

# MCF3MI

## Unit 8: Financial Applications

## UNIT 8: FINANCIAL APPLICATIONS – ESSENTIAL LEARNINGS

*You will have at least three opportunities to demonstrate the essential learnings in this course. The first opportunity will occur during the unit with formative quizzes and homework checks. Your second opportunity will be on the unit test. Your third opportunity will be on the exam. You need to demonstrate all essential learnings to earn this credit.*

*Please take the opportunity to keep track of the essential learnings you've demonstrated during this unit using the checklist below.*

<b><i>Essential Learnings: Solving Financial Problems</i></b>	<b><i>Homework</i></b>	<b><i>Associated Lessons</i></b>
<input type="checkbox"/> Solve problems involving simple interest	<i>pg. 352 – 354</i> #1, 2, 3a, 6, 7, 9, 11	Lesson 8.1
<input type="checkbox"/> Solve problems involving compound interest, both future and present, with real world scenarios	<i>pg. 359 – 361</i> #1, 2, 5, 7, 8, 10, 13	Lesson 8.2
	<i>pg. 365 – 366</i> #1ab, 2ab, 3, 5, 7, 9, 10	Lesson 8.3
<input type="checkbox"/> Solve problems involving annuities, including future value, present value, and payments with real world scenarios	<i>pg. 387 – 389</i> #4ab, 6abc, 7ab, 9abc, 10ab	Lesson 8.4
	<i>pg. 395 – 396</i> #4ab, 6, 7abc, 8, 9ab	Lesson 8.5
	<i>pg. 401 – 403</i> #3, 4ab, 6ab, 7ab, 8abc, 10	Lesson 8.6
<b><i>Unit Review:</i></b>	<i>worksheet</i>	

**Investigating Interest Rates – Simple and Compound Interest****A. Percents to Decimals**

The word “percent” means “out of 100”. To convert a percent to a decimal, simply divide the percent by 100, or “move” the decimal place two spaces to the left and remove the percent sign.

**Example:** Convert 64% to a decimal.

$$\begin{aligned} 64\% &= \frac{64}{100} \\ &= 0.64 \end{aligned}$$

or

$$64\% = 0. \overset{2 \text{ places}}{64}$$

**Practice Exercises:** Convert each percent to a decimal.

a) 7%

b)  $6\frac{1}{4}\%$

c) 0.5%

d) 0.008%

**Example:** Find 12% of 150.

$$\begin{aligned} 12\% \times 150 \\ &= 0.12 \times 150 \\ &= 18 \end{aligned}$$

**Recall:** The word “of” means to multiply

**Practice Exercises:** Find each value.

a) 7% of \$250

b) 3.5% of \$127.79

## B. Simple Interest

**Simple Interest** is the money earned (or owed) only on the original amount invested or borrowed. The original amount invested or borrowed is referred to as the **principal amount**, or simply the **principal**.

Let's examine this concept with an example. Suppose you invest \$1000 in a bank that offers you 5% simple interest. What is the ending balance after 5 years?

Let's begin by considering ONE YEAR...

$$5\% = \underline{\hspace{2cm}} \quad \text{and} \quad 1000(\underline{\hspace{2cm}}) = \underline{\hspace{2cm}}$$

Therefore, the **interest**,  $I = \underline{\hspace{2cm}}$  for one year.

Complete the chart below for this simple interest example.

Simple Interest Chart

Year	Starting Balance	Balance that Interest is Calculated On	Interest	Ending Balance
1	\$1000	\$1000	$I = (1000)(.05) = \$50$	\$1050
2	\$1050	\$1000	$I =$	
3				
4				
5				

In general, simple interest is calculated using the formula:  $I = Prt$

where,  $I$  – refers to the **interest earned** (in dollars)

$P$  – refers to the **principal** (or initial investment, in dollars)

$r$  – refers to the **interest rate per year** (as a decimal)

$t$  – refers to the length of **time** the money is invested (in years)

So, in the above example...

$$I = Prt$$

$$I = (\$1000)(0.05)(5)$$

$$I = \$250$$



### C. Compound Interest

**Compound Interest** is the money earned (or owed) on the original amount invested or borrowed as well as any interest that was previously earned or owed during the investment.

Let's examine this concept with an example similar to the one used above. Suppose you invest \$1000 in a bank that offers you 5% interest compounded annually. What is the ending balance after 5 years?

Complete the chart below for this compound interest example.

