Venn Diagrams

A. Even *More* Organized Counting!?

During the last unit, we looked at *tree diagrams* and *lists* to help us count the number of permutations for a given situation. A *Venn diagram* is an organized counting tool that helps us visualize relationships between sets of items, especially when the sets have some items in common. ☺

B. Definitions and Terminology

- **Set**
- **Element**
- **Universal set**
- **Empty set**
- ***n*( )**
- **Subset**
- **Venn Diagram**
- **Union**
- **Intersection**

C. Principle of Inclusion and Exclusion for Two Sets

Ex. 1: Of the 29 students taking drama, 18 have signed up for a play that will tour several local high schools, 12 have volunteered to help out in the drama festival, and 5 have agreed to do both.

a) Illustrate this solution using a Venn diagram.

b) How many students are involved in at least one of the productions?

c) How many students are involved in neither production?

Principle of Inclusion and Exclusion for Two Sets

n(A or B) = n(A) + n(B) – n(A and B) *or* ***$n(A \cup B) = n(A) + n(B) - n(A \cap B)$***

where n(X) represents the number of elements in set X

Ex.2: There are 8 students on the curling team, 12 students on the badminton team, and 3 students on both teams. How many students are on the two teams combined?

D. Principle of Inclusion and Exclusion for Three Sets

Ex. 3: A translation service has 40 employees, of whom 27 speak French, 11 speak Spanish, and 7 speak Hindi. Of the Spanish-speaking employees, 5 speak Hindi and 6 speak French. Three of the French-speaking employees also speak Hindi. Two employees speak all three languages.

a) Illustrate this solution using a Venn diagram.

b) How many employees speak French or Spanish or Hindi?

c) How many employees speak none of these languages?

Principle of Inclusion and Exclusion for Three Sets

$$n(A \text{ or } B \text{ or } C) = n(A) + n(B) + n(c) - n(A \text{ and } B) - n(A \text{ and } C) - n(B \text{ and } C) + n(A \text{ and } B \text{ and } C)$$

or

$$n(A \cup B \cup C) = n(A) + n(B) + n(c) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Ex. 4: All 24 condominium buildings in a retirement community have at least one of the following amenities: 12 have a swimming pool, 12 have a tennis court, and 15 have playing-card rooms. If 4 buildings have a pool and a tennis court, 5 have a pool and a card room, and 3 have all three, how many have **only** a tennis court and a card room?

CW: Venn Diagrams Worksheet; p. 271 # 2, 4, 6

Venn Diagrams Worksheet

1. Of the 25 students on School Council, 18 have signed up to help with spirit day, 10 have volunteered to help with a leadership conference and 5 students are involved with both projects. How many of the students on council do not have a project yet? **ANS: 2**

2. Of the 190 students graduating from Mathville High School, 111 played on a sports team, 26 served on the students' council and 67 were members of a club. Only 10 students did not participate in any of these activities. If 13 students were members of both a team and a club, 9 were on both a team and the students' council and 11 were members of both the students' council and a club, how many graduating students were members of all three groups? **ANS: 9**

3. A fabric store has 44 bolts of fabric on its clearance table. Of these bolts, 28 are broadcloth, 17 are floral prints, 20 have a blue background, 12 are floral prints on broadcloth, 10 are broadcloth with a blue background and 4 are floral prints with a blue background. If all but 2 of the clearance fabrics fit at least one of these categories, how many of the bolts are broadcloth floral prints with a blue background? **ANS: 3**

4. At Mathville High School, 125 students are taking university-preparation mathematics courses. Of these students, 64 take data management, 40 take calculus and 51 take geometry. There are 12 students who take both calculus and geometry, 11 who take both calculus and data and 8 who take all three courses. How many students take just geometry and data management? **ANS: 7**

a) How many students participated in the survey?

b) How many of these students play both soccer and baseball?

c) How many play only one sport?

d) How many play football and soccer?

e) How many play all three sports?

f) How many do not play soccer?

Apply, Solve, Communicate

B

3. Of the 220 graduating students in a school, 110 attended the semi-formal dance and 150 attended the formal dance. If 58 students attended both events, how many graduating students did not attend either of the two dances? Illustrate your answer with a Venn diagram.

4. Application A survey of 1000 television viewers conducted by a local television station produced the following data:
   - 40% watch the news at 12 00
   - 60% watch the news at 18 00
   - 50% watch the news at 23 00
   - 25% watch the news at 12 00 and at 18 00
   - 20% watch the news at 12 00 and at 23 00
   - 20% watch the news at 18 00 and at 23 00
   - 10% watch all three news broadcasts

   a) What percent of those surveyed watch at least one of these programs?

   b) What percent watch none of these news broadcasts?

   c) What percent view the news at 12 00 and at 18 00, but not at 23 00?

   d) What percent view only one of these shows?

   e) What percent view exactly two of these shows?

