Unit 1: Trigonometry

Learning Goals: We are learning to…

1. Identify and label the opposite, adjacent and hypotenuse sides in a right triangle
2. Determine the measure of an unknown side and angle in a right triangle
3. Apply right angle trigonometry to real-world application based problems including angles of elevation and depression
4. Investigate how the ratio of sine, cosine and tangent varies for different types of angles, specifically obtuse angles
5. Apply knowledge of quadrants to determine the quadrant a trig ratio lies in based on the size of the angle
6. Determine the angle of the acute and supplementary obtuse angle given the ratio
7. Determine the measure of an unknown side and/or angle using the Sine and Cosine Law
8. Apply the Sine and Cosine Law to real-world application based problems

Day	Topic	Expectations, Learning Goals	Practice/Homework
1	1.1 Trigonometry in Right Triangles	LG 1, 2 C3.1	Handout
2	1.2 Applications of Right Angle Trigonometry	LG 3 C3.1	Handout Extra in textbook: p.9 #8, p.10 #9, 10, 11
3	1.3 Obtuse Angles Investigation	LG 4 C3.2	Textbook: p.19 #2-6
4	1.4 Sine, Cosine and Tangent of Obtuse Angles	LG 5, 6 C3.3	Textbook: p.23 #2, 3abc, 4ab, 5ab, 6ab, 8, 10abc, 13, 14
5	1.5 The Sine Law **Quiz (1.1-1.3)**	LG 7 C3.4	Textbook: p.31 #3, 5, 8, 9, 10, 15, 17
6	1.6 The Cosine Law	LG 7 C3.4	Textbook: p.38 #3. 4, 6, 8a, 9, 11, 13
7	1.7 Applications of the Sine and Cosine Law	LG 8 C3.5	Textbook: p.38 #3, 6, 7, 11, 12, 14, 16 Extra practice on handouts
8	Review		See review package
9	**Unit 1 Test** **Date:**________________________		

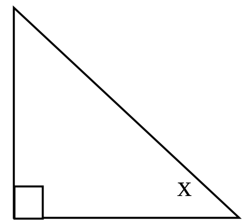
Subject to change based on school activities and class needs

1.1 Trigonometry in Right Triangles

Learning Goals: I am learning to…
- ☐ Identify and label the opposite, adjacent and hypotenuse sides in a right triangle
- ☐ Use the primary trigonometric ratios to determine an unknown side and/or angle in a right triangle
- ☐ Identify which ratio to used based on the given information

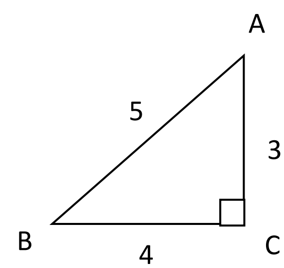
The **primary trigonometric ratios** are used to solve for any missing side or angle in a right triangle.

SOH	CAH	TOA
$\sin\theta = \dfrac{opp}{hyp}$	$\cos\theta = \dfrac{adj}{hyp}$	$\tan\theta = \dfrac{opp}{adj}$



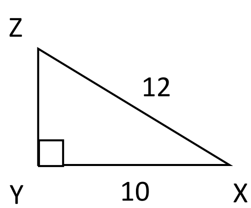
1. Always label the sides relative to the angle of interest (i.e opp, adj, hyp)
2. Decide which ratio (sin/cos/tan) you need to use based on what you have been given/need to find (use a process of elimination)
3. Set up the ratio with your known sides/angles
4. Solve for the unknown side or angle

Example 1: Determine the ratios of sin A, cos A, and tan A.



Example 2: Solve for ∠ X in each.

a)



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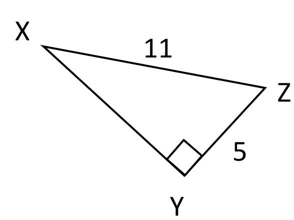
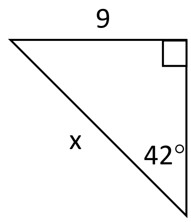


Example 3: Solve for each missing side.

a)



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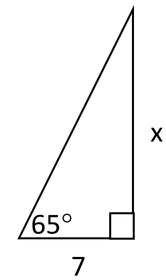
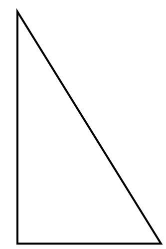
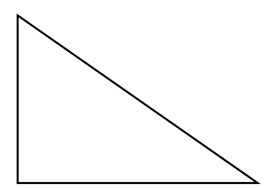


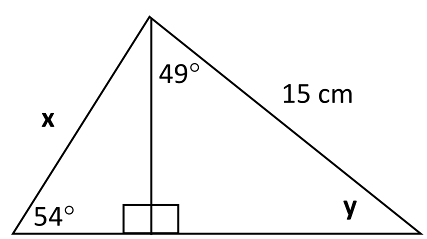
Example 4: A 9.5 m ladder leans against a vertical wall. If the foot of the ladder is 2 m from the base of the wall, what angle does the ladder make with the ground?