#### Compound Interest Chart

Year	Starting Balance	Balance that Interest is Calculated On	Interest $I = Prt$	Ending Balance
1	\$1000	\$1000	$I = (1000)(.05)(1) = \$50$	\$1050
2	\$1050	\$1050	$I =$	
3				
4				
5				

Which type of investment would earn you more money, simple or compound interest? **Explain.**

#### Summary of Terms and Concepts

- **Simple Interest:** Money earned (or owed) only on the original amount invested or borrowed.
- **Compound Interest:** Money earned (or owed) on the original amount invested or borrowed as well as any interest that was previously earned or owed during the investment.
- **Principal:** The original amount invested or borrowed.
- **Rate:** The interest rate per year (as a decimal)
- **Term:** The length of time money is invested or borrowed (in years).

#### D. Simple and Compound Interest: Problems

1. You invest \$250 at 4% per annum at a bank that pays **simple** interest.
  - a) How much simple interest would be earned each year?
  - b) If you kept your money invested for 8 years, what is the total simple interest that would be earned?
  - c) How much money would be in your bank account after the 8 years if you did not withdraw any money?

2. If you doubled the principal OR doubled the interest rate from question 1, would it double the total interest paid over 8 years?  
(COMPLETE FOR HOMEWORK)

3. You invest \$750 at 6% per annum at a bank that pays **compound** interest.
  - a) How much compound interest would be earned in the first year?
  - b) How much more compound interest would be earned in the second year?
  - c) If you kept your money invested for 4 years, what is the total compound interest that would be earned? (You may want to complete a chart similar to the one on page 3)
  - d) How much money would be in your bank account after 4 years if you did not withdraw any money?

4. If you doubled the principal OR doubled the interest rate from question 3, would it double the total interest paid over 4 years?  
(COMPLETE FOR HOMEWORK)

1. Calculate the simple interest earned on each investment.
  - a) \$5000 at 8% per year for 2 years
  - b) \$750 invested for 9 months at an annual interest rate of 5.75%
  - c) \$2500 earning 3.7% per year for 7 years
  - d) \$5000 earning 2.85% per year for 30 months
2. Determine the final amount of each investment in question 1.
3. Determine the amount of each compound interest investment.
  - a) \$2500 invested at 3.5% per year for 5 years with interest compounded annually
6. Natalie buys a \$3000 investment certificate that pays 4.8% simple interest per year. How much more would the investment earn in four years if it were invested at 4.8% annual interest, compounded annually?

7. **Use Technology** Consider the table of values.

Year	Value (\$)
0	500
1	525
2	550
3	575
4	600
5	625

- a) Create a scatter plot of the data.
- b) Calculate the first differences of the values.
- c) Determine the equation of a line of best fit.
- d) Describe an investment that might yield this table of values.

9. **Use Technology** Consider the table of values.

Year	Value (\$)
1	10 500.00
2	11 025.00
3	11 576.25
4	12 155.06
5	12 762.82
6	13 400.96
7	14 071.00
8	14 774.55
9	15 513.28
10	16 288.95

- a) Create a scatter plot of the data.
  - b) Determine the equation of a curve of best fit.
  - c) Describe an investment that might yield this table of values.
11. **Use Technology** Tess invests \$1000 in a GIC that pays simple interest at 7% per year. Jason invests \$1000 in a bond that pays compound interest at 7% per year, compounded annually.
    - a) Create a spreadsheet with appropriate headings and enter the formulas to show the amount of each investment over a period of 10 years.
    - b) Graph the data for the year and amount for both investments on the same pair of axes.
    - c) Classify each investment as linear, quadratic, or exponential. Explain how you know.

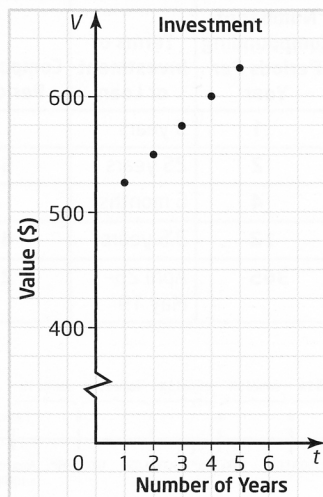
1. a) \$800                      b) \$32.34  
     c) \$647.50                d) \$356.25  
 2. a) \$5800                    b) \$782.34  
     c) \$3147.50                d) \$5356.25

3. a)

Year	Amount at End of Year (\$)
1	2587.50
2	2678.06
3	2771.79
4	2868.80
5	2969.20

