Organized Counting Principles

A. First of all, what is Combinatorics?

Combinatorics is a branch of mathematics that deals with ideas and methods for *counting*. This seems like a simple task, but it can get very complex! Consider counting how many possible license plates there are in Ontario, or how many 5-card hands are possible with a standard deck of cards!

Often we can use organized counting methods like tree diagrams, lists, charts, and Venn diagrams to help us count the possibilities in a given situation. Other times, these methods can be too time-consuming, so we choose to rely on other counting methods like permutations (unit 1) and combinations (unit2) to perform the calculations.

Permutations allow us to count how many *arrangements* can be made of a certain number of items when <u>order matters</u>. **Combinations** allow us to count how many *selections* can be made of a given number of items when <u>order is irrelevant</u>.

Basically, combinatorics provides us with systems and techniques that we can use to calculate possible outcomes in complex situations. Get ready – because it relies heavily on *logic* and only a few basic *counting principles*!

B. Making Choices at Different Stages

When making a *series* of choices, we want to find out how many different possible "paths" or outcomes there are.

Ex. 1: At breakfast, Bob can have tea, coffee, or orange juice to drink; he can have eggs, cereal, or pancakes as his main dish; and he can have either toast or hash browns as his side dish. Assuming he wants a drink, a main, and a side, how many possible breakfasts could Bob choose from?

We will make a **tree diagram** to visualize the outcomes, then count the number of "leaves" on the tree to find how many possible breakfasts Bob could have.

Ex. 2: Deirdre wishes to purchase a new car. She has the choice of 2-door or 4-door; standard or automatic; red, white or grey exterior. Draw a tree diagram to determine the number of different selections that she can make.

If, to reach a desired outcome, you need to make a *first choice* **and** a *second choice* **and** a *third choice*, **and** *so on*, then you **multiply** the number of choices for each stage to calculate the total number of possible outcomes. This is known as the Fundamental Counting Principle.

Fundamental (or Multiplicative) Counting Principle

If a task or process is made up of stages with separate choices at each stage, the total number of choices is $m \times n \times p \times \ldots$, where m is the number of choices in the first stage, n is the number of choices in the second stage, and p is the number of choices in the third stage, and so on.

Ex. 3: In your first year of university, you get a co-op job for the summer (yay! ☺). This job requires you to dress professionally each day (boo!). You do *no*t have a lot of clothing that suits your purposes. You *do*, however, have the following items in neutral colours (this means they match, right?): 4 different shirts, 3 pairs of dress pants, 5 ties, and 2 pairs of dress shoes.

a) If you must wear a shirt and a tie and pants and shoes (no one's looking at your socks!), how many possible outfits can you create to tide you over until you get your first paycheck?

b) What if the supervisor is a little more flexible and gives you the option of wearing the tie or *not* wearing the tie? Now how many options do you have?

With tie:

Without tie:

Since you can't simultaneously wear a tie while not wearing a tie, these events are considered to be **mutually exclusive** (cannot occur at the same time). When these mutually exclusive events are both desirable, we can **add** their outcomes together to find the number of total possibilities of one **or** the other occurring. This is known as the Additive Counting Principle.

∴ ________ + _________ = __________

Additive Counting Principle (or Rule of Sum)

*If one **mutually exclusive** event can occur in m ways, another in n ways, another in p ways, and so on, then the total number of ways these events can occur is* $m + n + p + \ldots$

Ex. 4: In a standard deck of cards there are 52 cards and 4 suits. How many ways are there to select:

a) a card that is either a red face card or a black ace?

b) a red face card and a black ace, one after the other?

c) a heart or a queen?

Ex. 5: Using the digits 1, 2, 3, and 4, allowing for repetition:

a) How many different 2-digit numbers are possible?

b) How many different 3-digit numbers are possible?

c) How many different 2-digit **or** 3-digit numbers are possible?

Ex. 6: Using all digits from 0 to 9, allowing for repetition:

a) How many different 3-digit numbers are possible?

b) How many 3-digit numbers are possible in which there are no repeated digits?

c) How many 3-digit numbers are possible that contain at least one 5?

CW: p. 229-230 #1, 3, 4, 5ac, 6, 8, 9, 15, 16

Communicate Your Understanding

1. Explain the fundamental counting principle in your own words and give an example of how you could apply it.
2. Are there situations where the fundamental counting principle does not apply? If so, give one example.
3. Can you always use a tree diagram for organized counting? Explain your reasoning.