5. Suppose the Canadian Embassy in the Netherlands has 32 employees, all of whom speak both French and English. In addition, 22 of the employees speak German and 15 speak Dutch. If there are 10 who speak both German and Dutch, how many of the employees speak neither German nor Dutch? Illustrate your answer with a Venn diagram.

6. Application There are 900 employees at CantoCrafts Inc. Of these, 615 are female, 345 are under 35 years old, 482 are single, 295 are single females, 187 are singles under 35 years old, 190 are females under 35 years old, and 120 are single females under 35 years old. Use a Venn diagram to determine how many employees are married males who are at least 35 years old.

7. Communication A survey of 100 people who volunteered information about their reading habits showed that
   - 75 read newspapers daily
   - 35 read books at least once a week
   - 45 read magazines regularly
   - 25 read both newspapers and books
   - 15 read both books and magazines
   - 10 read newspapers, books, and magazines

   a) Construct a Venn diagram to determine the maximum number of people in the survey who read both newspapers and magazines.

   b) Explain why you cannot determine exactly how many of the people surveyed read both newspapers and magazines.

Intro to Combinations

Permutations vs. Combinations

Recall that for permutations we were counting possible *arrangements* where ORDER MATTERS (*time* order or *spatial* order). In this unit, we will look at combinations in which we are making *selections* where ORDER DOES NOT MATTER.

Ex. 1: In how many ways can you select 3 letters from the word *help*?

a) How many ways can we arrange each of these selections?

b) Lets look at how many ways we can arrange 3 letters from the word *help*.

c) What's the relationship between the answers above?

The number of combinations of r objects chosen from a set of n distinct objects is:

$$_nC_r = C(n, r) = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

Ex. 2: In how many ways can the following cards be chosen from a deck of playing cards if order of selection does not matter?

a) Any four cards

b) Four diamonds

c) Two kings and three queens

d) Two spades or three hearts

e) 4 aces, 3 kings, 2 queens, 1 jack, and 1 other card.

Ex.3: Eunice, Beauregard, Clyde, Hank, and Delilah are being considered for a committee.

a) In how many ways could they be chosen to fill the positions of president, vice president, and treasurer?

b) In how many ways could these same 5 students form an executive committee of three members?

CW: p. 279-281 #3-7, 11-13, 16, 18, 22

Communicate Your Understanding

1. Explain why n objects have more possible permutations than combinations. Use a simple example to illustrate your explanation.
2. Explain whether you would use combinations, permutations, or another method to calculate the number of ways of choosing
   a) three items from a menu of ten items
   b) an appetizer, an entrée, and a dessert from a menu with three appetizers, four entrées, and five desserts
3. Give an example of a combination expression you could calculate
   a) by hand
   b) algebraically
   c) only with a calculator or computer

Practise

A

1. Evaluate using a variety of methods. Explain why you chose each method.
   a) ${}_{21}C_{19}$ b) ${}_{30}C_{28}$
   c) ${}_{18}C_5$ d) ${}_{16}C_3$
   e) ${}_{19}C_4$ f) ${}_{25}C_{20}$
2. Evaluate the following pairs of combinations and compare their values.
   a) ${}_{11}C_1$, ${}_{11}C_{10}$
   b) ${}_{11}C_2$, ${}_{11}C_9$
   c) ${}_{11}C_3$, ${}_{11}C_8$

Apply, Solve, Communicate

B

3. Communication In how many ways could you choose 2 red jellybeans from a package of 15 red jellybeans? Explain your reasoning.
4. How many ways can 4 cards be chosen from a deck of 52, if the order in which they are chosen does not matter?
5. How many groups of 3 toys can a child choose to take on a vacation from a toy box containing 11 toys?
6. How many sets of 6 questions for a test can be chosen from a list of 22 questions?
7. In how many ways can a teacher select 5 students from a class of 23 to make a bulletin-board display? Explain your reasoning.
8. As a promotion, a video store decides to give away posters for recently released movies.
   a) If posters are available for 27 recent releases, in how many ways could the video-store owner choose 8 different posters for the promotion?
   b) Are you able to calculate the number of ways mentally? Why or why not?

9. **Communication** A club has 11 members.
   a) How many different 2-member committees could the club form?
   b) How many different 3-member committees could the club form?
   c) In how many ways can a club president, treasurer, and secretary be chosen?
   d) By what factor do the answers in parts b) and c) differ? How do you account for this difference?

10. Fritz has a deck of 52 cards, and he may choose any number of these cards, from none to all. Use a spreadsheet or Fathom™ to calculate and graph the number of combinations for each of Fritz's choices.