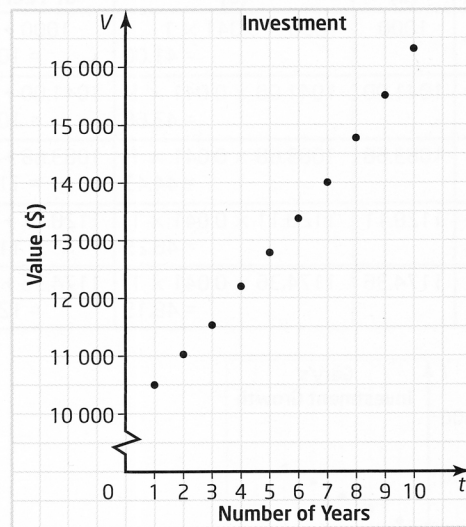
6. \$42.82

7. a)



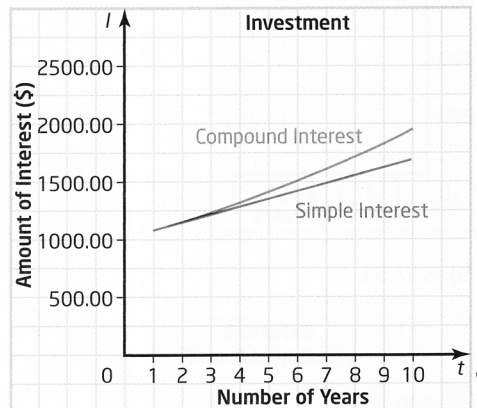
- b) \$25  
 c)  $V = 25t + 500$   
 d) This equation could describe the accumulated value of an investment earning simple interest at a rate of 5% per year to be paid at the end of  $t$  years.

9. a)



- b)  $V = 10\,000(1.05)^x$     c) Answers may vary.

11. b)



- c) The simple interest investment is a linear function because the first differences in the amounts are constant at \$70. The compound interest investment is exponential because the common ratio for each successive amount is 1.07.

## Compound Interest – Future Value

### A. Simple vs. Compound Interest: What's the Difference?

**Simple Interest:** Complete the *difference* column.

Year	Starting Balance	Balance that Interest is Calculated on	Interest	Ending Balance	First Differences
1	\$1000	\$1000	\$50	\$1050	
2	\$1050	\$1000	\$50	\$1100	
3	\$1100	\$1000	\$50	\$1150	
4	\$1150	\$1000	\$50	\$1200	
5	\$1200	\$1000	\$50	\$1250	
6	\$1250	\$1000	\$50	\$1300	
7	\$1300	\$1000	\$50	\$1350	
8	\$1350	\$1000	\$50	\$1400	
9	\$1400	\$1000	\$50	\$1450	

**Compound Interest:** Complete the *y-ratios* column.

Year	Starting Balance	Balance that Interest is Calculated on	Interest	Ending Balance	Y-Ratios
1	\$1000	\$1000	\$50	\$1050	
2	\$1050	\$1050	\$52.50	\$1102.50	
3	\$1102.50	\$1102.50	\$55.13	\$1157.63	
4	\$1157.63	\$1157.63	\$57.88	\$1215.51	
5	\$1215.51	\$1215.51	\$60.78	\$1276.28	
6	\$1276.28	\$1276.28	\$63.81	\$1340.10	
7	\$1340.10	\$1340.10	\$67.00	\$1407.10	
8	\$1407.10	\$1407.10	\$70.36	\$1477.46	
9	\$1477.46	\$1477.46	\$73.87	\$1551.33	

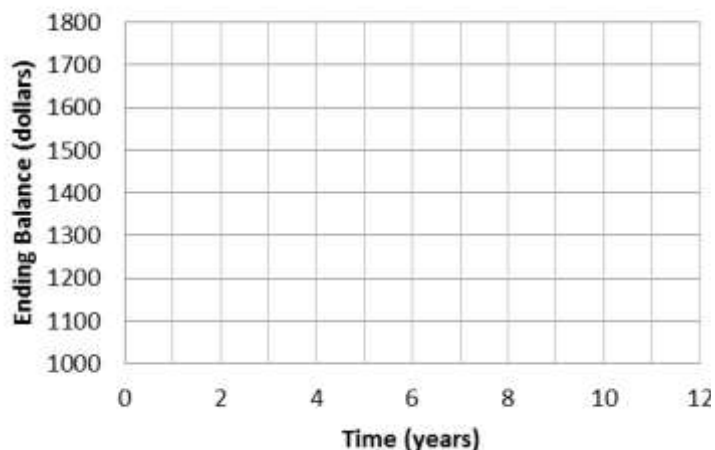
**Summary:**

For **Simple Interest**, the relationship between time and the ending balance is \_\_\_\_\_

