Probability - Basic Principles

A. First of all, what is probability?
Probability is the branch of mathematics that attempts to *predict* answers to questions about the likelihood or chances of an event occurring.

An **event** is a collection of outcomes satisfying a particular condition. The probability of an event can range from **0 (impossible) to 1 (certain)** - probabilities less than 0 or greater than 1 have no meaning**.**

The probability of an event, ***E***, is denoted as **P(*E*).** It is a measure of the *likelihood* that the event will occur. Probability can be expressed as a fraction, decimal, or percent – *i.e.* ½ or 0.5 or 50%

There are three basic types of probability:

- **Empirical or Experimental Probability**

Ex. 1: Calculate the *experimental probability* of rolling a “1” on a six-sided die.
(Roll the die 6 times [the number of *trials* in your experiment] and record the outcome for each roll.)

**Experimental probability approaches theoretical probability as the #of trials approaches _____________.

- **Theoretical Probability**

The **theoretical probability** of an event, *E*, is given by: $\boldsymbol{P(E)} = \frac{\boldsymbol{n(E)}}{\boldsymbol{n(S)}}$

where $0 \le P(E) \le 1$

Ex. 2: Calculate the *theoretical probability* of rolling a “1” on a six-sided die.

**The sum of the probabilities of all possible events = ______

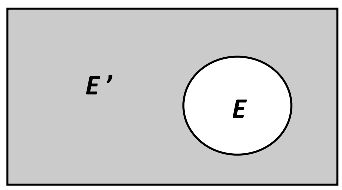
- **Subjective Probability**

Ex. 3: *Estimate* the probability that the next pair of shoes you buy will be the same size as the last pair you bought. P(*same size*) = _________

B. The Complement of an Event

In probability, the **complement** of an event *E* is written as *E'* (pronounced *E-prime* or *not-E*). *E'* is the event where "event *E* does *not* occur". Thus, whichever outcomes make up event *E*, all the *other* outcomes make up *E'*. Because *E* and *E'* together make up all possible outcomes, *E* + *E'* = 1.

The probability of the **complement** of an event, *E'*, is given by: $\boldsymbol{P(E') = 1 - P(E)}$



Ex. 4: What is the probability that a randomly drawn integer between 1 and 40 is *not* a perfect square?

Ex. 5: Determine the probability of rolling a sum of 7 with a pair of dice.

Ex. 6: Use a tree diagram to illustrate flipping a coin 3 times.
a) Determine the probability of getting HHH ∴ *P*(HHH) = _________
b) Determine the probability of getting at least one T ∴ *P*(at least one T) =________

CW: p. 312-313 #1abcef, 2, 4, 5 (each team has a 50% chance of winning any individual game), 6, 7

Practise

A

1. Determine the probability of
   a) tossing heads with a single coin
   b) tossing two heads with two coins
   c) tossing at least one head with three coins
   d) rolling a composite number with one die
   e) not rolling a perfect square with two dice
   f) drawing a face card from a standard deck of cards

2. Estimate a subjective probability of each of the following events. Provide a rationale for each estimate.
   a) the sun rising tomorrow
   b) it never raining again
   c) your passing this course
   d) your getting the next job you apply for

3. Recall the sum/product game at the beginning of this section. Suppose that the game were altered so that the slips of paper showed the numbers 2, 3, and 4, instead of 1, 2, and 3.
   a) Identify all the outcomes that will produce each of the three possible events
      i) $p > s$ ii) $p < s$ iii) $p = s$
   b) Which player has the advantage in this situation?

Apply, Solve, Communicate

4. The town planning department surveyed residents of a town about home ownership. The table shows the results of the survey.

Residents	At Address Less Than 2 Years	At Address More Than 2 Years	Total for Category
Owners	2000	8000	10 000
Renters	4500	1500	6 000
Total	6500	9500	16 000

Determine the following probabilities.
   a) *P*(resident owns home)
   b) *P*(resident rents and has lived at present address less than two years)
   c) *P*(homeowner has lived at present address more than two years)

B

5. **Application** Suppose your school's basketball team is playing a four-game series against another school. So far this season, each team has won three of the six games in which they faced each other.
   a) Draw a tree diagram to illustrate all possible outcomes of the series.
   b) Use your tree diagram to determine the probability of your school winning exactly two games.
   c) What is the probability of your school sweeping the series (winning all four games)?
   d) Discuss any assumptions you made in the calculations in parts b) and c).

6. **Application** Suppose that a graphing calculator is programmed to generate a random natural number between 1 and 10 inclusive. What is the probability that the number will be prime?

7. **Communication**
   a) A game involves rolling two dice. Player A wins if the throw totals 5, 7, or 9. Player B wins if any other total is thrown. Which player has the advantage? Explain.
   b) Suppose the game is changed so that Player A wins if 5, 7, or doubles (both dice showing the same number) are thrown. Who has the advantage now? Explain.
   c) Design a similar game in which each player has an equal chance of winning.

8. **a)** Based on the randomly tagged sample, what is the empirical probability that a deer captured at random will be a doe?

b) If ten deer are captured at random, how many would you expect to be bucks?

C

9. Inquiry/Problem Solving Refer to the prime number experiment in question 6. What happens to the probability if you change the upper limit of the sample space? Use a graphing calculator or appropriate computer software to investigate this problem. Let A be the event that the random natural number will be a prime number. Let the random number be between 1 and n inclusive. Predict what you think will happen to $P(A)$ as n increases. Investigate $P(A)$ as a function of n, and reflect on your hypothesis. Did you observe what you expected? Why or why not?