Example 5: A 200 m cable attached to the top of an antenna makes an angle of 37° with the ground. How tall is the antenna?



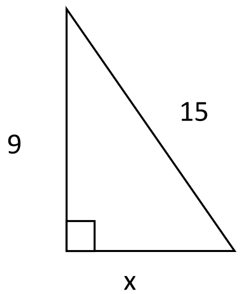
Example 6: Solve for x and y.



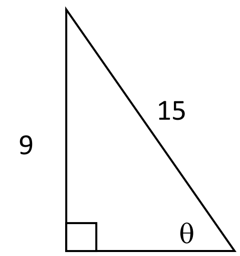
1.1 SOH CAH TOA – Practice

For each triangle below, determine the unknown side or angle.

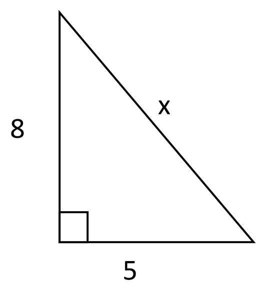
1.



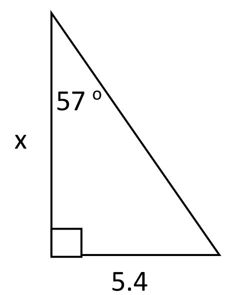
2.



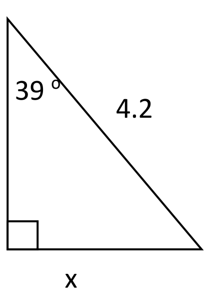
3.



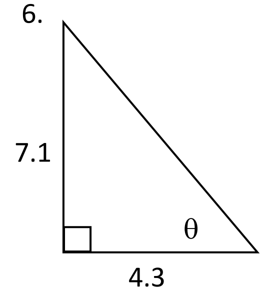
4.



5.



6.



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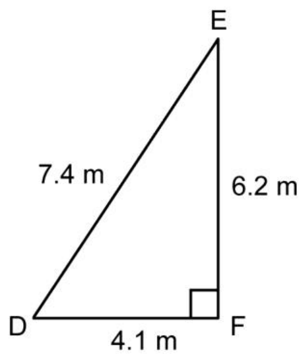
BLM 8–1

SOH CAH TOA

Primary Trigonometric Ratios

1. a) Find the primary trigonometric ratios for ∠D in △DEF.

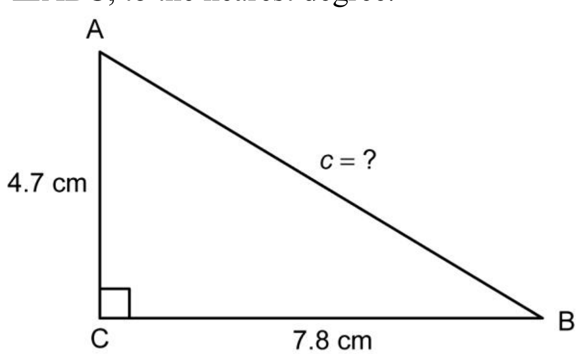
b) Find the measures of the acute angles in △DEF, to the nearest degree.



2. a) Find the length of side *c* in △ABC, to the nearest tenth of a centimetre, by applying the Pythagorean theorem.

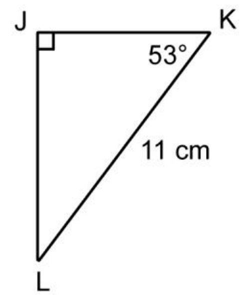
b) Find the primary trigonometric ratios for ∠B in △ABC.

c) Find the measures of the acute angles in △ABC, to the nearest degree.



Solve Triangles

3. Solve △JKL. Round side lengths to the nearest tenth of a centimetre.



4. In △PQR, ∠Q = 90°, p = 3.5 cm, and r = 4.8 cm.

a) Draw and label △PQR.

b) Solve △PQR. Round the side length to the nearest tenth of a centimetre and the angles to the nearest degree.