Practise

A

1. Construct a tree diagram to illustrate the possible contents of a sandwich made from white or brown bread, ham, chicken, or beef, and mustard or mayonnaise. How many different sandwiches are possible?
2. In how many ways can you roll either a sum of 4 or a sum of 11 with a pair of dice?
3. In how many ways can you draw a 6 or a face card from a deck of 52 playing cards?
4. How many ways are there to draw a 10 or a queen from the 24 cards in a euchre deck, which has four 10s and four queens?
5. Use tree diagrams to answer the following:
   a) How many different soccer uniforms are possible if there is a choice of two types of shirts, three types of shorts, and two types of socks?
   b) How many different three-scoop cones can be made from vanilla, chocolate, and strawberry ice cream?
   c) Suppose that a college program has six elective courses, three on English literature and three on the other arts. If the college requires students to take one of the English courses and one of the other arts courses, how many pairs of courses will satisfy these requirements?

Apply, Solve, Communicate

6. Ten different books and four different pens are sitting on a table. One of each is selected. Should you use the rule of sum or the product rule to count the number of possible selections? Explain your reasoning.

B

7. **Application** A grade 9 student may build a timetable by selecting one course for each period, with no duplication of courses. Period 1 must be science, geography, or physical education. Period 2 must be art, music, French, or business. Periods 3 and 4 must each be mathematics or English.
   a) Construct a tree diagram to illustrate the choices for a student's timetable.
   b) How many different timetables could a student choose?
8. A standard die is rolled five times. How many different outcomes are possible?
9. A car manufacturer offers three kinds of upholstery material in five different colours for this year's model. How many upholstery options would a buyer have? Explain your reasoning.
10. **Communication** In how many ways can a student answer a true-false test that has six questions. Explain your reasoning.

11. The final score of a soccer game is 6 to 3. How many different scores were possible at half-time?

12. A large room has a bank of five windows. Each window is either open or closed. How many different arrangements of open and closed windows are there?

13. **Application** A Canadian postal code uses six characters. The first, third, and fifth are letters, while the second, fourth, and sixth are digits. A U.S.A. zip code contains five characters, all digits.

a) How many codes are possible for each country?

b) How many more possible codes does the one country have than the other?

14. When three-digit area codes were introduced in 1947, the first digit had to be a number from 2 to 9 and the middle digit had to be either 1 or 0. How many area codes were possible under this system?

15. Asha builds new homes and offers her customers a choice of brick, aluminium siding, or wood for the exterior, cedar or asphalt shingles for the roof, and radiators or forced-air for the heating system. How many different configurations is Asha offering?

16. **a)** In how many ways could you choose two fives, one after the other, from a deck of cards?

b) In how many ways could you choose a red five and a spade, one after the other?

c) In how many ways could you choose a red five or a spade?

d) In how many ways could you choose a red five or a heart?

e) Explain which counting principles you could apply in parts a) to d).

17. Chapter Problem Ten students have been nominated for a students' council executive. Five of the nominees are from grade 12, three are from grade 11, and the other two are from grades 9 and 10.

a) In how many ways could the nominees fill the positions of president and vice-president if all ten are eligible for these senior positions?

b) How many ways are there to fill these positions if only grade 11 and grade 12 students are eligible?

18. **Communication**

a) How many different licence plates could be made using three numbers followed by three letters?

b) In 1997, Ontario began issuing licence plates with four letters followed by three numbers. How many different plates are possible with this new system?

c) Research the licence plate formats used in the other provinces. Compare and contrast these formats briefly and suggest reasons for any differences between the formats.

19. In how many ways can you arrange the letters of the word *think* so that the *t* and the *h* are separated by at least one other letter?

20. **Application** Before the invention of the telephone, Samuel Morse (1791–1872) developed an efficient system for sending messages as a series of dots and dashes (short or long pulses). International code, a modified version of Morse code, is still widely used.

a) How many different characters can the international code represent with one to four pulses?

b) How many pulses would be necessary to represent the 72 letters of the Cambodian alphabet using a system like Morse code?