11. **Application** A track club, a swim club, and a cycling club are forming a joint committee to organize a triathlon. The committee will have two members from each club. In how many ways can the committee be formed if ten runners, eight swimmers, and seven cyclists volunteer to serve on it?

12. In how many ways can a jury of 6 men and 6 women be chosen from a group of 10 men and 15 women?

13. **Inquiry/Problem Solving** There are 15 technicians and 11 chemists working in a research laboratory. In how many ways could they form a 5-member safety committee if the committee
   a) may be chosen in any way?
   b) must have exactly one technician?
   c) must have exactly one chemist?
   d) must have exactly two chemists?
   e) may be all technicians or all chemists?

14. Jeffrey, a DJ at a local radio station, is choosing the music he will play on his shift. He must choose all his music from the top 100 songs for the week and he must play at least 12 songs an hour. In his first hour, all his choices must be from the top-20 list.
   a) In how many ways can Jeffrey choose the songs for his first hour if he wants to choose exactly 12 songs?
   b) In how many ways can Jeffrey choose the 12 songs if he wants to pick 8 of the top 10 and 4 from the songs listed from 11 to 20 on the chart?
   c) In how many ways can Jeffrey choose either 12 or 13 songs to play in the first hour of his shift?
   d) In how many ways can Jeffrey choose the songs if he wants to play up to 15 songs in the first hour?

15. The game of euchre uses only 24 of the cards from a standard deck. How many different five-card euchre hands are possible?

16. **Application** A taxi is shuttling 11 students to a concert. The taxi can hold only 4 students. In how many ways can 4 students be chosen for
   a) the taxi's first trip?
   b) the taxi's second trip?

17. Diane is making a quilt. She needs three pieces with a yellow undertone, two pieces with a blue undertone, and four pieces with a white undertone. If she has six squares with a yellow undertone, five with a blue undertone, and eight with a white undertone to choose from, in how many ways can she choose the squares for the quilt?

18. Inquiry/Problem Solving At a family reunion, everyone greets each other with a handshake. If there are 20 people at the reunion, how many handshakes take place?

ACHIEVEMENT CHECK

Knowledge/ Understanding	Thinking/Inquiry/ Problem Solving	Communication	Application

19. A basketball team consists of five players—one centre, two forwards, and two guards. The senior squad at Vennville Central High School has two centres, six forwards, and four guards.

a) How many ways can the coach pick the two starting guards for a game?

b) How many different starting lineups are possible if all team members play their specified positions?

c) How many of these starting lineups include Dana, the team's 185-cm centre?

d) Some coaches designate the forwards as power forward and small forward. If all six forwards are adept in either position, how would this designation affect the number of possible starting lineups?

e) As the league final approaches, the centre Dana, forward Ashlee, and guard Hollie are all down with a nasty flu. Fortunately, the five healthy forwards can also play the guard position. If the coach can assign these players as either forwards or guards, will the number of possible starting lineups be close to the number in part b)? Support your answer mathematically.

f) Is the same result achieved if the forwards play their regular positions but the guards can play as either forwards or guards? Explain your answer.

20. In the game of bridge, each player is dealt a hand of 13 cards from a standard deck of 52 cards.

a) By what factor does the number of possible bridge hands differ from the number of ways a bridge hand could be dealt to a player? Explain your reasoning.

b) Use combinations to write an expression for the number of bridge hands that have exactly five clubs, two spades, three diamonds, and three hearts.

c) Use combinations to write an expression for the number of bridge hands that have exactly five hearts.

d) Use software or a calculator to evaluate the expressions in parts b) and c).

C

21. There are 18 students involved in the class production of *Arsenic and Old Lace*.

a) In how many ways can the teacher cast the play if there are five male roles and seven female roles and the class has nine male and nine female students?

b) In how many ways can the teacher cast the play if Jean will play the young female part only if Jovane plays the male lead?

c) In how many ways can the teacher cast the play if all the roles could be played by either a male or a female student?

22. A large sack contains six basketballs and five volleyballs. Find the number of combinations of four balls that can be chosen from the sack if

a) they may be any type of ball

b) two must be volleyballs and two must be basketballs

c) all four must be volleyballs

d) none may be volleyballs

All Possible Combinations!?

In the previous lesson, we looked at the number of ways of choosing ***r*** items from a set of *n* distinct items, $_nC_r$. Today, we will investigate how to count the number of combinations of *any size* (**all** *r* values from 0 to *n*). In this way, we will be finding the total number of subsets given of a set of items.

A. All Possible Combinations of *n* Distinct Items

Ex. 1: How many different sums of money can you make with a penny, a nickel, a dime, and a quarter?

"All possible combinations of 4 distinct items" implies that we could choose subsets of 0 or 1 or 2 or 3 or 4 of the items (*i.e.* all r values from 0 to *n*). To make a sum of money, however, we need at least 1 coin, so we will leave out the subset where $r = 0$.