Angles of Elevation and Depression

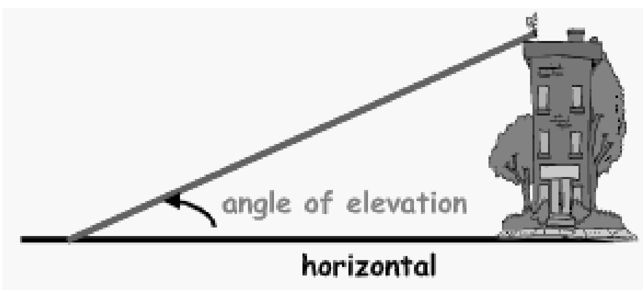
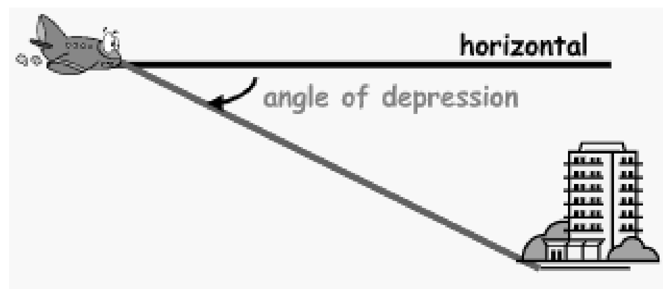
5. From a point 35 m from the base of a building, the angle of elevation of the top of the building is 32°. Apply the tangent ratio to find the height of the building, to the nearest tenth of a metre.

6. From the top of a cliff that is 50 m high, at the edge of a lake, a sailboat is spotted at an angle of depression of 23°. How far from the base of the cliff is the sailboat, to the nearest metre?

1.2 Applications of Right Triangles

Learning Goals: I am learning to…
- ☐ Apply right angle trigonometry to solve real-world application problems
- ☐ Set up diagrams and solve problems involving the angles of elevation and depression

Angles of Elevation and Depression

Angle of Elevation	Angle of Depression
The **Angle of Elevation** is the angle measured from the horizontal looking ______.	The **Angle of Depression** is the angle measured from the horizontal looking ________.
	
$\boldsymbol{Angle\ of\ elevation = Angle\ of\ Depression}$ Recall: Alternating angles (the z pattern)	

Do not forget the following when solving an application problem:
- Always draw a well-labeled diagram, especially if one has not bee included in the initial problem
- If you are introducing a new variable (unknown) define this using a let statement
- Label everything you know as well as everything you want to determine
- Remember to include units and a therefore statement at the end of your solution

Example 1: A plane is coming into land at Pearson airport. The angle of depression is 22° and the plane is 350m from the ground. Determine the distance from the plane to the airport.

Example 2: A weather balloon is tethered to a rope that is 600m long.
a) What is the angle of elevation of the balloon, when it is 400m high?
b) What do you have to assume about the rope in order to answer this question?

Example 3: An observer in a 55m tall lighthouse spots a ship in distress 68 m from shore. What is the angle of depression from the lighthouse to the ship?

Example 4: A wheelchair ramp must have an angle of elevation of no more than 12°. A wooden ramp is available that is 8.0m long and 1.5 m high. Will this ramp meet the requirements?

Example 5: A carpenter leans a ladder against a wall at an angle of 68°. The distance from the foot of the ladder to the wall is 36 inches.

- **a)** How long is the ladder in inches?
- **b)** How high up the wall does the ladder reach in inches?

1.2 Angles of Elevation and Depression Practice

Round all lengths to 1 decimal place and all angles to the nearest degree!

1. Suppose a tree 50 feet in height casts a shadow of length 60 feet. What is the angle of elevation from the end of the shadow to the top of the tree with respect to the ground?

2. John knows the tree is 9.5 m tall. He walks exactly 10 m from the base of the tree and looks up. Determine the angle of elevation from the ground to the top of the tree.

3. A building is 50 feet high. An observer is 21 feet away. What is the angle of elevation from the observer to the top of the building?

4. An airplane is flying at a height 2 miles above the ground. The distance along the ground from the airplane to the airport is 5 miles. What is the angle of depression from the airplane to the airport?

5. A bird sits on top of a lamppost. The lamppost is 8 m high. The distance from the bird to an observer is 25 m. What is the angle of depression from the bird to the feet of an observer standing away from the lamppost?

6. The wires supporting a flagpole are anchored 7 m from the base of the flagpole at an angle of 52° with the ground. What is the total length of wire needed to support this flagpole?