10. Suppose that the Toronto Blue Jays face the New York Yankees in the division final. In this best-of-five series, the winner is the first team to win three games. The games are played in Toronto and in New York, with Toronto hosting the first, second, and if needed, fifth games. The consensus among experts is that Toronto has a 65% chance of winning at home and a 40% chance of winning in New York.

a) Construct a tree diagram to illustrate all the possible outcomes.

b) What is the chance of Toronto winning in three straight games?

c) For each outcome, add to your tree diagram the probability of that outcome.

d) Communication Explain how you found your answers to parts b) and c).

11. Communication Prior to a municipal election, a public-opinion poll determined that the probability of each of the four candidates winning was as follows:

Jonsson 10%

Trimble 32%

Yakamoto 21%

Audette 37%

a) How will these probabilities change if Jonsson withdraws from the race after ballots are cast?

b) How will these probabilities change if Jonsson withdraws from the race before ballots are cast?

c) Explain why your answers to a) and b) are different.

12. Inquiry/Problem Solving It is known from studying past tests that the correct answers to a certain university professor's multiple-choice tests exhibit the following pattern.

Correct Answer	Percent of Questions
A	15%
B	25%
C	30%
D	15%
E	15%

a) Devise a strategy for guessing that would maximize a student's chances for success, assuming that the student has no idea of the correct answers. Explain your method.

b) Suppose that the study of past tests revealed that the correct answer choice for any given question was the same as that of the immediately preceding question only 10% of the time. How would you use this information to adjust your strategy in part a)? Explain your reasoning.

Probabilities Using Counting Techniques

In many situations, possible outcomes are not easy or convenient to count individually. In these cases, the counting techniques involving permutations and combinations can be helpful for calculating theoretical probabilities. Yay!

Recall: $P(E) = \frac{n(E)}{n(S)}$, $0 \le P(E) \le 1$, $1 - P(E) = P(E')$

A. Using Permutations

Recall that when using permutations, **order matters** – whether it is time order or spatial order.

Ex. 1: Alphonse and Beauregard enter a race with 5 friends. The racers draw straws to determine their starting positions. What is the probability that Alphonse will start in lane 1 and Beauregard will start in lane 2?

Ex. 2: A bag contains 26 tiles, each marked with a different letter of the alphabet. What is the probability that a student will take out 4 consecutive letters (without replacement) and have them spell out the word M-A-T-H?

B. Using Combinations

When using combinations, **order is irrelevant** – we are selecting or choosing or "grabbing" some items!

Ex. 3: Suppose you randomly draw 2 marbles, without replacement, from a bag containing six green, four red, and three blue marbles.

a) Determine the probability that both marbles are red.

b) Determine the probability that you pick at least one green marble.

Ex. 4: A focus group of 3 members is to be randomly selected from a medical team consisting of 5 doctors and 7 technicians.

a) What is the probability that the focus group will be comprised entirely of doctors?

b) What is the probability that the focus group will *not* be comprised entirely of doctors?

c) What is the probability that the group will be made up of 1 doctor and 2 technicians?

Ex. 5: In a game of poker, each player is dealt a 5-card hand from a deck of 52 shuffled playing cards.

a) Find the probability of receiving a hand containing a spade flush (all 5 cards are spades).

b) Find the probability of receiving a hand containing a full house of 3 kings and 2 fives.

CW: p. 324-325 #3, 5, 6, 7a, 8, 9, 11

Key Concepts

- In probability experiments with many possible outcomes, you can apply the fundamental counting principle and techniques using permutations and combinations.
- Permutations are useful when order is important in the outcomes; combinations are useful when order is not important.

Communicate Your Understanding

1. In the game of bridge, each player is dealt 13 cards out of the deck of 52. Explain how you would determine the probability of a player receiving
   a) all hearts
   b) all hearts in ascending order
2. a) When should you apply permutations in solving probability problems, and when should you apply combinations?
   b) Provide an example of a situation where you would apply permutations to solve a probability problem, other than those in this section.
   c) Provide an example of a situation where you would apply combinations to solve a probability problem, other than those in this section.

Practise

A

1. Four friends, two females and two males, are playing contract bridge. Partners are randomly assigned for each game. What is the probability that the two females will be partners for the first game?
2. What is the probability that at least two out of a group of eight friends will have the same birthday?
3. A fruit basket contains five red apples and three green apples. Without looking, you randomly select two apples. What is the probability that
   a) you will select two red apples?
   b) you will not select two green apples?
4. Refer to Example 1. What is the probability that the two brothers will start beside each other in any pair of lanes?

Apply, Solve, Communicate

B

5. An athletic committee with three members is to be randomly selected from a group of six gymnasts, four weightlifters, and eight long-distance runners. Determine the probability that
   a) the committee is comprised entirely of runners
   b) the committee is represented by each of the three types of athletes

6. A messy drawer contains three black socks, five blue socks, and eight white socks, none of which are paired up. If the owner grabs two socks without looking, what is the probability that both will be white?

7. a) A family of nine has a tradition of drawing two names from a hat to see whom they will each buy presents for. If there are three sisters in the family, and the youngest sister is always allowed the first draw, determine the probability that the youngest sister will draw both of the other two sisters' names. If she draws her own name, she replaces it and draws another.
   b) Suppose that the tradition is modified one year, so that the first person whose name is drawn is to receive a "main" present, and the second a less expensive, "fun" present. Determine the probability that the youngest sister will give a main present to the middle sister and a fun present to the eldest sister.