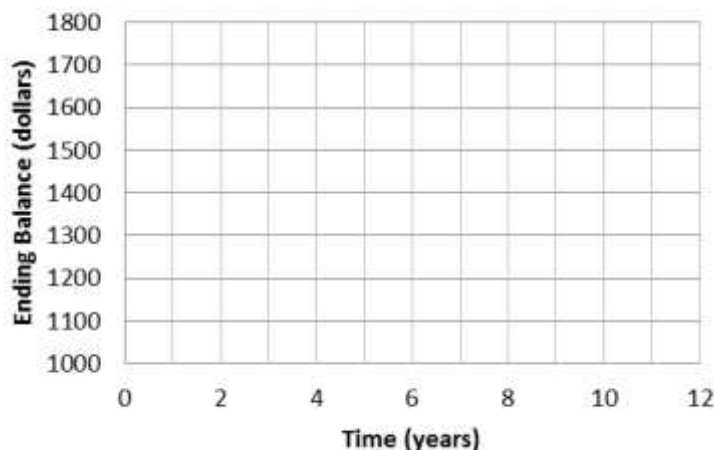
For **Compound Interest**, the relationship between time and the ending balance is \_\_\_\_\_

1. a) **Graph** the relationship between “year” and “ending balance” for each table of data on page 1.

**Simple Interest on \$1000**



**Compound Interest on \$1000**



- b) Use the patterns in both tables to graph the ending balances for each account after 12 years.

2. a) What is the value of the **common (“y”) ratio** for the **Compound Interest** example? \_\_\_\_\_

- b) What seems to be the relationship between the **common ratio** and the **compound interest rate**?

3. If a bank advertises a savings account interest rate of 6% (compounded), what would the **common ratio** be?

4. **Recall:** A general formula for exponential growth is ...  $y = a(b)^x$ , where  **$b$  = the growth factor** and  **$b - 1$  = the growth rate (as a decimal)**.

Write a **specific** formula for the **Compound Interest** example above: \_\_\_\_\_.

Now, use the following variables to **generalize** it...

**$A$**  → final **amount**

**$P$**  → Initial amount (**Principal** amount)

**$i$**  → the **interest rate** as a decimal

**$n$**  → the **number of times** interest is paid.



## B. Compound Interest - Explained

When money is borrowed or invested, the interest is added on to the principal after a previously set period of time. After the same period of time, interest is calculated again. This time, it is calculated on the new amount, which includes the principal and interest. This procedure continues for the entire term of the loan or investment.

### Compound Interest Formulas

$$A = P(1 + i)^n$$

or

$$FV = PV(1 + i)^n$$

$$A = P(1 + i)^n$$

**Amount, A**, remaining after  $n$  compounding periods; also called **Future Value (FV)**

**Principal, P**, or initial amount invested; also called **Present Value (PV)**

**Total number of compounding periods,  $n$** ; calculated by multiplying the number of compounding periods per year,  $N$ , by the number of years,  $y$

$$\therefore n = Ny$$

**Interest rate,  $i$** , per compounding period (as a decimal); calculated by taking the annual interest rate,  $r$ , and dividing by the number of compounding periods per year,  $N$

$$\therefore i = \frac{r}{N}$$

Where  **$N$**  is the number of times interest is paid per year... called the **compounding period**:

if the **period** is ... **annual**  $\Rightarrow N = 1$  (interest paid **once** a year)

if the **period** is ... **semi-annual**  $\Rightarrow N = 2$  (interest paid **twice** a year)

if the **period** is ... **quarterly**  $\Rightarrow N = 4$  (interest paid **4 times** a year)

if the **period** is ... **monthly**  $\Rightarrow N = 12$  (interest paid **12 times** a year)

if the **period** is ... **bi-weekly**  $\Rightarrow N = 26$  (interest paid **26 times** a year)

if the **period** is ... **weekly**  $\Rightarrow N = 52$  (interest paid **52 times** a year)

if the **period** is ... **daily**  $\Rightarrow N = 365$  (interest paid **365 times** a year)

**Ex. 1:** Determine the values of  $P$ ,  $n$ , and  $i$  in the formula  $A = P(1 + i)^n$  for each of the following:

- a) Mary invested \$500 for 5 years at 8%, compounded **semi-annually**.

$$P = \qquad i = \qquad n =$$

- b) Raymond borrowed \$300 for 3 years at 5%, compounded **quarterly**.

$$P = \qquad i = \qquad n =$$

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**Ex. 2:** Renata invested \$500 at 5% compounded **monthly** for 4 years. What will the investment be worth at the end of the 4-year term?