ACHIEVEMENT CHECK

Knowledge/ Understanding	Thinking/ Inquiry/ Problem Solving	Communication	Application

21. Ten finalists are competing in a race at the Canada Games.

a) In how many different orders can the competitors finish the race?

b) How many ways could the gold, silver, and bronze medals be awarded?

c) One of the finalists is a friend from your home town. How many of the possible finishes would include your friend winning a medal?

d) How many possible finishes would leave your friend out of the medal standings?

e) Suppose one of the competitors is injured and cannot finish the race. How does that affect your previous answers?

f) How would the competitor's injury affect your friend's chances of winning a medal? Explain your reasoning. What assumptions have you made?

C

22. A locksmith has ten types of blanks for keys. Each blank has five different cutting positions and three different cutting depths at each position, except the first position, which only has two depths. How many different keys are possible with these blanks?

23. **Communication** How many 5-digit numbers are there that include the digit 5 and exclude the digit 8? Explain your solution.

24. **Inquiry/ Problem Solving** Your school is purchasing a new type of combination lock for the student lockers. These locks have 40 positions on their dials and use a three-number combination.

a) How many combinations are possible if consecutive numbers cannot be the same?

b) Are there any assumptions that you have made? Explain.

c) Assuming that the first number must be dialled clockwise from 0, how many different combinations are possible?

d) Suppose the first number can also be dialled counterclockwise from 0. Explain the effect this change has on the number of possible combinations.

e) If you need four numbers to open the lock, how many different combinations are possible?

25. **Inquiry/ Problem Solving** In chess, a knight can move either two squares horizontally plus one vertically or two squares vertically plus one horizontally.

a) If a knight starts from one corner of a standard 8×8 chessboard, how many different squares could it reach after

i) one move?

ii) two moves?

iii) three moves?

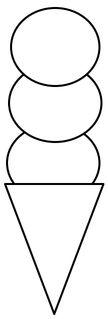
b) Could you use the fundamental counting principle to calculate the answers for part a)? Why or why not?

More Organized Counting!

A. Restrictions

When we count the number of outcomes **without restrictions**, we are able to use *all* of the possible options available, as well as use them *more than once* (unless otherwise stated). Restrictions are placed to limit which options we can choose from and/or to limit the number of times we can use them. In permutations, we may also have restrictions on which items can/cannot be grouped together in an arrangement.

Ex. 1: a) How many different 3-scoop cones can be made from vanilla, chocolate, and strawberry ice cream?



b) How many different cones can be made if each flavor can only be used once (i.e. each scoop is unique)?

c) How many cones can be made where at least 2 scoops are the same flavour?

B. Indirect Methods

In part c) above, we could have used an **indirect method** to complete our calculations. Sometimes it is easier to count the number of outcomes that do NOT meet the conditions set, and then subtract this value from the total number of possible outcomes (without restrictions):

Total # of cones possible (without restrictions) – # of cones where each scoop is unique =
# of cones where at least 2 scoops are the same flavour

∴ ____________ – _____________ = ______________

Ex.2: If we revisit example 6c) from last class, we can use an indirect method here as well: "Using digits from 0 to 9, how many 3-digit numbers are possible that contain at least one 5?"

C. More Advanced Counting Problems

Ex. 3: How many 6-digit even numbers over 300 000 can be formed using the digits 0-5 without repetition?

Ex. 4: Brittany is writing a multiple choice quiz consisting of 6 test questions with 4 possible answers each. How many ways can she complete the quiz (assuming that she answers each question)?

Ex. 5: The score at half-time for a football game is 5-2. How many different scores were possible after the first quarter?

Ex. 6: Student council is forming a committee with three distinct positions: president, vice president, and treasurer. The student council members include four grade 12s, five grade 11s, three grade 10s, and two grade 9s.

a) How many different committees are possible?

b) How many different committees are possible in which there are two senior students and one junior student?

Ex. 7: Eight dogs are competing in the Doggy Olympics.

a) In how many different orders can the 8 dogs finish the race?

b) How many different ways could gold, silver, and bronze collars be awarded?

c) One of the doggies is your Chihuahua, Chuck. How many possible finishes would include Chuck winning a collar?

d) How many possible finishes would leave Chuck out of the "collar" standings?

CW: p. 229-231 #10, 11, 12, 13, 14, 17, 18ab, 19, 23

Factorials and Permutations

A. Factorials!

Many counting and probability calculations involve multiplying a *series* of consecutive *natural numbers*.