Another way to think about this is that for each coin, we have the choice to either *include* it or *not include* it. This strategy creates one possible combination where all 4 coins are excluded, so we will subtract this 1 from our total.

Since you can't make a "sum of money" without any coins (*i.e.* r = 0), we subtracted this set (called the *null* set) from the total. **Be careful with this because every case is different. Sometimes it's okay to include the null set!

A set with n distinct elements has 2^n subsets, including the null set.

{ If the null set (r =0) is to be excluded, then the number of distinct subsets is $2^n - 1$ }.

Ex. 2: A cartoonist has 5 pens, each with a different colour of ink. How many different colour combinations could the cartoonist create within one panel of his graphic novel?

B. All Possible Combinations of *n* Items, with <u>Some Identical Items</u>

Ex. 3: A florist has six tulips, four daffodils, eight crocuses, and two irises to choose from to make a flower bouquet. How many different flower combinations are available for the bouquet?

We have 4 different *kinds* of flowers. For each kind of flower, we can include anywhere from **0** to **all** of them in the bouquet. This will include a subset where each kind has 0 items, so we will subtract this 1 subset from our total. After all, it's not a bouquet if there aren't ANY flowers!

To count all possible combinations of items when some are alike, we use the following formula:

*The total number of selections (subsets) of **at least one item** that can be made from a set of p items of one kind, q items of another kind, and r items of another kind, and so on, is:*

$$(p + 1)(q + 1)(r + 1) \ldots - 1$$

Ex. 4: How many different sums of money can be made from four pennies, two nickels, and six quarters?

Ex. 5: Rachel walks into a bookstore looking for six books on her list. How many possible combinations of these books could she find?

Ex. 6: Rachel walks out of the book store having made a purchase based on the six books on her list. How many possible purchases could she have made?

CW: p. 286-288 #1, 2, 4, 7, 13

Key Concepts

- Use the formula $(p + 1)(q + 1)(r + 1) \ldots - 1$ to find the total number of selections of at least one item that can be made from p items of one kind, q of a second kind, r of a third kind, and so on.
- A set with n distinct elements has 2^n subsets including the null set.
- For combinations with some identical elements, you often have to consider all possible cases individually.
- In a situation where you must choose *at least* one particular item, either consider the total number of choices available minus the number without the desired item or add all the cases in which it is possible to have the desired item.

Communicate Your Understanding

1. Give an example of a situation where you would use the formula $(p + 1)(q + 1)(r + 1) \ldots - 1$. Explain why this formula applies.
2. Give an example of a situation in which you would use the expression $2^n - 1$. Explain your reasoning.
3. Using examples, describe two different ways to solve a problem where *at least* one particular item must be chosen. Explain why both methods give the same answer.

Practise

A

1. How many different sums of money can you make with a penny, a dime, a one-dollar coin, and a two-dollar coin?
2. How many different sums of money can be made with one \$5 bill, two \$10 bills, and one \$50 bill?
3. How many subsets are there for a set with
   a) two distinct elements?
   b) four distinct elements?
   c) seven distinct elements?
4. In how many ways can a committee with eight members form a subcommittee with at least one person on it?

B

5. Determine whether the following questions involve permutations or combinations and list any formulas that would apply.
   a) How many committees of 3 students can be formed from 12 students?
   b) In how many ways can 12 runners finish first, second, and third in a race?
   c) How many outfits can you assemble from three pairs of pants, four shirts, and two pairs of shoes?
   d) How many two-letter arrangements can be formed from the word *star*?

Apply, Solve, Communicate

6. Seven managers and eight sales representatives volunteer to attend a trade show. Their company can afford to send five people. In how many ways could they be selected

a) without any restriction?

b) if there must be at least one manager and one sales representative chosen?

7. Application A cookie jar contains three chocolate-chip, two peanut-butter, one lemon, one almond, and five raisin cookies.

a) In how many ways can you reach into the jar and select some cookies?

b) In how many ways can you select some cookies, if you must include at least one chocolate-chip cookie?

8. A project team of 6 students is to be selected from a class of 30.

a) How many different teams can be selected?

b) Pierre, Gregory, and Miguel are students in this class. How many of the teams would include these 3 students?

c) How many teams would not include Pierre, Gregory, and Miguel?

9. The game of euchre uses only the 9s, 10s, jacks, queens, kings, and aces from a standard deck of cards. How many five-card hands have

a) all red cards?

b) at least two red cards?

c) at most two red cards?

10. If you are dealing from a standard deck of 52 cards,

a) how many different 4-card hands could have at least one card from each suit?

b) how many different 5-card hands could have at least one spade?

c) how many different 5-card hands could have at least two face cards (jacks, queens, or kings)?