7. Montreal's Marathon building is 195 m tall. From a point 48 m away from the base of the building, what is the angle of elevation to the top of the building?

Extra practice from textbook: p.9 #8, p.10 #9, 10, 11

1.3 Obtuse Angles Investigation

Learning Goals: I am learning to…

- ☐ Make connections between the primary trigonometric ratios of obtuse angles and their supplementary angles

Before we can complete the investigation, it is important to understand some of the different types of angles.

Terminology	Definition	Diagram
Acute Angle		
Right Angle		
Obtuse Angle		
Supplementary Angle		

Part A: Evaluating Angles

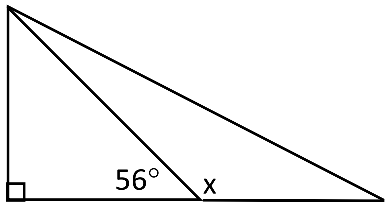
Using a scientific calculator, determine the ratio of each angle below. Round all answers to four decimal places.

Angle	Sine	Cosine	Tangent
20°			
41°			
88°			
92°			
139°			
160°			

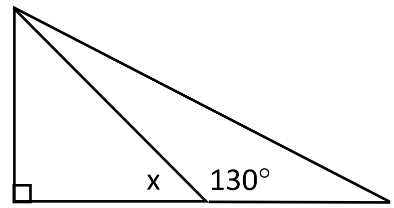
Part B: Investigating Obtuse and Supplementary Angles

1) Determine the measure of the unknown angle, x in each of the triangles below.

a)



b)



2) Complete the chart for each set of supplementary angles (acute and obtuse). Round each ratio to four decimals.

a)

Trig Ratio	Acute Angle (56°)	Obtuse Angle (x =)
Sine		
Cosine		
Tangent		

b)

Trig Ratio	Acute Angle (56°)	Obtuse Angle (x =)
Sine		
Cosine		
Tangent		

3) What do you notice about each ratio in the charts above?

4) Looking back at the chart in part A on the previous page, which pairs of angles are supplementary?

5) In general, what did you notice was the same and what was different about the trigonometric ratios of the supplementary angles?

Summary of Key Concepts

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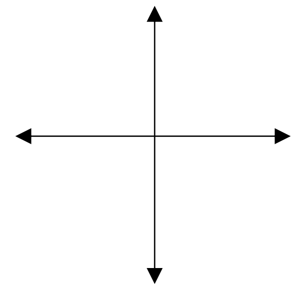
1.4 Sine, Cosine and Tangent of Obtuse Angles

Learning Goals: I am learning to…

- ☐ Identify the sign of the three primary trig ratios in quadrant I and II
- ☐ Determine the properties of supplementary angles given the trig ratio

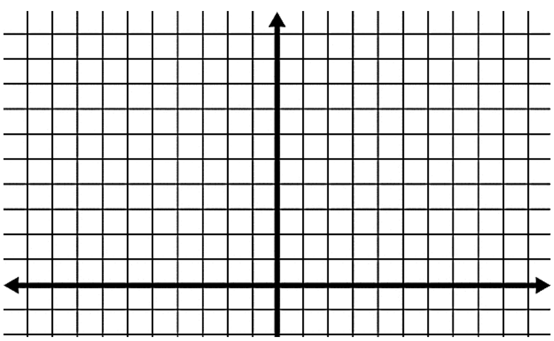
In lesson 1.3, we concluded that by knowing what quadrant an angle is in, we can determine the sign of the trigonometric ratio.



Recall: The Cartesian plane is divided into four quadrants, numbered in a counter-clockwise direction using Roman numerals. Starting at 0° on the x-axis, any oblique triangle can be defined, since the angles must be less than 180°.

Defining Trigonometric Ratios

Any trigonometric ratio can be defined given a point, $P(x, y)$ on a Cartesian plane, by making a right triangle with the x-axis. The hypotenuse can also be found by using Pythagorean Theorem.



General case: In triangle PBA,

$\mathbf{sin}\, \boldsymbol{A} =$

$\mathbf{cos}\, \boldsymbol{A} =$

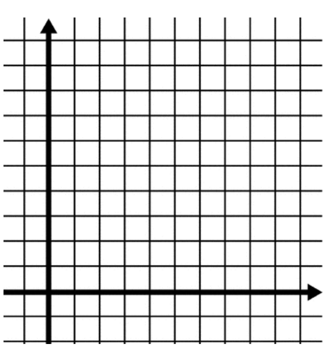
$\mathbf{tan}\, \boldsymbol{A} =$

In general, for point, $P(x, y)$.