Ex. 1: Five friends are posing for a photo you are taking. If they stand next to one another facing you, how many possible arrangements could you have?

What if you had fifteen friends posing for this picture? Or 25 friends? You would probably not be inclined to multiply out the entire product of 25 x 24 x 23 x . . . x 2 x 1! Instead, you could use **factorial notation** to help you calculate this value more easily ☺ :

Factorial Notation

In general, for any natural number n:

n! = n x (n – 1) x (n – 2) x (n – 3) x (n – 4) x . . . x 3 x 2 x 1

(where the expression "n!" is read " n factorial")

Ex.2: Evaluate each factorial expression.

a) 5!

b) 12!

c) (– 2)!

d) $\frac{10!}{5!}$

e) $\frac{83!}{79!}$

Ex.3: Simplify each factorial expression.

a) 5 x 4 x 3!

b) 30 x 4!

B. Permutations

A **permutation** is a distinct *arrangement* of a certain number of items in a definite order. There are *n* possible ways of selecting the first item, *(n – 1)* ways of selecting the second item, *(n – 2)* ways of selecting the third item, *and so on*.

I. Arranging *all n* items

Ex. 4: There are 10 songs on the MGMT album "Oracular Spectacular". If I play the album on shuffle using my iPod, in how many different orders could the 10 songs be played without repetition?

Using the Fundamental Counting Principle, the total number of ways of arranging **all *n* distinct items** is:
$n \times (n-1) \times (n-2) \times (n-3) \times (n-4) \times \ldots \times 3 \times 2 \times 1$. *This can be written as* $\boldsymbol{n!}$ *or* $\boldsymbol{{}_nP_n}$ *or* $\boldsymbol{P(n, n)}$.

Permutations** of **n distinct items** chosen **n at a time** in a **definite order:

$${}_nP_n = P(n, n) = n! = n \times (n-1) \times (n-2) \times (n-3) \times (n-4) \times \ldots \times 3 \times 2 \times 1$$

∴ __________ = __________ = __________= __________

Ex. 5: a) In how many ways can you arrange the letters of the word *trickle*?

b) What if the two vowels must be adjacent?

c) What if the *k* must be first and the *t* must be last?

d) What if the *i* must NOT be in the middle?

II. Arranging *r* of the *n* items

Ex. 6: Someone is trying to crack the combination for your locker. The combination consists of 3 distinct numbers selected from 60 numbers on the face of the lock. How many permutations are possible?

So, what if we don't want to select all (*n)* of the items? We want to arrange a few (*r*) items at a time. We are looking at a scenario where we have *n* items and only *r* positions available to arrange them in. The total number of ways of arranging ***r of the n* items** is: $n \times (n-1) \times (n-2) \times \ldots \times (n-r+1)$. *This can be written as* $_nP_r$ *or* ***P(n, r).***

Permutations** of **n distinct items** chosen **r at a time** in a **definite order:

$$_nP_r = P(n, r) = \frac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \ldots \times (n-r+1)$$

∴ _________ = _________ = _________ = _________= _________ = _________

Ex. 7: a) How many 4-letter arrangements can you make using the letters from the word *potluck*?

b) What if you must include the letter *c*?

c) What if you must NOT include the vowels?

Ex. 8: In how many ways could 10 different questions be arranged on your next quiz if:

a) there are no restrictions?

b) the easiest question must be first?

c) the easiest question must be first and the most difficult question must be last?

d) the easiest question and the most difficult question must be side-by-side?

e) the easiest question and the most difficult question must NOT be side-by-side?

CW: p. 239-240 #1-4, 6abc, 7, 9 – 11, 14

Practise

A

1. Express in factorial notation.

a) $6 \times 5 \times 4 \times 3 \times 2 \times 1$

b) $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

c) $3 \times 2 \times 1$

d) $9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

2. Evaluate.

a) $\frac{7!}{4!}$ **b)** $\frac{11!}{9!}$

c) $\frac{8!}{5!\,2!}$ **d)** $\frac{15!}{3!\,8!}$

e) $\frac{85!}{82!}$ **f)** $\frac{14!}{4!\,5!}$

3. Express in the form ${}_nP_r$.

a) $6 \times 5 \times 4$

b) $9 \times 8 \times 7 \times 6$

c) $20 \times 19 \times 18 \times 17$

d) $101 \times 100 \times 99 \times 98 \times 97$

e) $76 \times 75 \times 74 \times 73 \times 72 \times 71 \times 70$

4. Evaluate without using technology.

a) $P(10, 4)$ **b)** $P(16, 4)$ **c)** ${}_5P_2$

d) ${}_9P_4$ **e)** $7!$

5. Use either a spreadsheet or a graphing or scientific calculator to verify your answers to question 4.

Apply, Solve, Communicate

6. a) How many ways can you arrange the letters in the word *factor*?