11. The number 5880 can be factored into prime divisors as $2 \times 2 \times 2 \times 3 \times 5 \times 7 \times 7$.

a) Determine the total number of divisors of 5880.

b) How many of the divisors are even?

c) How many of the divisors are odd?

12. Application A theme park has a variety of rides. There are seven roller coasters, four water rides, and nine story rides. If Stephanie wants to try one of each type of ride, how many different combinations of rides could she choose?

13. Shuwei finds 11 shirts in his size at a clearance sale. How many different purchases could Shuwei make?

14. Communication Using the summary on page 285, draw a flow chart for solving counting problems.

15. **a)** How many different teams of 4 students could be chosen from the 15 students in the grade-12 Mathematics League?

b) How many of the possible teams would include the youngest student in the league?

c) How many of the possible teams would exclude the youngest student?

16. Inquiry/Problem Solving

a) Use combinations to determine how many diagonals there are in

i) a pentagon **ii)** a hexagon

b) Draw sketches to verify your answers in part a).

17. A school is trying to decide on new school colours. The students can choose three colours from gold, black, green, blue, red, and white, but they know that another school has already chosen black, gold, and red. How many different combinations of three colours can the students choose?

18. Application The social convenor has 12 volunteers to work at a school dance. Each dance requires 2 volunteers at the door, 4 volunteers on the floor, and 6 floaters. Joe and Jim have not volunteered before, so the social convenor does not want to assign them to work together. In how many ways can the volunteers be assigned?

Chapter Problem

19. Jeffrey is a DJ at a local radio station. For the second hour of his shift, he must choose all his music from the top 100 songs for the week. Jeffery will play exactly 12 songs during this hour.

a) How many different stacks of discs could Jeffrey pull from the station's collection if he chooses at least 10 songs that are in positions 15 to 40 on the charts?

b) Jeffrey wants to start his second hour with a hard-rock song and finish with a pop classic. How many different play lists can Jeffrey prepare if he has chosen 4 hard rock songs, 5 soul pieces, and 3 pop classics?

c) Jeffrey has 8 favourite songs currently on the top 100 list. How many different subsets of these songs could he choose to play during his shift?

ACHIEVEMENT CHECK

Knowledge/ Understanding	Thinking/Inquiry/ Problem Solving	Communication	Application

20. There are 52 white keys on a piano. The lowest key is A. The keys are designated A, B, C, D, E, F, and G in succession, and then the sequence of letters repeats, ending with a C for the highest key.

a) If five notes are played simultaneously, in how many ways could the notes all be

i) As? **ii)** Gs?

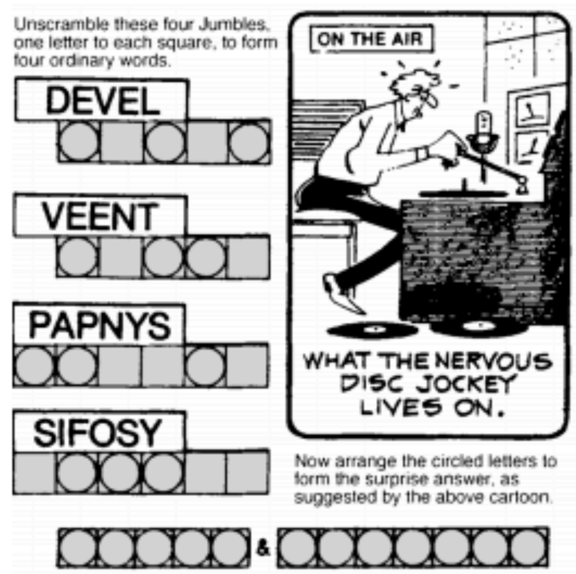
iii) the same letter? **iv)** different letters?

b) If the five keys are played in order, how would your answers in part a) change?

21. Communication

a) How many possible combinations are there for the letters in the three circles for each of the clue words in this puzzle?

b) Explain why you cannot answer part a) with a single ${}_nC_r$ calculation for each word.



22. Determine the number of ways of selecting four letters, without regard for order, from the word *parallelogram*.

C

23. Inquiry/Problem Solving Suppose the artist in Example 1 of this section had two apples, two oranges, and two pears in his refrigerator. How many combinations does he have to choose from if he wants to paint a still-life with

a) two pieces of fruit?

b) three pieces of fruit?

c) four pieces of fruit?

24. How many different sums of money can be formed from one \$2 bill, three \$5 bills, two \$10 bills, and one \$20 bill?

Problem-Solving With Permutations *and* Combinations!