8. Application
   a) Laura, Dave, Monique, Marcus, and Sarah are going to a party. What is the probability that two of the girls will arrive first?
   b) What is the probability that the friends will arrive in order of ascending age?
   c) What assumptions must be made in parts a) and b)?

9. A hockey team has two goalies, six defenders, eight wingers, and four centres. If the team randomly selects four players to attend a charity function, what is the likelihood that
   a) they are all wingers?
   b) no goalies or centres are selected?

10. Application A lottery promises to award ten grand-prize trips to Hawaii and sells 5 400 000 tickets.
    a) Determine the probability of winning a grand prize if you buy
       i) 1 ticket
       ii) 10 tickets
       iii) 100 tickets
    b) Communication How many tickets do you need to buy in order to have a 5% chance of winning a grand prize? Do you think this strategy is sensible? Why or why not?
    c) How many tickets do you need to ensure a 50% chance of winning?

11. Suki is enrolled in one data-management class at her school and Leo is in another. A school quiz team will have four volunteers, two randomly selected from each of the two classes. Suki is one of five volunteers from her class, and Leo is one of four volunteers from his. Calculate the probability of the two being on the team and explain the steps in your calculation.

Probability Practise Worksheet

A. Basic Probability Concepts

1. A coin is tossed 3 times.
   a. What is the probability of tossing 3 heads in succession?
   b. What is the probability of tossing exactly 2 heads?
   c. What is the probability of tossing *at least* 2 heads?

2. Two standard dice are rolled.
   a. What is the probability of rolling a pair (both the same number)?
   b. What is the probability of rolling a sum of 9?
   c. What is the probability of rolling a sum that is *less than* 6?
   d. What is the probability that a sum *less than* 7 is *not* rolled?

3. If the probability of rain tomorrow is 65%, what is the probability that it will *not* rain tomorrow?

4. What is the probability of drawing a red face card from a standard deck of playing cards?

5. Use a tree diagram to illustrate the possible outcomes for a couple having 4 children, assuming that the probability of having a boy equals the probability of having a girl. What is the probability that the family will have all 4 boys *or* all 4 girls?

6. A stable has 15 horses available for trail rides. Of these horses, 6 are brown, 5 are mainly white, and the rest are black. If Zeljana selects one at random, what is the probability that her horse will:
   a. be black?
   b. *not* be black?
   c. be either black *or* brown?

7. A number is chosen randomly from the numbers 1 – 20. If event A = {a multiple of 5}, what is the value of $P(A')$?

8. *Match each of these terms with the phrases below.*

a. sample space	b. empirical probability	c. complement	d. subjective probability
e. outcomes	f. theoretical probability	g. trials	h. event

i. deduced from analysis of all possible outcomes _______
ii. the set of all possible outcomes _______
iii. repetitions of a probability experiment _______
iv. the outcomes not included in a particular event _______
v. based upon intuition and previous experience _______
vi. a specific group of outcomes that is being investigated _______
vii. experimental probability _______
viii. the different results of a probability experiment _______

Answers:

1a) 1/8	1b) 3/8	1c) 1/2	2a) 1/6	2b) 1/9	2c) 5/18	2d) 7/12	3) 35%	4) 3/26	5) 1/8
6a) 4/15	6b) 11/15	6c) 2/3	7) 4/5						
8a) ii	8b) vii	8c) iv	8d) v	8e) viii	8f) i	8g) iii	8h) vi		

B. Probabilities Using Counting Techniques

1. A group of volleyball players consists of four grade 11 students and six grade 12 students. If six players are chosen at random to start a match, what is the probability that three will be from each grade?

2. If a bowl contains 10 hazelnuts and 8 almonds, what is the probability that 4 nuts randomly selected from the bowl will all be hazelnuts?

3. Without looking, Adam randomly selects 2 socks from a drawer containing 4 blue, 3 white, and 5 black socks, none of which are paired up. What is the probability that he chooses two socks of the same colour?

4. A four-member curling team is randomly chosen from six grade 10 students and nine grade 11 students. What is the probability that the team has *at least* one grade 10 student?

5. Sam has 5 white and 6 grey huskies in her kennel. If a wilderness expedition chooses a team of 6 sled dogs at random from Sam's kennel, what is the probability that the team will consist of:
   a. all white huskies?
   b. all grey huskies?
   c. 3 of each colour?

6. You need to visit the bank, the book store, the pharmacy, and the video store. You make a random choice of the order in which you visit the four places. Find the probability of each of the following events:
   a. you visit the bank first
   b. you visit the pharmacy second and the book store last
   c. you visit the bank *right* before the pharmacy

Answers:

1) 8/21	2) 7/102	3) 19/66	4) 59/65	5a) 0	5b) 1/462
5c) 100/231	6a) 1/4	6b) 1/12	6c) 1/4		

Independent Events

A. Compound Events

For the past 2 lessons, we have been finding probabilities for *simple events (events that consist of only one outcome).* However, there are times when we may deal with probabilities involving **two or more separate events**. For example, flipping a coin and then rolling a die is an example of two separate events, known as **compound events**.

B. Independent Events

In some situations involving compound events, the occurrence of one event, *A,* has *no effect* on the occurrence of another event, *B*. In such cases, events *A* and *B* are **independent**.

Ex. 1: A coin is tossed 4 times and turns up heads each time. What is the probability that the fifth toss will be heads?

Does the result of the fifth trial depend on the results of the previous 4 trials? No – the coin has no "memory" of the past 4 trials, so each toss is independent!