b) How many ways can Ismail arrange four different textbooks on the shelf in his locker?

c) How many ways can Laura colour 4 adjacent regions on a map if she has a set of 12 coloured pencils?

B

7. Simplify each of the following in factorial form. Do not evaluate.

a) $12 \times 11 \times 10 \times 9!$

b) $72 \times 7!$

c) $(n + 4)(n + 5)(n + 3)!$

8. Communication Explain how a factorial is an iterative process.

9. Seven children are to line up for a photograph.

a) How many different arrangements are possible?

b) How many arrangements are possible if Brenda is in the middle?

c) How many arrangements are possible if Ahmed is on the far left and Yen is on the far right?

d) How many arrangements are possible if Hanh and Brian must be together?

10. A 12-volume encyclopedia is to be placed on a shelf. How many incorrect arrangements are there?

11. In how many ways can the 12 members of a volleyball team line up, if the captain and assistant captain must remain together?

12. Ten people are to be seated at a rectangular table for dinner. Tanya will sit at the head of the table. Henry must not sit beside either Wilson or Nancy. In how many ways can the people be seated for dinner?

13. Application Joanne prefers classical and pop music. If her friend Charlene has five classical CDs, four country and western CDs, and seven pop CDs, in how many orders can Joanne and Charlene play the CDs Joanne likes?

14. In how many ways can the valedictorian, class poet, and presenter of the class gift be chosen from a class of 20 students?

15. **Application** If you have a standard deck of 52 cards, in how many different ways can you deal out

a) 5 cards?
b) 10 cards?
c) 5 red cards?
d) 4 queens?

16. **Inquiry/ Problem Solving** Suppose you are designing a coding system for data relayed by a satellite. To make transmissions errors easier to detect, each code must have no repeated digits.

a) If you need 60 000 different codes, how many digits long should each code be?
b) How many ten-digit codes can you create if the first three digits must be 1, 3, or 6?

17. Arnold Schoenberg (1874–1951) pioneered serialism, a technique for composing music based on a tone row, a sequence in which each of the 12 tones in an octave is played only once. How many tone rows are possible?

18. Consider the students' council described on page 223 at the beginning of this chapter.

Chapter Problem

a) In how many ways can the secretary, treasurer, social convenor, and fundraising chair be elected if all ten nominees are eligible for any of these positions?
b) In how many ways can the council be chosen if the president and vice-president must be grade 12 students and the grade representatives must represent their current grade level?

19. **Inquiry/ Problem Solving** A student has volunteered to photograph the school's championship basketball team for the yearbook. In order to get the perfect picture, the student plans to photograph the ten players and their coach lined up in every possible order. Determine whether this plan is practical.

ACHIEVEMENT CHECK

Knowledge/ Understanding	Thinking/ Inquiry/ Problem Solving	Communication	Application

20. Wayne has a briefcase with a three-digit combination lock. He can set the combination himself, and his favourite digits are 3, 4, 5, 6, and 7. Each digit can be used at most once.

a) How many permutations of three of these five digits are there?
b) If you think of each permutation as a three-digit number, how many of these numbers would be odd numbers?
c) How many of the three-digit numbers are even numbers and begin with a 4?
d) How many of the three-digit numbers are even numbers and do *not* begin with a 4?
e) Is there a connection among the four answers above? If so, state what it is and why it occurs.

C

21. TI-83 series calculators use the definition $\left(-\frac{1}{2}\right)! = \sqrt{\pi}$. Research the origin of this definition and explain why it is useful for mathematical calculations.

22. **Communication** How many different ways can six people be seated at a round table? Explain your reasoning.

23. What is the highest power of 2 that divides evenly into 100! ?

24. A committee of three teachers are to select the winner from among ten students nominated for special award. The teachers each make a list of their top three choices in order. The lists have only one name in common, and that name has a different rank on each list. In how many ways could the teachers have made their lists?