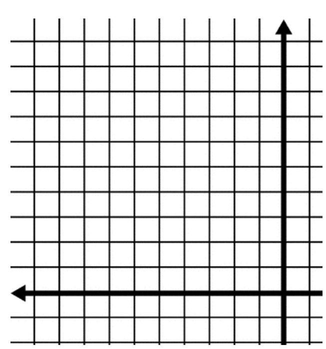
Trigonometric Ratio	Quadrant I	Quadrant II
Sine		
Cosine		
Tangent		

Example 1: Determine the trigonometric ratios for angles A and B, given point P.

a) $P(3,4)$



b) $P(-3,4)$



Properties of Supplementary Angles

By knowing what quadrant an angle lies in, we can derive a general rule for the properties of an obtuse angle given the acute angle A.

- $\sin A =$
- $\cos A =$
- $\tan A =$

Example 2: Determine the measure of each **supplementary obtuse angle**.

a) The sine of acute $< P$ is 0.65

Method 1	Method 2
Determine the measure of the acute angle first	Determine the measure of the obtuse angle based on the supplementary angle property

b) The cosine of acute $< R$ is 0.22

c) The tangent of acute $< S$ is 0.44

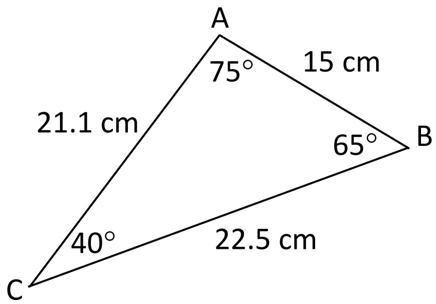
1.5 The Sine Law

Learning Goals: I am learning to…

- ☐ Determine the measure of an unknown angle and/or side using the Sine Law
- ☐ Understand and explain when to use the Sine Law

The three primary trigonometric ratios (SOH CAH TOA) can only be used for right angle triangles, however, the Sine Law can be used for any type of triangle to find an unknown side or angle.

Investigation: Given the triangle below with all angles and side lengths known, find each ratio in the table below. Round each ratio to **4 decimal places**.

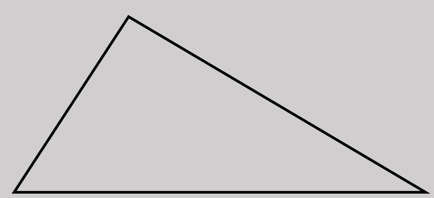


$\dfrac{\sin A}{a} =$	$\dfrac{\sin B}{b} =$	$\dfrac{\sin C}{c} =$
$\dfrac{a}{\sin A} =$	$\dfrac{b}{\sin B} =$	$\dfrac{c}{\sin C} =$

What do you notice about the ratios in the table above?

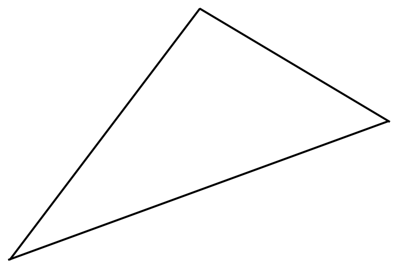
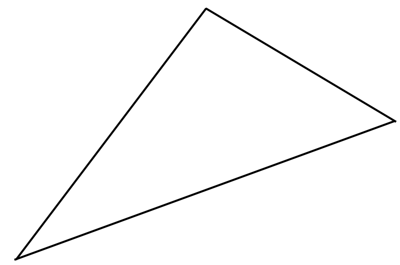
Sine Law:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \textbf{OR} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

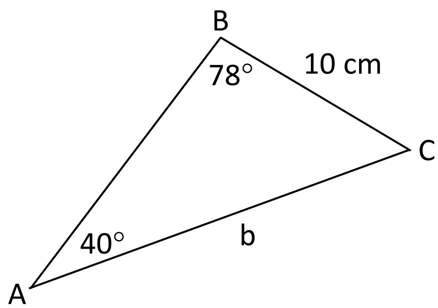


- Note: An angle is labeled with a capital letter while a side is labeled with a lower case letter. Angles and sides are labeled opposite each other.
- The equation has three parts, but only two parts are used at the same time
- To find an unknown, you need one complete pair (side and opposite angle) plus one extra side or angle.