C. Product Rule for Independent Events

A compound probability asks us to find the likelihood that event A *and* event B will occur. Like the *Fundamental Counting Principle* we used with permutations and combinations, compound probabilities can be found by multiplying the probabilities of each independent event together.

In general, the compound probability of two independent events can be calculated using the product rule for independent events:

P(*A* and *B*) = P(*A*) x P(*B*)

Ex. 2: A coin is flipped while a six-sided die is rolled. What is the probability of flipping heads *and* rolling a 5 in a single trial?

Ex. 3: A coin is tossed 6 times. What is the probability that *all six tosses* will be tails?

Ex. 4: A die is rolled and a card is drawn from a standard 52-card deck. What is the probability that a red face card will be drawn and a number greater than 2 will be rolled?

Ex. 5: At an athletic event, athletes are tested for steroids using two different tests. The first test has a 93% probability of giving accurate results, while the second test is accurate 87% of the time. If both tests are used on a sample that does contain steroids, what is the probability that:

a) neither test shows that steroids are present?

b) both tests show that steroids are present?

c) at least one of the tests detects the steroids?

CW: p. 334 # 1-4, 15, 16

Practise

A

1. Classify each of the following as independent or dependent events.

	First Event	Second Event
a)	Attending a rock concert on Tuesday night	Passing a final examination the following Wednesday morning
b)	Eating chocolate	Winning at checkers
c)	Having blue eyes	Having poor hearing
d)	Attending an employee training session	Improving personal productivity
e)	Graduating from university	Running a marathon
f)	Going to a mall	Purchasing a new shirt

2. Amitesh estimates that he has a 70% chance of making the basketball team and a 20% chance of having failed his last geometry quiz. He defines a "really bad day" as one in which he gets cut from the team and fails his quiz. Assuming that Amitesh will receive both pieces of news tomorrow, how likely is it that he will have a really bad day?

3. In the popular dice game Yahtzee®, a Yahtzee occurs when five identical numbers turn up on a set of five standard dice. What is the probability of rolling a Yahtzee on one roll of the five dice?

Apply, Solve, Communicate

B

4. There are two tests for a particular antibody. Test A gives a correct result 95% of the time. Test B is accurate 89% of the time. If a patient is given both tests, find the probability that
   a) both tests give the correct result
   b) neither test gives the correct result
   c) at least one of the tests gives the correct result

5. a) Rocco and Biff are two koala bears participating in a series of animal behaviour tests. They each have 10 min to solve a maze. Rocco has an 85% probability of succeeding if he can smell the eucalyptus treat at the other end. He can smell the treat 60% of the time. Biff has a 70% chance of smelling the treat, but when he does, he can solve the maze only 75% of the time. Neither bear will try to solve the maze unless he smells the eucalyptus. Determine which koala bear is more likely to enjoy a tasty treat on any given trial.
   b) Communication Explain how you arrived at your conclusion.

6. Shy Tenzin's friends assure him that if he asks Mikala out on a date, there is an 85% chance that she will say yes. If there is a 60% chance that Tenzin will summon the courage to ask Mikala out to the dance next week, what are the odds that they will be seen at the dance together?

7. When Ume's hockey team uses a "rocket launch" breakout, she has a 55% likelihood of receiving a cross-ice pass while at full speed. When she receives such a pass, the probability of getting her slapshot away is $\frac{1}{3}$. Ume's slapshot scores 22% of the time. What is the probability of Ume scoring with her slapshot when her team tries a rocket launch?

8. Inquiry/Problem Solving Show that if A and B are dependent events, then the conditional probability $P(A \mid B)$ is given by

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}.$$

9. A consultant's study found Megatran's call centre had a 5% chance of transferring a call about schedules to the lost articles department by mistake. The same study shows that, 1% of the time, customers calling for schedules have to wait on hold, only to discover that they have been mistakenly transferred to the lost articles department. What are the chances that a customer transferred to lost articles will be put on hold?

10. Pinder has examinations coming up in data management and biology. He estimates that his odds in favour of passing the data-management examination are 17:3 and his odds against passing the biology examination are 3:7. Assume these to be independent events.
   a) What is the probability that Pinder will pass both exams?
   b) What are the odds in favour of Pinder failing both exams?
   c) What factors could make these two events dependent?

11. Inquiry/Problem Solving How likely is it for a group of five friends to have the same birth month? State any assumptions you make for your calculation.

12. Determine the probability that a captured deer has the bald patch condition.

Chapter Problem

13. Communication Five different CD-ROM games, Garble, Trapster, Zoom!, Bungie, and Blast 'Em, are offered as a promotion by SugarRush cereals. One game is randomly included with each box of cereal.
   a) Determine the probability of getting all 5 games if 12 boxes are purchased.
   b) Explain the steps in your solution.
   c) Discuss any assumptions that you make in your analysis.

14. Application A critical circuit in a communication network relies on a set of eight identical relays. If any one of the relays fails, it will disrupt the entire network. The design engineer must ensure a 90% probability that the network will not fail over a five-year period. What is the maximum tolerable probability of failure for each relay?

C

15. a) Show that if a coin is tossed n times, the probability of tossing n heads is given by $P(A) = \left(\frac{1}{2}\right)^n$.
   b) What is the probability of getting at least one tail in seven tosses?

16. What is the probability of not throwing 7 or doubles for six consecutive throws with a pair of dice?

17. Laurie, an avid golfer, gives herself a 70% chance of breaking par (scoring less than 72 on a round of 18 holes) if the weather is calm, but only a 15% chance of breaking par on windy days. The weather forecast gives a 40% probability of high winds tomorrow. What is the likelihood that Laurie will break par tomorrow, assuming that she plays one round of golf?