Permutations With Some Identical Items

So far, permutations have been used to count the number of *distinguishable arrangements* when we choose *r* of *n* **distinct** items. What if some of the items are **not** distinct? What happens to our permutations when some items are **identical**?

Ex. 1: List all permutations of the "word" LOL:

a) if you can use a subscript to *differentiate* between the two Ls.

b) if you can ***not*** use a subscript to differentiate between the two Ls (i.e. they are indistinguishable!)

In how many ways can you arrange 2 distinct items?

Ex.2: How many distinguishable 4-letter arrangements (permutations) are there of each of the following words?

MASE MASS SASS

Permutations With Some Identical Items

*The number of permutations of a set of **n** objects containing **a** identical objects of one type, **b** identical objects of another type, **c** identical objects of a third type, and so on is*

$$\frac{n!}{a!\,b!\,c!\,\ldots}$$

Ex.3: Calculate the number of permutations of the letters in the following words:

a) WATERLOO

b) TORONTO

c) MISSISSAUGA

Ex. 4: Ms. Hughes' parrots played 11 tricks at their last training session. If they hopped 4 times, turned around 2 times, "shook hands" 3 times, and gave 2 kisses, in how many ways could this session have occurred?

Ex. 5: How many permutations of the word TENNESSEE do NOT start with the letter N?

CW: p. 245-246 #1ab, 2-4, 6, 7, 11, 12

Practise

A

1. Identify the indistinguishable items in each situation.
   a) The letters of the word *mathematics* are arranged.
   b) Dina has six notebooks, two green and four white.
   c) The cafeteria prepares 50 chicken sandwiches, 100 hamburgers, and 70 plates of French fries.
   d) Thomas and Richard, identical twins, are sitting with Marianna and Megan.

2. How many permutations are there of all the letters in each name?
   a) Inverary b) Beamsville
   c) Mattawa d) Penetanguishene

3. How many different five-digit numbers can be formed using three 2s and two 5s?

4. How many different six-digit numbers are possible using the following numbers?
   a) 1, 2, 3, 4, 5, 6 b) 1, 1, 1, 2, 3, 4
   c) 1, 3, 3, 4, 4, 5 d) 6, 6, 6, 6, 7, 8

Apply, Solve, Communicate

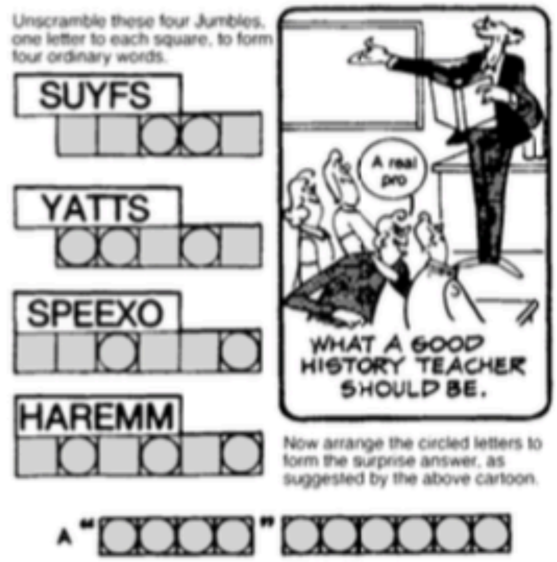
B

5. **Communication** A coin is tossed eight times. In how many different orders could five heads and three tails occur? Explain your reasoning.

6. **Inquiry/Problem Solving** How many 7-digit even numbers less than 3 000 000 can be formed using all the digits 1, 2, 2, 3, 5, 5, 6?

7. Kathryn's soccer team played a good season, finishing with 16 wins, 3 losses, and 1 tie. In how many orders could these results have happened? Explain your reasoning.

8. a) Calculate the number of permutations for each of the jumbled words in this puzzle.
   b) Estimate how long it would take to solve this puzzle by systematically writing out the permutations.



WEB CONNECTION

www.mcgrawhill.ca/links/MDM12

For more word jumbles and other puzzles, visit the above web site and follow the links. Find or generate two puzzles for a classmate to solve.