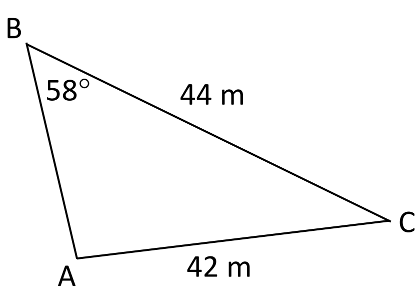
The Sine Law can be used for any type of triangle, but it depends on what you are given as to whether you can use it or not. The Sine Law works with pairs made up of an angle and its opposite side. We must know at least one full pair (side and opposite angle) in order to use the Sine Law. It is always important to check before applying the Sine Law.

Finding an Unknown Angle	Finding an Unknown Side
If we know 2 sides and 1 angle	**If we know 2 angles and 1 side**
	

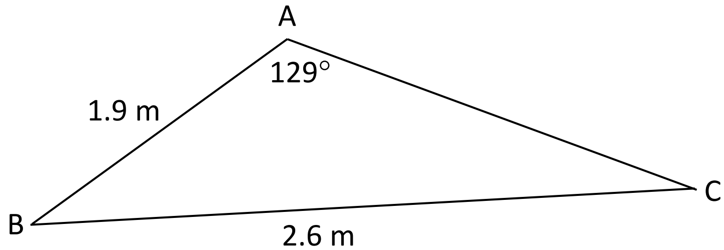
Example 1: Use the Sine Law to find the unknown side length, b.



Example 2: Use the Sine Law to find the unknown angle, A.



Example 3: Determine the measure of angle B.

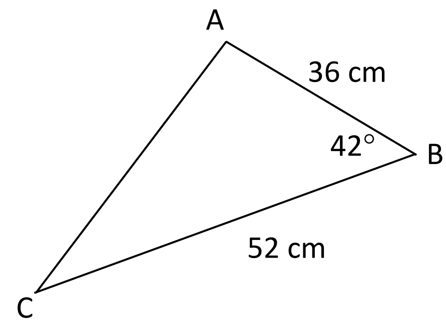


1.6 The Cosine Law

Learning Goals: I am learning to…

- ☐ Explain and identify when to use the Cosine Law
- ☐ Solve for an unknown side and/or angle using the Cosine Law

Warm-up: Given the triangle below, you are asked to find side b.



1) Why can't you use the Pythagorean theorem?

2) Why can't you use SOH CAH TOA?

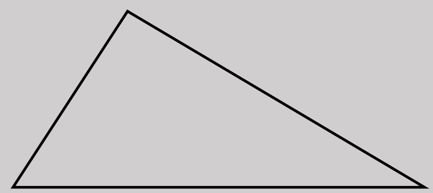
3) Why can't you use the Sine Law?

Cosine Law:

$$c^2 = a^2 + b^2 - 2ab(\cos C) \qquad \cos C = \frac{a^2+b^2-c^2}{2ab}$$

To use cosine law you need to know either:

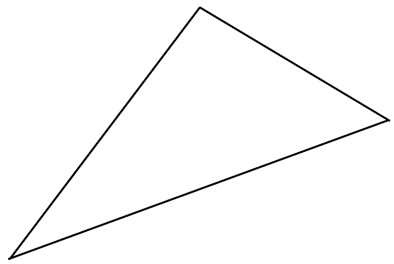
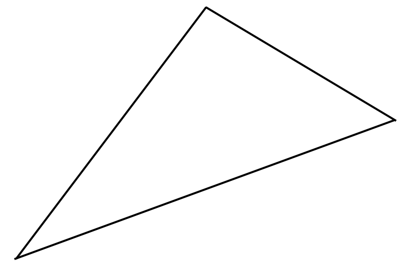
1) All three side lengths (SSS)
2) Two sides and the contained angle (SAS)



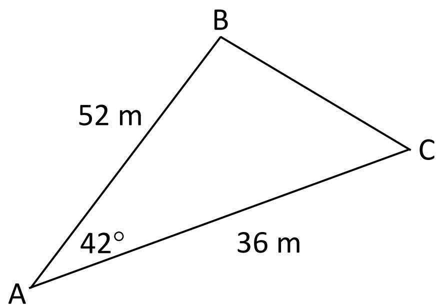
Try rewriting the Cosine Law for all other possible unknown sides and angles. Are there any patterns you notice?

Unknown Side	Unknown Angle
$c^2 = a^2 + b^2 - 2ab(\cos C)$	$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
Side a?	Angle A?
Side b?	Angle B?