18. Application The Tigers are leading the Storm one game to none in a best-of-five playoff series. After a playoff win, the probability of the Tigers winning the next game is 60%, while after a loss, their probability of winning the next game drops by 5%. The first team to win three games takes the series. Assume there are no ties. What is the probability of the Storm coming back to win the series?

Dependent Events

A. Recall Independent Events

When the occurrence of one event, *A,* has *no effect* on the occurrence of another event, *B*, events *A* and *B* are said to be **independent**. The product rule for independent events states: **P(*A* and *B*) = P(*A*) x P(*B*)**

Ex. 1: Suppose you simultaneously roll a standard die and spin a spinner with eight equal sectors numbered 1 – 8. What is the probability of rolling an even number and spinning an odd number?

Rolling a die and spinning a spinner are *independent events* because the occurrence of one does not affect the occurrence of the other. We now turn our attention to cases where the probability of one event occurring *does affect* the outcome of another event.

B. Dependent Events

When the probable outcome of an event, *B*, depends directly on the outcome of another event, *A*, the events are said to be **dependent.** The **conditional probability** of *B*, written as P(*B* given *A*) or P(*B*|*A*), is the probability that *B* will occur *given* that *A* has already occurred.

The product rule for two **dependent** events is the probability of the first event times the conditional probability of the second event given the first has occurred:

P(*A* and *B*) = P(*A*) x P(*B given A*) or **P(*A* and *B*) = P(*A*) x P(*B*|*A*)**

Ex. 2: The probability that Mary will go to U of T is 0.2. The probability that she will go to another university is 0.5. If Mary goes to U of T, the probability that her partner Taylor will follow her and go to U of T is 0.75. What is the probability that both Mary and Taylor will attend U of T?

Ex. 3: In your pocket, you have a penny, a nickel and a dime. Determine the probability of reaching into your pocket and selecting a:

a) dime

b) dime then a nickel (without replacing the dime)

Ex. 4: The KCI Girls hockey team has eight wingers. Three of these wingers scored 4+ goals. If you choose two wingers at random, what is the probability of them both being 4+ goal scorers?

Ex. 5: A card is drawn from a standard deck of 52 cards. What is the probability that it is a jack, given that it is a face card?

CW: p. 334 # 5, 6; p. 358 # 9-11

6.4 Dependent and Independent Events

Refer to the Key Concepts on page 333.

9. Classify each of the following pairs of events as independent or dependent.

	First Event	Second Event
a)	Hitting a home run while at bat	Catching a pop fly while in the field
b)	Staying up late	Sleeping in the next day
c)	Completing your calculus review	Passing your calculus exam
d)	Randomly selecting a shirt	Randomly selecting a tie

10. Bruno has just had job interviews with two separate firms: Golden Enterprises and Outer Orbit Manufacturing. He estimates that he has a 40% chance of receiving a job offer from Golden and a 75% chance of receiving an offer from Outer Orbit.

a) What is the probability that Bruno will receive both job offers?

b) Is Bruno applying the concept of theoretical, empirical, or subjective probability? Explain.

11. Karen and Klaus are the parents of James and twin girls Britta and Kate. Each family member has two shirts in the wash. If a shirt is pulled from the dryer at random, what is the probability that the shirt belongs to

a) Klaus, if it is known that the shirt belongs to one of the parents?

b) Britta, if it is known that the shirt is for a female?

c) Kate, if it is known that the shirt belongs to one of the twins?

d) Karen or James

12. During a marketing blitz, a telemarketer conducts phone solicitations continuously from 16 00 until 20 00. Suppose that you have a 20% probability of being called during this blitz. If you generally eat dinner between 18 00 and 18 30, how likely is it that the telemarketer will interrupt your dinner?

6.5 Mutually Exclusive Events

Refer to the Key Concepts on page 340.

13. Classify each pair of events as mutually exclusive or non-mutually exclusive.

	First Event	Second Event
a)	Randomly selecting a classical CD	Randomly selecting a rock CD
b)	Your next birthday occurring on a Wednesday	Your next birthday occurring on a weekend
c)	Ordering a hamburger with cheese	Ordering a hamburger with no onions
d)	Rolling a perfect square with a die	Rolling an even number with a die

14. a) Determine the probability of drawing either a 5 or a black face card from a standard deck of cards.

b) Illustrate this situation with a Venn diagram.

15. In a data management class of 26 students, there are 9 with blonde hair, 7 with glasses, and 4 with blonde hair and glasses.

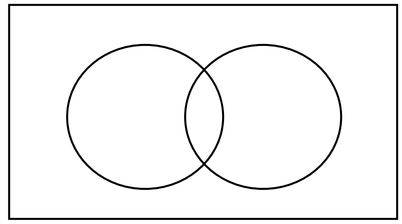
a) Draw a Venn diagram to illustrate this situation.

b) If a student is selected at random, what is the probability that the student will have either blonde hair or glasses?

Mutually Exclusive and Non-Mutually Exclusive Events

A. Recall Venn Diagrams

When we looked at two events, *A* and *B,* that had some "overlap" (*intersection*) we noted that we could count the total number in A *or* B *or* Both (*union*) two ways:



1) *n*(A or B) = *n*(A *only*) + *n*(B *only*) + *n*(A *and* B)

2) *n*(A or B) = *n*(A total) + *n*(B total) – *n*(A *and* B)

These methods helped us avoid double-counting the number of outcomes in the event where A *and* B could occur together. We will use the same methods to help us avoid double-counting *probabilities* where two events could occur simultaneously!