9. **Application** Roberta is a pilot for a small airline. If she flies to Sudbury three times, Timmins twice, and Thunder Bay five times before returning home, how many different itineraries could she follow? Explain your reasoning.

10. After their training run, six members of a track team split a bag of assorted doughnuts. How many ways can the team share the doughnuts if the bag contains
    a) six different doughnuts?
    b) three each of two varieties?
    c) two each of three varieties?

11. As a project for the photography class, Haseeb wants to create a linear collage of photos of his friends. He creates a template with 20 spaces in a row. If Haseeb has 5 identical photos of each of 4 friends, in how many ways can he make his collage?

12. **Communication** A used car lot has four green flags, three red flags, and two blue flags in a bin. In how many ways can the owner arrange these flags on a wire stretched across the lot? Explain your reasoning.

13. **Application** Malik wants to skateboard over to visit his friend Gord who lives six blocks away. Gord's house is two blocks west and four blocks north of Malik's house. Each time Malik goes over, he likes to take a different route. How many different routes are there for Malik if he only travels west or north?

ACHIEVEMENT CHECK

Knowledge/ Understanding	Thinking/ Inquiry/ Problem Solving	Communication	Application

14. Fran is working on a word puzzle and is looking for four-letter "scrambles" from the clue word *calculate*.
    a) How many of the possible four-letter scrambles contain four different letters?
    b) How many contain two *a*s and one other pair of identical letters?
    c) How many scrambles consist of any two pairs of identical letters?
    d) What possibilities have you not yet taken into account? Find the number of scrambles for each of these cases.
    e) What is the total number of four-letter scrambles taking all cases into account?

Chapter Problem

15. Ten students have been nominated for the positions of secretary, treasurer, social convenor, and fundraising chair. In how many ways can these positions be filled if the Norman twins are running and plan to switch positions on occasion for fun since no one can tell them apart?

16. **Inquiry/ Problem Solving** In how many ways can all the letters of the word *CANADA* be arranged if the consonants must always be in the order in which they occur in the word itself?

C

17. Glen works part time stocking shelves in a grocery store. The manager asks him to make a pyramid display using 72 cans of corn, 36 cans of peas, and 57 cans of carrots. Assume all the cans are the same size and shape. On his break, Glen tries to work out how many different ways he could arrange the cans into a pyramid shape with a triangular base.
    a) Write a formula for the number of different ways Glen could stack the cans in the pyramid.
    b) Estimate how long it will take Glen to calculate this number of permutations by hand.
    c) Use computer software or a calculator to complete the calculation.

18. How many different ways are there of arranging seven green and eight brown bottles in a row, so that exactly one pair of green bottles is side-by-side?

19. In how many ways could a class of 18 students divide into groups of 3 students each?

Unit 1 Review of Key Concepts

- ✓ **Tree diagrams** are useful tools for organized counting with a small number of options.

- ✓ If you can choose from *m* items of one type and *n* items of another, there are $m \times n$ ways to choose one item of each type (**fundamental counting principle**).

- ✓ If you can choose from either *m* items of one type or *n* items of another type (and both events are **mutually exclusive**), then the total number of ways you can choose an item is $m + n$ (**additive counting principle**).

- ✓ Sometimes an **indirect method** provides an easier way to solve a problem.

- ✓ When given **restrictions**, you must consider ***cases***. Set up your cases using one of the restrictions, continue to fill in values based on the other restrictions, then finally assign values to the unrestricted positions.

- ✓ A **factorial** indicates the multiplication of consecutive natural numbers:

$$n! = n(n-1)(n-2)\times\ldots\times 1$$

- ✓ The number of permutations of ***n* distinct items chosen *n* at a time** in a definite order is:

$$_{n}P_{n} = n!$$

- ✓ The number of permutations of ***r* items taken from *n* distinct items** is:

$$_{n}P_{r} = P(n,r) = \frac{n!}{(n-r)!}$$

- ✓ When dealing with permutations of *n* items that include *a* **identical items** of one type, *b* identical items of another type, *c* identical items of another type, *and so on*, you can use the formula:

$$\frac{n!}{a!b!c!\ldots}$$

Note: using this formula implies that **all *n* items** are being arranged. If you only need to arrange a *subset* of the items, you must consider each ***case*** individually.

Permutations Test Review!

