semicircle design has a greater area; about 69.2 sq. ft.

17. Rectangle

18. Maximum area: 67 500 m^2

2.7 3D Optimization

Learning Goals: I am learning to…

- ☐ Investigate the properties of 3D optimization and determine the dimensions of an object that would produce the maximum volume and minimum surface area

Optimization is the process of finding the most efficient use of available materials with given constraints. (i.e. finding the maximum or minimum for a specific variable)

For 2D optimization, we concluded that to optimize both perimeter and area, a **square** produces the optimal dimensions. What about 3D objects? How could we extend this?

1. For a rectangular prism, with a given surface area, a _________ has the **maximum volume.**
2. For a rectangular prism, with a given volume, a _________ has the **minimum surface area.**

Volume:

Surface Area:

Example 1: What dimensions produce a minimum surface area of a rectangular prism with a volume of
1000 cm^3

Example 2: What dimensions of a rectangular prism will produce a maximum volume if the surface area is
486 cm^2

3D Optimization with Restrictions

Similar to 2D optimization, there may certain situations where it may not be possible to form a cube because of certain restrictions, such as:

- The dimensions need to be whole numbers or multiples of other numbers
- Sometimes more surfaces are missing or are blocked by other barriers

Example 3: Jeff is designing a glass candle holder. It will be a square-based rectangular prism with an outer surface area of, 225 cm^2 and no top. Determine the **maximum volume** of the candle holder to the nearest cm^3. What are the dimensions of the candle holder?

Base Length (cm)	Height (cm)	Volume (m^3)	Surface Area (cm^2)
1			225
2			225
3			225
4			225
5			225
6			225
7			225
8			225
9			225
10			225

2.7 3D Optimization Homework

1. Yasmin is constructing a rectangular prism using exactly 96 cm^2 of cardboard. The prism will have the greatest possible volume.

a) Describe the prism. What will be its dimensions?

b) What will be its volume?

2. Mathew is constructing a rectangular prism with volume exactly 729 cubic inches. It will have the least possible surface area.

a) Describe the prism. What will be its dimensions?

b) What will be its surface area?

3. The dimensions of two rectangular prisms with volume 240 cm^3 are given. Sketch each prism and predict which will have less surface area. Check your prediction.

a) 10 cm by 6 cm by 4 cm

b) 12 cm by 10 cm by 2 cm

4. Krikor has to design and build a box with the greatest volume possible. The box is a rectangular prism. For each surface area, what will be the dimensions of the box?

a) 600 square inches b) 1350 cm^2 c) 2400 square inches

5. Tanya is designing a storage box. It will be a rectangular prism with the least possible surface area. For each volume, what will be the dimensions of the box?

a) 1 m^3 b) 125 000 cm^3 c) 8 cubic feet

8. An electrical transformer box is a rectangular prism constructed from sheet metal. It must have volume at least 274 625 cm^3 to hold all the necessary equipment.

a) What dimensions for the box require the least area of sheet metal?

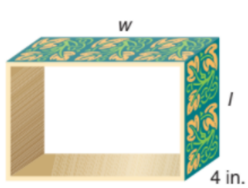
b) What area of sheet metal is needed to build the box?

c) Tony has 20 square feet of sheet metal. Will this be enough to construct the box? Justify your answer.

9. Tori is designing a hanging shelf. It has volume 400 cubic inches and depth 4 inches. She will paint a design that will cover the four outside faces.

a) Determine the minimum area she will paint.

b) What are the dimensions of the shelf with the minimum area to paint?



12. Filip is designing a can for a new vegetable product. The can should hold 750 mL of vegetables. To reduce waste, he wants the surface area of the can to be as small as possible.

a) What dimensions should Filip use?

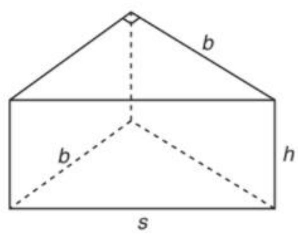
b) What will the surface area be?

13. A cylindrical storage tank holds 1800 cubic feet of gasoline. Determine the minimum amount of material needed to build this tank.

14. Look at the dimensions of the optimal cylinders in *Example 3* and questions 12 and 13. How do the diameter and height appear to be related?

15. **Assessment Focus** A beverage company is investigating containers that can hold 512 mL of juice. They are debating whether to use a rectangular prism or a cylinder. Which object would require less material? Justify your answer.

16. Courtney is designing a gift box. It will be a triangular prism with surface area 220 cm^2. She decides the box should have a right isosceles triangular base to make it easy to package.



a) Determine the length of the hypotenuse of the base, *s*.

b) Determine the maximum possible volume of the box.

c) What are the dimensions of the box with the maximum volume?

Answers:

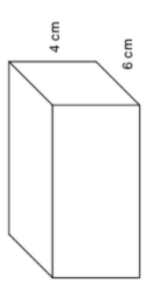
1. a) The rectangular prism is a cube with edge length 4 cm.

b) $V = 64$ cm^3

2. a) The rectangular prism is a cube with edge length 9 in.

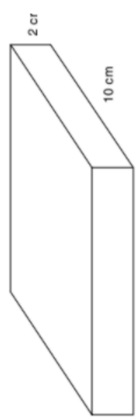
b) $SA = 486$ sq. in.

3. a)



$SA = 248$ cm^2

b)



$SA = 328$ cm^2

Prism a has the least surface area.

4. a) 10 in. by 10 in. by 10 in.

b) 15 cm by 15 cm by 15 cm

c) 20 in. by 20 in. by 20 in.

5. a) 1 m by 1 m by 1 m

b) 50 cm by 50 cm by 50 cm

c) 2 ft. by 2 ft. by 2 ft.

8. a) A cube with edge length 65 cm

b) 25 350 cm^2

c) No

9. a) 160 sq. in. **b)** 10 in. by 10 in. by 4 in.

12. a) The dimensions of the cylinder should be $r \doteq 4.9$ cm and $h \doteq 9.94$ cm.

b) $SA \doteq 456.98$ cm^2

13. About 819.3 sq. ft.

15. Cylinder

16. a) $s = b\sqrt{2}$

b) $V \doteq 184.9$ cm^3

c) $b = 8.6$ cm, $h = 5.0$ cm