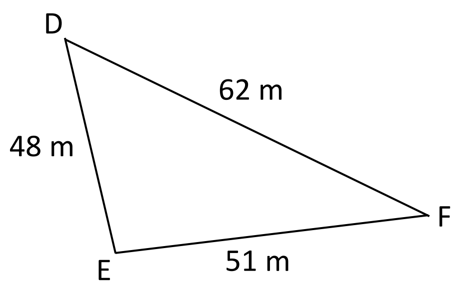
As with the Sine Law, there are conditions on when to use the Cosine Law.

Finding an Unknown Angle	Finding an Unknown Side
	

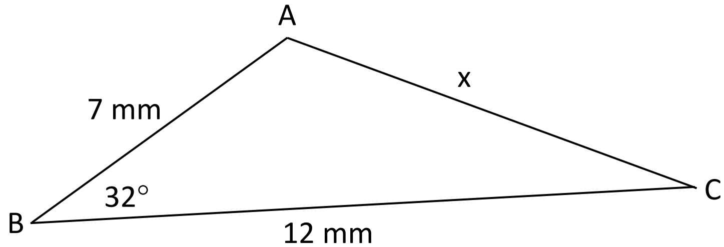
Example 1: Use the Cosine Law to find the unknown side length, a.



Example 2: Use the Cosine Law to find the unknown angle, D.



Example 3: Determine the measure of side, x.



1.7 Applications of the Sine and Cosine Law

Learning Goals: I am learning to…
- ☐ Solve application based problems involving the Sine and Cosine Law
- ☐ Apply a variety of trigonometry solving techniques to real-world problems
- ☐ Identify when to use different trigonometry skills

Primary Trig Ratios:

$\sin\theta = \frac{opp}{hyp}$ (SOH)

$\cos\theta = \frac{adj}{hyp}$ (CAH)

$\tan\theta = \frac{opp}{adj}$ (TOA)

Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

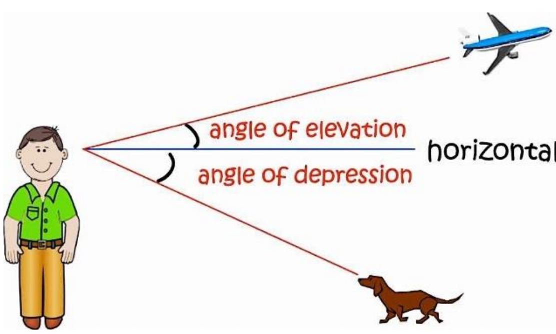
*c is always the longest side (hypotenuse)

Sum of Interior Angles:

$$< A + < B + < C = 180°$$

Angle of Elevation (Inclination):
The angle formed by the horizontal and the line of sight to an object **above** the horizontal.

Angle of Depression:
The angle between the horizontal line and the line of sight to an object **below** the horizontal.



Sine Law:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Law:

$$c^2 = a^2 + b^2 - 2ab\ \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Trigonometric application problems can be solved by applying a variety of different skills and problem solving techniques.

1. Sketch a well-labeled diagram to model the situation (if appropriate).
2. Apply the appropriate trig strategy or strategies to solve.
   - Pythagorean Theorem $a^2 + b^2 = c^2$
   - Sum of angles is equal to 180°
   - Primary trig rations – (SOH CAH TOA)
   - Sine or cosine law
3. Conclude the problem and explain what the solution means, including units!

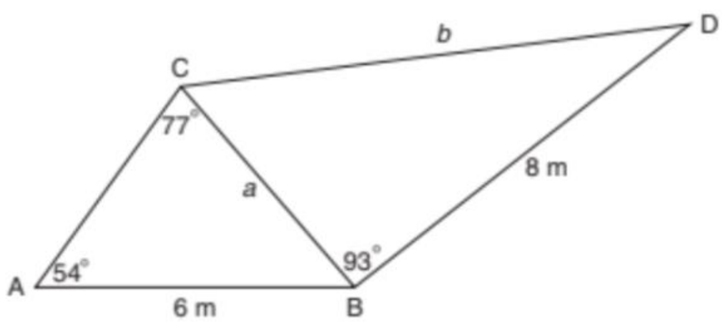
Example 1: Lynn and Fred standing 2000 m apart, spotted a hot air balloon at angles of elevation of 50° and 70° respectively. The hot air balloon is located between them. What is the distance from Fred directly to the hot air balloon? Show all your work.