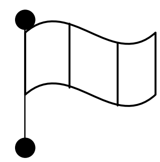
1. A PIN number for a bank card consists of 4-6 digits, with no restrictions. How many such PIN numbers are possible? [Hint: consider 3 cases]

2. How many different 8-digit binary codes are possible? [Binary code is made up of 0s and 1s without restrictions]

3. The Peppler family have a new baby girl. They want to give her a first name after her grandmother, her mother, or her aunt, and a middle name from a 'short list' of 5 names. How many such name combinations are the Pepplers able to select from?

4. How many different 7-digit telephone numbers can a given area code have if the first digit cannot be zero?

5. In how many ways can the flag to the right be coloured if you have 10 colours to choose from, and no two adjacent regions can be the same colour?



6. Below is a "jargon generator". You use it by selecting one word, in order, from each of columns A, B, and C. How many phrases can be constructed using this jargon generator?

Column A	**Column B**	**Column C**
Quasi	Permanent	Behavior
Semi	Affordable	Product
Tentatively	Stoical	Acceptance
Ongoing	Temperate	Wonderment
Differentiated	Aberrant	Attainment
Polymorphous		
inconsequential		

7. Determine the values of *n* and *r* for each of the following to be true:
   a) nPr = 6 x 5 x 4 x 3 x 2 x 1
   b) nPr = 8 x 7 x 6
   c) nPr = 510 x 509 x 508 x 507
   d) nPr = 9 x 8 x 7 x 6 x 5 x 4 x 3
   e) nPr = a(a – 1)(a – 2) (a – 3)(a – 4)

8. Students are asked to solve the quadratic equation $ax^2 + bx + c = 0$ where the coefficients *a, b,* and *c* can be any value of 1, 2, 3, 4, or 5, without repetition. How many different quadratic equations could the students have to solve?

9. You are given the word HEXAGON. How many ways can these letters be arranged if:
   a) there are no restrictions
   b) the vowels must remain together
   c) a vowel must be in the first and second positions, and X must be last
   d) O and N must NOT be together

10. How many four-digit numbers can be made **without repeating any digits** if:
    a) we can only use the digits 1 – 8
    b) we can use the digits 0 – 9
    c) we can only use odd digits
    d) the final number must be odd (using digits 0 – 9 without repetition)
    e) the number must be even and greater than 6000 (using digits 0 – 9 without repetition)

11. Evaluate each of the following without using factorials on your calculator:
    a) $\frac{20!}{18!}$
    b) $\frac{10!}{5!4!}$
    c) 56 x 6!
    d) $\frac{8!}{8 \times 7 \times 6!}$

12. In how many ways can a person answer a 10-question True/False test if they are permitted to leave any question blank?

13. In lottery A you must pick 4 numbers in order out of 40 numbers to win. In lottery B you must pick 3 numbers in order out of 60 numbers to win (repetition is not possible). Which lottery is easier to win?

14. In how many different ways can you stack 2 dimes, 4 nickels, and 2 quarters?

15. A team finishes a 20-game season with a record of 12 wins, 6 losses, and 2 ties. In how many different ways could this have occurred?

16. Invent a permutations problem to correspond with the following answer: $\frac{12!}{4!3!}$

17. How many numbers greater than 3 000 000 can be formed from the digits: 1, 2, 3, 3, 4, 4, 4

18. A coin is tossed 9 times. In how many ways could the results be 6 heads and 3 tails?

19. How many permutations are there of the letters in the word BASKETBALL:
    a) without restrictions
    b) K must be first
    c) the two Ls must be together

20. In which word will you find the greatest number of arrangements of all of its letters: BINGO, AARDVARK, or DEEDED?

1) 1 110 000	2) 256	3) 15	4) 9 000 000	5) 810	6) 175
7 a) *n =6, r =6*	7 b) *n =8, r =3*	7 c) *n =510, r = 4*	7 d) *n = 9, r = 7*	7 e) *n =a, r = 5*	8) 60
9 a) 5040	9 b) 720	9 c) 144	9 d) 3600	10 a) 1680	10 b) 4536
10 c) 120	10 d) 2240	10 e) 1008	11 a) 380	11 b) 1260	11 c) 8!
11 d) 1	12) 59049	13) lottery B	14) 420	15) 3 527 160	16) answers vary
17) 300	18) 84	19 a) 453 600	19 b) 45 360	19 c) 90 720	20) AARDVARK