B. Mutually Exclusive Events

Mutually exclusive events are events that **cannot occur simultaneously**. If one has occurred, it is not possible that the other occurred at the same time. Recall the tie example from *Unit 1*: since you cannot simultaneously *wear a tie* while *not wearing a tie*, these events are mutually exclusive. More examples of mutually exclusive events include:

- getting a 1 and a 2 on a single roll of a die
- getting a sum of 3 and two of a kind when rolling a pair of dice once
- removing a queen and an ace when taking one card from a standard deck

The addition rule for **mutually exclusive events** is (the probability of the first event) + (the probability of the second event), given there is no intersection of the events:

P(*A* or *B*) = P(*A*) + P(*B*)

Ex. 1: If two standard dice are rolled, what is the probability of rolling doubles *or* a total of 11?

Ex. 2: If a committee of five is to be randomly chosen from six males and eight females, what is the probability that the committee will be either all male or all female?

C. Non-Mutually Exclusive Events

Non-mutually exclusive events are events that **may occur simultaneously**. If one has occurred, it is quite possible that the other occurred at the same time. Recall the **Q♥** example from *Unit 1*: we were looking for the number of ways to select a queen *or* a heart from a standard deck of cards. Since you **could** select one or the other or both simultaneously, these events are *not mutually exclusive*. More examples of non-mutually exclusive events include:

- getting a 1 or a 2 on two rolls of a die
- getting a sum of 4 or two of a kind when rolling a pair of dice once
- removing a king or a spade when taking one card from a standard deck

The addition rule for **non-mutually exclusive events** is (the probability of the first event) + (the probability of the second event) – (the probability of both events):

P(*A* or *B*) = P(*A*) + P(*B*) – P(A *and* B)

Ex. 3: If two standard dice are rolled, what is the probability of rolling doubles *or* a total of 10?

Ex. 4: A card is randomly selected from a standard deck of cards. What is the probability that either a red card pr a face card will be selected?

Ex. 5: In Statsville, the probability of a teenager listening to the classic rock station is 42%, while the probability of listening to the New Country station is 36%. If 13% of the teenagers surveyed listen to both stations, what is the probability that a given student listens to neither of the stations?

Ex. 6: At Waterloo West Animal Hospital, where Mr. Jackson takes his puppy, the vet has found that Pipper will require his teeth cleaned with a probability of 0.6, his fur clipped with a probability of 0.3, and both with a probability of 0.1. (Hint: a Venn diagram is helpful . . .)

a) What is the probability that Pipper needs either his teeth cleaned or his fur clipped?
b) What is the probability that Pipper needs his teeth cleaned but not his fur clipped?
c) What is the probability that Pipper will require neither?

CW: p. 340-343 # 1 - 5, 11, 13, 15.

Key Concepts

- If A and B are mutually exclusive events, then the probability of either A or B occurring is given by $P(A \text{ or } B) = P(A) + P(B)$.
- If A and B are non-mutually exclusive events, then the probability of either A or B occurring is given by $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Communicate Your Understanding

1. Are an event and its complement mutually exclusive? Explain.
2. Explain how to determine the probability of randomly throwing either a composite number or an odd number using a pair of dice.
3. a) Explain the difference between independent events and mutually exclusive events.
   b) Support your explanation with an example of each.
   c) Why do you add probabilities in one case and multiply them in the other?

Practise

A

1. Classify each pair of events as mutually exclusive or non-mutually exclusive.

	Event A	Event B
a)	Randomly drawing a grey sock from a drawer	Randomly drawing a wool sock from a drawer
b)	Randomly selecting a student with brown eyes	Randomly selecting a student on the honour roll
c)	Having an even number of students in your class	Having an odd number of students in your class
d)	Rolling a six with a die	Rolling a prime number with a die
e)	Your birthday falling on a Saturday next year	Your birthday falling on a weekend next year
f)	Getting an A on the next test	Passing the next test
g)	Calm weather at noon tomorrow	Stormy weather at noon tomorrow
h)	Sunny weather next week	Rainy weather next week

2. Nine members of a baseball team are randomly assigned field positions. There are three outfielders, four infielders, a pitcher, and a catcher. Troy is happy to play any position except catcher or outfielder. Determine the probability that Troy will be assigned to play
   a) catcher
   b) outfielder
   c) a position he does not like

3. A car dealership analysed its customer database and discovered that in the last model year, 28% of its customers chose a 2-door model, 46% chose a 4-door model, 19% chose a minivan, and 7% chose a 4-by-4 vehicle. If a customer was selected randomly from this database, what is the probability that the customer
   a) bought a 4-by-4 vehicle?
   b) did not buy a minivan?
   c) bought a 2-door or a 4-door model?
   d) bought a minivan or a 4-by-4 vehicle?

Apply, Solve, Communicate

B

4. As a promotion, a resort has a draw for free family day-passes. The resort considers July, August, March, and December to be "vacation months."

a) If the free passes are randomly dated, what is the probability that a day-pass will be dated within

i) a vacation month?

ii) June, July, or August

b) Draw a Venn diagram of the events in part a).

5. A certain provincial park has 220 campsites. A total of 80 sites have electricity. Of the 52 sites on the lakeshore, 22 of them have electricity. If a site is selected at random, what is the probability that

a) it will be on the lakeshore?

b) it will have electricity?

c) it will either have electricity or be on the lakeshore?

d) it will be on the lakeshore and not have electricity?