Law used:

Example 2: A triangular piece of land is surrounded by 32 m of brick wall, 50 m of fencing, and 28 m of road frontage. What angle does the fence make with the road? Show all your work!

Law used:

Example 3: Determine side lengths, a and b. Round to one decimal place.



Applications of Sine and Cosine Law – Practice

1. A movie screen is 12 m wide. Tanya's seat is 16 m from one end of the screen and 14 m from the other end. John's seat is 13 m from one end of the screen and 15 m from the other. Who has the greater angle of view to the screen? (Hint: You need two separate triangles) **[John]**

2. Mira and Raz went out in two separate boats to place markers for a boat race. Their paths formed an angle of 85°. Mira rowed 85 m and Raz rowed 102 m to place their markers. How far apart are their markers? **[127.0 m]**

3. Allison is flying a kit and has let out the entire 150 m ball of kite string. She notices that the string forms an angle of 70° with the ground. Marc is on the other side of the kite and sights the kite at an angle of elevation of 30°. How far is Marc from Allison? **[295.4 m]**

4. At an evacuation site, points A, B and C have been marked. The measure of angle C is 132°. Points A and C are 4.2 m apart. Points B and C are 5.7 m apart. How far apart are points A and B? **[9.1 m]**

5. A hot air balloon is being observed. Two tracking stations are 20 km apart from each other. From station A, the angle of elevation is 41° and from station B it is 75°. What is the distance between the balloon and station B? **[14.6 km]**

6. A pendulum on a clock is 94.5 cm long. When the pendulum swings from one side to the other, the horizontal separation is 15.3 cm. Determine the angle through which the pendulum swings. **[9.3°]**

Sine and Cosine Law – Mixed Practice

1. From a point C, both ends A and B of a proposed railroad tunnel are visible. If AC=165m, BC=115m, and C=74°, find the length of the tunnel.

2. A triangular lot has sides 215m, 185m, and 125m.
   a. Find the measure of the smallest angle.
   b. Find the area of the lot.

3. The distances from a boat to two seagulls on the shore are 100m and 80m respectively. If the angle between the two lines of sight is 55°, how far would one seagull have to walk to meet the other seagull?

4. Two ships leave port at the same time and sail on straight paths making an angle of 60° with each other. How far apart are the ships at the end of 1 hour if the speed of one ship is 25km/h and that of the other is 15 km/h?

5. The sides of a triangle have lengths 10cm, 9cm, and 3cm.
   a. Find the largest angle.
   b. Find the area of the triangle.

6. An engineering student was given an assignment to construct a triangular model of the three steel girders. Two of the girders measured 20cm and 15cm, and the angle opposite the 15cm girder was 21°. How long did the third girder have to be?

7. A ship at sea is 70 miles from one radio transmitter and 130 miles from another. The measurement of the angle between the signals is 130°. How far apart are the transmitters?

8. A building 60ft tall is on top of a hill. A surveyor is at a point on the hill and observes that the angle of elevation to the top of the building measures 42° and to the bottom of the building is 18°. How far is the surveyor from the building?

9. Ana Galo needs to draw a triangle for her geometry class. She makes one side 40mm long and another side 32mm long with opposite angle measuring 48°. What would be the length of the third side?

10. A triangular piece of land has boundaries
    70 ft. and 85 ft. If the angle between these boundaries is 36°, what is the area of the land?

11. Two angles of a triangle measure 38° and 87°. If the longest side is 24cm, find the length of the shortest side to the nearest tenth.

12. Two motorists start at the same point and travel in two straight courses. The courses diverge by 75°. If one is traveling at 55mph and the other is traveling at 60mph, how far apart will they be after 2 hours?

13. The lengths of the boundaries of a triangular plot of land are 80ft, 100ft, and 120 ft. Find the area of the land.

14. Two ranger stations are 100miles apart. A forest fire is spotted. The angle measured at Station 1 between Station 2 and the fire is 61°. The angle measured at Station 2 between Station 1 and the fire is 48°. How far is Station 1 from the fire?

15. Two trains leave from the same point and travel along straight tracks that differ in direction by 65°. If their speeds are 100mph and 80mph respectively, approximate how far apart they will be after 30 minutes.

16. The angles of elevation of a balloon from the two points on level ground are 24° and 47° respectively. If the points are 8.4 miles apart and the balloon is between the points, in the same vertical plane, approximate, to the nearest tenth of a mile, the height of the balloon above the ground.

17. A triangular plot of land has two sides with lengths of 100m and 150m that intersect at an angle of 38°.
    a. Find the third.
    b. Find the area of the land.