1. Identify whether each of the following problems is a permutation or a combination. Then, solve the problem! ☺

a. How many five-digit numbers can be formed using each of the digits 1, 2, 3, 4, 5?

b. In how may ways can six runners finish a race?

c. Three of five people are chosen to go on a trip. In how many ways can they be chosen?

d. A bridge hand consists of 13 of 52 cards. How many different bridge hands are there?

e. You want to order a medium-sized 3-topping pizza. There are eight toppings available. How many different pizzas can you order if you choose 3 different toppings?

f. How many arrangements of the word *FIELD* are possible?

g. Ten people shake hands with each other. How many handshakes are possible if no pair shakes hands twice?

h. A committee of 4 people is to be chosen from a group of 12. How many committees can be formed?

i. A committee consisting of President, Vice President, Treasurer, and Secretary is to be chosen from a group of 12. How many ways can this be done?

j. In how many ways can ten people be divided into two teams of 5 each?

2. Five cards are dealt from a deck of 52 cards.

a. How many different hands can be dealt?

b. How many hands will contain all face cards?

c. How many hands will contain no face cards?

d. How many hands will contain only spades or clubs?

e. How many hands will contain only hearts?

f. How many will contain at least 1 queen?

g. How many will contain exactly two 4s?

h. How many will contain a jack or a king?

i. How many will contain 2 jacks and 2 aces?

CW: **p. 280 #9; p. 286-288 #5, 6, 8, 9, 10, 12, 15**

Combinations Test Review!

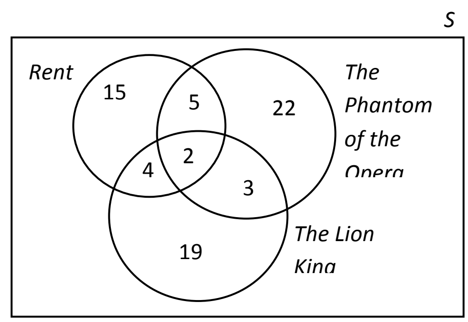
1. A survey of 200 grade 12 students produced the following data:
   - 101 had a computer
   - 73 had a DVD player
   - 98 had a CD player
   - 41 had both a computer and DVD player
   - 33 had both a DVD player and a CD player
   - 21 had both a computer and a CD player
   - 12 had all three items.

   Construct a Venn diagram to illustrate the survey results.

   a) Using your Venn diagram, determine how many people in the survey have none of the three items.
   b) How many had a computer and a DVD player but NOT a CD player?

2. The Venn diagram represents the number of actors who performed in the three shows, *The Lion King*, *Rent*, and *The Phantom of the Opera.*

   a) How many actors participated in the survey?
   b) How many of the actors were in both *Rent* and *The Phantom of the Opera*?
   c) How many of the actors were in only one show?
   d) How many actors were in *Rent* but not in *The Lion King*?
   e) How many of the actors were in all three shows?



3. There are seven teams competing in the Waterloo District Mathematics Challenge. In how many ways can you choose the first, second, and third place team?

4. In how many ways can you form a recycling team consisting of three teachers and nine students if there are seven teachers and 21 students interested in being on the team?

5. There are 15 boys and 19 girls in a room.
   a) How many groups of 9 would contain exactly four girls and five boys?
   b) How many groups of 14 would have an equal number of boys and girls?
   c) How many groups of 5 have more boys than girls? (*Hint*: Consider three cases.)

6. How many different sums of money can be made with a \$1 coin, a \$2 coin, four \$5 bills, and a \$50 bill?

7. When preparing a bouquet, a florist can choose from seven roses, six carnations, and four chrysanthemums. How many bouquets can the florist make if the bouquet must have at least one flower?

8. The school band contains 11 members. In how many ways can a committee of at least one member be formed from the school band?

9. Suppose fifteen people qualify for a college cheerleading squad, six women and nine men.
   a) How many six-member squads can be selected?
   b) Suppose that exactly two members of the six-member squad must be male. How many six-member squads can be selected?

10. Mr. Jones owns 4 pairs of pants, 7 shirts, and 3 sweaters. In how many ways may he choose 2 of the pairs of pants, 3 of the shirts, and 1 of the sweaters to pack for a trip?

11. An art collector, who owns 10 original paintings, is preparing a will. In how many ways may the collector leave these paintings to only three heirs? (*Hint*: Think of each painting as a stage in the decision-making process.)

12. The school board has eight members
   a) The board must have three officers: a chairperson, an assistant chairperson, and a secretary. How many different sets of these officers can be formed from this board?
   b) How many executive three-person committees can be formed from this board?
   c) Is part (a) asking for a number of *permutations* (P) or a number of *combinations* (C)? What about part (b)?

For questions 13-20, determine whether each situation involves a permutation (P) or a combination (C). Then find the number of possibilities.