6. A market-research firm monitored 1000 television viewers, consisting of 800 adults and 200 children, to evaluate a new comedy series that aired for the first time last week. Research indicated that 250 adults and 148 children viewed some or all of the program. If one of the 1000 viewers was selected, what is the probability that

a) the viewer was an adult who did not watch the new program?

b) the viewer was a child who watched the new program?

c) the viewer was an adult or someone who watched the new program?

7. Application In an animal-behaviour study, hamsters were tested with a number of intelligence tasks, as shown in the table below.

Number of Tests	Number of Hamsters
0	10
1	6
2	4
3	3
4 or more	5

If a hamster is randomly chosen from this study group, what is the likelihood that the hamster has participated in

a) exactly three tests?

b) fewer than two tests?

c) either one or two tests?

d) no tests or more than three tests?

8. Communication

a) Prove that, if A and B are non-mutually exclusive events, the probability of either A or B occurring is given by
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

b) What can you conclude if $P(A \text{ and } B) = 0$? Give reasons for your conclusion.

9. Inquiry/Problem Solving Design a game in which the probability of drawing a winning card from a standard deck is between 55% and 60%.

10. Chapter Problem: Determine the probability that a captured deer has either cross-hatched antlers or bald patches. Are these events mutually exclusive? Why or why not?

11. The eight members of the debating club pose for a yearbook photograph. If they line up randomly, what is the probability that

a) either Hania will be first in the row or Aaron will be last?

b) Hania will be first and Aaron will not be last?

ACHIEVEMENT CHECK

Knowledge/Understanding	Thinking/Inquiry/Problem Solving	Communication	Application

12. Consider a Stanley Cup playoff series in which the Toronto Maple Leafs hockey team faces the Ottawa Senators. Toronto hosts the first, second, and if needed, fifth and seventh games in this best-of-seven contest. The Leafs have a 65% chance of beating the Senators at home in the first game. After that, they have a 60% chance of a win at home if they won the previous game, but a 70% chance if they are bouncing back from a loss. Similarly, the Leafs' chances of victory in Ottawa are 40% after a win and 45% after a loss.

a) Construct a tree diagram to illustrate all the possible outcomes of the first three games.

b) Consider the following events:

A = {Leafs lose the first game but go on to win the series in the fifth game}

B = {Leafs win the series in the fifth game}

C = {Leafs lose the series in the fifth game}

Identify all the outcomes that make up each event, using strings of letters, such as *LLSLL*. Are any pairs from these three events mutually exclusive?

c) What is the probability of event A in part b)?

d) What is the chance of the Leafs winning in exactly five games?

e) Explain how you found your answers to parts c) and d).

C

13. A grade 12 student is selected at random to sit on a university liaison committee. Of the 120 students enrolled in the grade 12 university-preparation mathematics courses,

- 28 are enrolled in data management only
- 40 are enrolled in calculus only
- 15 are enrolled in geometry only
- 16 are enrolled in both data management and calculus
- 12 are enrolled in both calculus and geometry
- 6 are enrolled in both geometry and data management
- 3 are enrolled in all three of data management, calculus, and geometry

a) Draw a Venn diagram to illustrate this situation.

b) Determine the probability that the student selected will be enrolled in either data management or calculus.

c) Determine the probability that the student selected will be enrolled in only one of the three courses.

14. Application For a particular species of cat, the odds against a kitten being born with either blue eyes or white spots are 3:1. If the probability of a kitten exhibiting only one of these traits is equal and the probability of exhibiting both traits is 10%, what are the odds in favour of a kitten having blue eyes?

15. Communication

a) A standard deck of cards is shuffled and three cards are selected. What is the probability that the third card is either a red face card or a king if the king of diamonds and the king of spades are selected as the first two cards?

b) Does this probability change if the first two cards selected are the queen of diamonds and the king of spades? Explain.

Probability Practise Worksheet II

A. Dependent and Independent Events

1. What is the probability that three standard dice rolled simultaneously will all land with the same number facing up?
2. Suppose you simultaneously roll a standard die and spin a spinner divided into 10 equal sectors, numbered 1 through 10. What is the probability of getting a 4 on both the die and the spinner?
3. If Aaron does his math homework today, the probability that he will do it tomorrow is 0.6. The probability that he will do it today is 0.4. What is the probability that he will do it both today and tomorrow?
4. What is the probability of rolling a total of 7 in two rolls of a standard die if you get an even number on the first roll??
5. A bag contains three green marbles and four black marbles. If Cristobal randomly picks two marbles from the bag at the same time, what is the probability that both marbles will be black?
6. If a satellite launch has a 97% chance of success, what is the probability of three consecutive successful launches?
7. A survey at a school asked students if they were ill with a cold or flu during the last month. The results were as follows. None of the students had both a cold and the flu.

	Cold	**Flu**	**Healthy**
Females	32	18	47
Males	25	19	38

Use these results to estimate the probability that:

a. A randomly selected student had a cold last month
b. A randomly selected female student was healthy last month
c. A randomly selected student who had the flu last month was male
d. A randomly selected male student had either a cold or the flu last month

Answers:

1) 1/36	2) 1/60	3) 0.24	4) 1/12	5) 2/7
6) 91.3%	7a) 57/179	7b) 47/97	7c) 19/37	7d) 22/41

B. Mutually Exclusive and Non-Mutually Exclusive Events

1. What is the probability of randomly selecting either a club *or* a non-face card from a standard deck of cards?

2. In a particular class, the probability of a student having blue eyes is 0.3, of having both blue eyes and blonde hair is 0.2, and of having neither blue eyes nor blonde hair is 0.5. What is the probability that a student in this class has blonde hair?