13. The winner and first, second, and third runners-up in a contest with 10 finalists.

14. Selecting two of eight employees to attend a business seminar.

15. An arrangement of the letters in the word “algebra”.

16. Placing an algebra book, a geometry book, a chemistry book, an English book, and a health book on a shelf.

17. Selecting nine books to check out of the library from a reading list of twelve.

18. Selecting three of fifteen flavors of ice cream at the grocery store.

19. Choosing a hand of five cards from a standard deck of cards consisting of four cards from one suit and one card from another suit. (*Hint*: First choose 1 of the 4 suits and then choose 4 cards from that suit. Next, choose 1 of the remaining 3 suits and 1 card from that suit.)

20. Seating five men and five women alternately in a row, beginning with a woman.

1a) 11	1b) 29	2a) 70	2b) 7	2c) 56	2d) 20
2e) 2	3) 210	4) 10 287 550	5a) 11 639 628	5b) 324 246 780	5c) 106 743
6) 39	7) 279	8) 2047	9a) 5005	9b) 540	10) 630
11) 59 049	12a) 336	12b) 56	12c) (*a*)=P, (*b*)=C	13) P; 720	14) C; 28
15) P; 2520	16) P; 120	17) C; 220	18) C; 455	19) C; 111 540	20) P; 14 400

Textbook Review: p. 296 #1 – 3, 5 – 13; p. 298 #5 – 8

Review of Key Concepts

5.1 Organized Counting With Venn Diagrams

Refer to the Key Concepts on page 270.

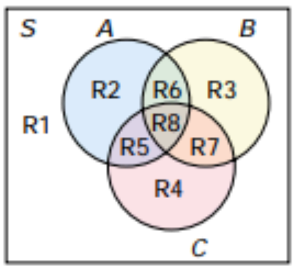
1. Which regions in the diagram below correspond to

a) the union of sets A and B?

b) the intersection of sets B and C?

c) $A \cap C$?

d) either B or S?



2. a) Write the equation for the number of elements contained in either of two sets.

b) Explain why the principle of inclusion and exclusion subtracts the last term in this equation.

c) Give a simple example to illustrate your explanation.

3. A survey of households in a major city found that

- 96% had colour televisions
- 65% had computers
- 51% had dishwashers
- 63% had colour televisions and computers
- 49% had colour televisions and dishwashers
- 31% had computers and dishwashers
- 30% had all three

a) List the categories of households not included in these survey results.

b) Use a Venn diagram to find the proportion of households in each of these categories.

5.2 Combinations

Refer to the Key Concepts on page 278.

4. Evaluate the following and indicate any calculations that could be done manually.

a) ${}_{41}C_8$ **b)** ${}_{33}C_{15}$

c) ${}_{25}C_{17}$ **d)** ${}_{50}C_{10}$

e) ${}_{10}C_8$ **f)** ${}_{15}C_{13}$

g) ${}_{5}C_4$ **h)** ${}_{25}C_{24}$

i) ${}_{15}C_{11}$ **j)** ${}_{25}C_{20}$

k) ${}_{16}C_8$ **l)** ${}_{30}C_{26}$

5. A track and field club has 12 members who are runners and 10 members who specialize in field events. The club has been invited to send a team of 3 runners and 2 field athletes to an out-of-town meet. How many different teams could the club send?

6. A bridge hand consists of 13 cards. How many bridge hands include 5 cards of one suit, 6 cards of a second, and 2 cards of a third?

7. Explain why combination locks should really be called permutation locks.

5.3 Problem Solving With Combinations

Refer to the Key Concepts on page 286.

8. At Subs Galore, you have a choice of lettuce, onions, tomatoes, green peppers, mushrooms, cheese, olives, cucumbers, and hot peppers on your submarine sandwich. How many ways can you "dress" your sandwich?

9. Ballots for municipal elections usually list candidates for several different positions. If a resident can vote for a mayor, two councillors, a school trustee, and a hydro commissioner, how many combinations of positions could the resident choose to mark on the ballot?

10. There are 12 questions on an examination, and each student must answer 8 questions including at least 4 of the first 5 questions. How many different combinations of questions could a student choose to answer?

11. Naomi invites eight friends to a party on short notice, so they may not all be able to come. How many combinations of guests could attend the party?

12. In how many ways could 15 different books be divided equally among 3 people?

13. The camera club has five members, and the mathematics club has eight. There is only one member common to both clubs. In how many ways could a committee of four people be formed with at least one member from each club?

5.4 The Binomial Theorem

Refer to the Key Concepts on page 293.