3. The probability that Jack will play golf today is 60%, the probability that he will play golf tomorrow is 75%, and the probability that he will play both days is 50%. What is the probability that he does *not* play golf on either day?

4. If 28% of the residents of Statsville wear contact lenses, 9% have blue eyes *and* wear contact lenses, and 44% have *neither* blue eyes *nor* wear contact lenses, what is the probability that a randomly selected resident has blue eyes?

5. If a survey of teenage readers of popular magazines shows that 38% subscribe to *Teen People*, 47% subscribe to *Cool Life*, and 35% subscribe to neither magazine, what is the probability that a randomly selected teenager:

    a. subscribes to both magazines?
    b. subscribes to either one magazine or both magazines?
    c. subscribes to only one of the magazines?

6. A survey of 50 female high-school athletes collected the following data:

Team	**Number of Athletes**
Field Hockey	23
Volleyball	16
Rugby	29
Both rugby and field hockey	8
Both rugby and volleyball	9
Both field hockey and volleyball	7
All three teams	6

    a. Draw a Venn diagram to illustrate the data.
    b. What is the probability that a randomly selected athlete will play on *only one* of the teams?
    c. What is the probability that a randomly selected rugby player also plays volleyball?
    d. What is the probability that a randomly selected athlete who does not play rugby is on the field hockey team?

Answers:

1) 43/52	2) 0.2			3) 15%	4) 37%
5a) 20%	5b) 65%	5c) 45%	6b) 19/25	6c) 9/29	6d) 15/21

MDM 4UI Name: ______________________________

Sample TEST – Unit 3
Probability

INSTRUCTIONS
- ✓ Only neat, complete and organized solutions will receive full marks.
- ✓ Show all your work.
- ✓ All solutions must have accompanying event definitions, unless otherwise stated.
- ✓ Total Marks = 40

Useful formulas:

$$P(A)=\frac{n(A)}{n(S)} \qquad P(A \text{ and } B)=P(A)\times P(B) \qquad P(A \text{ and } B)=P(A)\times P(B\,|\,A)$$

$$P(A \text{ or } B)=P(A)+P(B) \qquad P(A \text{ or } B)=P(A)+P(B)-P(A \text{ and } B)$$

Question 1 (2 marks)

Chris' sock drawer contains three pairs of grey socks, two pairs of white socks and four pairs of black socks. The socks are not matched or organized in any special way.

a) In a mad dash to get to school (and not walk into class late) Chris randomly selects a sock from his drawer without looking. What is the probability that he selects a grey sock?
b) Once this sock has been drawn, and discovered to be grey, what is the probability that Chris will select another grey sock to make a matching pair?

Question 2 (2 marks)

Keanan is immersed in an intense game of Yahtzee. Assuming the game is played with fair dice, find the probability of rolling:
a) a three or a five with one die. b) two fives with two dice.

Question 3 (4 marks)

In Yu-Jin's bag of Smarties there are six red, five brown and three purple candies remaining. Suppose she selects two Smarties, one after the other, without replacing them. Find the probability that:
a) both Smarties are brown.
b) *at least* one Smartie is purple.

Question 4 (4 marks)

A six member work group is being created to plan details about KCI's new sports field. Five coaches and nine students are being considered for the committee. If the group is randomly selected, find the probability that it will include *at least* two coaches.

Question 5 (5 marks)

If a survey on teenage readers of popular magazines shows that 38% subscribe to *Teen People*, 47% subscribe to *Cool Life*, and 20% subscribe to both magazines, what is the probability that a randomly selected teenager subscribes to neither magazine?

Question 6 (4 marks)

Matt estimates that he has a 95% chance of passing Physics and a 99% chance of passing Data Management. Assuming that these are independent events,

a) Find the probability that Matt will pass both courses.
b) Find the probability that Matt will pass only one of the two courses.

Question 7 (4 marks)

A certain student determines that there is a 60% chance he will gather the courage to ask his sweetie to the prom. This student's friends assure him that if he asks, there is an 85% chance his sweetie will say yes. What is the probability our shy student will be seen at the prom with his sweetie?

Question 8 (4 marks)

A euchre deck contains 24 cards, the 9, 10, jack, queen, king and ace from each suit. If you were to deal out five cards from this deck, what is the probability that they will be a 10, jack, queen, king and ace all from the same suit?

Question 9 (5 marks)

When the KCI Senior Boys Football team has possession of the ball, the following empirical probabilities have been determined:

- The probability that the quarterback completes a pass is 0.6
- The probability that the quarterback completes a pass and they score a touchdown is 0.01

Find the probability that they score a touchdown given that the quarterback completes a pass.

Question 10 (6 marks)

The Blue Jays are underdogs in a best-of-five playoff series against Boston. The probability of the Jays winning each game is 0.375. If the Jays win the first game in this series, what is the probability they will win the series? (Hint: draw a tree diagram and consider each case that would lead to the Jays winning the series.)

Answers:

1a) 1/3	1b) 5/17	2a) 1/3	2b) 1/36	3a) 11%	3b) 40%	4) 76%
5) 35%	6a) 94%	6b) 5.9%	7) 51%	8) 0.009%	9) 1.66%	10) 48%

HW: p. 357 # 1 – 3, 6 – 8, 13 – 16; p. 360 # 1 – 5, 7 - 9