14. Without expanding $(x + y)^5$, determine
  a) the number of terms in the expansion
  b) the value of k in the term $10x^ky^2$

15. Use Pascal's triangle to expand
  a) $(x + y)^8$
  b) $(4x - y)^6$
  c) $(2x + 5y)^4$
  d) $(7x - 3)^5$

16. Use the binomial theorem to expand
  a) $(x + y)^6$
  b) $(6x - 5y)^4$
  c) $(5x + 2y)^5$
  d) $(3x - 2)^6$

17. Write the first three terms of the expansion of
  a) $(2x + 5y)^7$
  b) $(4x - y)^6$

18. Describe the steps in the binomial expansion of $(2x - 3y)^6$.

19. Find the last term in the binomial expansion of $\left(\frac{1}{x^2} + 2x\right)^5$.

20. Find the middle term in the binomial expansion of $\left(\sqrt{x} + \frac{5}{\sqrt{x}}\right)^8$.

21. In the expansion of $(a + x)^6$, the first three terms are $1 + 3 + 3.75$. Find the values of a and x.

22. Use the binomial theorem to expand and simplify $(y^2 - 2)^6(y^2 + 2)^6$.

23. Write $1024x^{10} - 3840x^8 + 5760x^6 - 4320x^4 + 1620x^2 - 243$ in the form $(a + b)^n$. Explain your steps.

Chapter Test

ACHIEVEMENT CHART				
Category	Knowledge/ Understanding	Thinking/Inquiry/ Problem Solving	Communication	Application
Questions	All	12	6, 12	5, 6, 7, 8, 9

1. Evaluate each of the following. List any calculations that require a calculator.

a) ${}_{25}C_{25}$

b) ${}_{52}C_{1}$

c) ${}_{12}C_{3}$

d) ${}_{40}C_{15}$

2. Rewrite each of the following as a single combination.

a) ${}_{10}C_{7} + {}_{10}C_{8}$

b) ${}_{23}C_{15} - {}_{22}C_{14}$

3. Use Pascal's triangle to expand

a) $(3x - 4)^4$

b) $(2x + 3y)^7$

4. Use the binomial theorem to expand

a) $(8x - 3)^5$

b) $(2x - 5y)^6$

5. A student fundraising committee has 14 members, including 7 from grade 12. In how many ways can a 4-member subcommittee for commencement awards be formed if

a) there are no restrictions?

b) the subcommittee must be all grade-12 students?

c) the subcommittee must have 2 students from grade 12 and 2 from other grades?

d) the subcommittee must have no more than 3 grade-12 students?

6. A track club has 20 members.

a) In how many ways can the club choose 3 members to help officiate at a meet?

b) In how many ways can the club choose a starter, a marshal, and a timer?

c) Should your answers to parts a) and b) be the same? Explain why or why not.

7. Statistics on the grade-12 courses taken by students graduating from a secondary school showed that

- 85 of the graduates had taken a science course
- 75 of the graduates had taken a second language
- 41 of the graduates had taken mathematics
- 43 studied both science and a second language
- 32 studied both science and mathematics
- 27 had studied both a second language and mathematics
- 19 had studied all three subjects

a) Use a Venn diagram to determine the minimum number of students who could be in this graduating class.

b) How many students studied mathematics, but neither science nor a second language?

8. A field-hockey team played seven games and won four of them. There were no ties.

   a) How many arrangements of the four wins and three losses are possible?

   b) In how many of these arrangements would the team have at least two wins in a row?

9. A restaurant offers an all-you-can-eat Chinese buffet with the following items:

   - egg roll, wonton soup
   - chicken wings, chicken balls, beef, pork
   - steamed rice, fried rice, chow mein
   - chop suey, mixed vegetables, salad
   - fruit salad, custard tart, almond cookie

   a) How many different combinations of items could you have?

   b) The restaurant also has a lunch special with your choice of one item from each group. How many choices do you have with this special?

10. In the expansion of $(1 + x)^n$, the first three terms are $1 - 0.9 + 0.36$. Find the values of x and n.

11. Use the binomial theorem to expand and simplify $(4x^2 - 12x + 9)^3$.

12. A small transit bus has 8 window seats and 12 aisle seats. Ten passengers board the bus and select seats at random. How many seating arrangements have all the window seats occupied if which passenger is in a seat

    a) does not matter? b) matters?

ACHIEVEMENT CHECK

Knowledge/Understanding	Thinking/Inquiry/Problem Solving	Communication	Application

13. The students' council is having pizza at their next meeting. There are 20 council members, 6 of whom are vegetarian. A committee of 3 will order six pizzas from a pizza shop that has a special price for large pizzas with up to three toppings. The shop offers ten different toppings.

    a) How many different pizza committees can the council choose if there must be at least one vegetarian and one non-vegetarian on the committee?

    b) In how many ways could the committee choose *exactly* three toppings for a pizza?

    c) In how many ways could the committee choose *up to* three toppings for a pizza?

    d) The committee wants as much variety as possible in the toppings. They decide to order each topping exactly once and to have at least one topping on each pizza. Describe the different cases possible when distributing the toppings in this way.

    e) For one of these cases, determine the number of ways of choosing and distributing the ten toppings.