**Solution:**  $A = P(1 + i)^n$

$$A =$$

$$P =$$

$$i =$$

$$n =$$

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**Ex. 3:** Find the future value of a \$100 000 investment made at 6.2% annual interest, compounded semi-annually for 3 years.

**Solution:**  $FV = PV(1 + i)^n$

$$FV =$$

$$PV =$$

$$i =$$

$$n =$$



1. Copy and complete the table.

Annual Interest Rate	Compounding Frequency	Number of Compounding Periods Per Year, $n$	Interest Rate Per Compounding Period, $i$ (expressed as a decimal)
a) 5%	annually		
b) 6.25%	semi-annually		
c) 0.9%	quarterly		
d) 9.5%	monthly		
e) 28.8%	daily		

2. Copy and complete the table.

Compounding Frequency	Number of Compounding Periods Per Year	Term of Investment or Loan	Compounding Periods, $n$
a) annually		3 years	
b) semi-annually		25 years	
c) quarterly		6 months	
d) monthly		3.5 years	
e) daily		April 25 to May 19, inclusive	

5. Determine the amount if \$2000 is invested at 5% per year, for 2 years at each compounding frequency.
  - a) annually
  - b) semi-annually
  - c) quarterly
  - d) monthly
7. Use the compound interest formula to calculate each final amount.
  - a) \$500 invested at 3.85% per year, compounded semi-annually for 4 years
  - b) \$1200 loan at 4.9% per year, compounded monthly for 2.5 years
  - c) \$18 000 investment at 3.25% per year, compounded daily from September 4th to October 22nd, inclusive
8. To purchase a car, Jeremy borrows \$2900 from his parents at 3.2% interest per year, compounded semi-annually. The loan must be paid in full at the end of 2 years.
  - a) How much money will Jeremy need to repay his parents?
  - b) How much interest, in dollars, will Jeremy pay?
  - c) The value of the vehicle depreciates by 20% each year. What will be the value of the car when Jeremy has paid off his loan?
10. Refer to the previous question. Goh recently purchased a laptop computer with a before-tax price of \$1199. Goh paid for the computer with a credit card. Interest is 28.8% per year and is compounded daily on an overdue balance.
  - a) Calculate the after-tax cost of the computer if PST is 8% and GST is 6%.
  - b) How much interest will be charged to Goh's credit card if the payment is 30 days overdue from the date of sale?
  - c) Explain why Goh should withdraw money from his savings account and pay for the computer on or before the payment due date.
13. **Chapter Problem** When Suzanne was born, her parents set up a fund to help pay for her post-secondary education. Calculate the value of the following investments on Suzanne's 17th birthday.
  - a) On the day she was born, Suzanne's parents invested \$100 that has averaged 8% growth per year with interest compounded annually.
  - b) On Suzanne's 1st birthday, her parents invested another \$100 into the same fund as part a).
  - c) On Suzanne's 2nd birthday, her parents invested \$500 into a fund that has averaged a 6.9% return per year, with interest compounded quarterly.
  - d) When Suzanne turned 9, she inherited \$5000 from a family member. This money was invested in a fund that has monthly compounding and has paid a 7.5% annual rate of return.
  - e) Determine the current total amount of the investments.

1.

Annual Interest Rate	Compounding Frequency	Number of Compounding Periods Per Year	Interest Rate Per Compounding Period, $i$
a) 5%	annually	1	0.05
b) 6.25%	semi-annually	2	0.031 25
c) 0.9%	quarterly	4	0.002 25
d) 9.5%	monthly	12	0.007 916 7
e) 28.8%	daily	365	0.000 789 04

2.

Compounding Frequency	Number of Compounding Periods Per Year	Terms of Investment or Loan	Compounding Periods, $n$
a) annually	1	3 years	3
b) semi-annually	2	25 years	50
c) quarterly	4	6 months	2
d) monthly	12	3.5 years	42
e) daily	365	April 25-May 19	25

5. a) \$2205

b) \$2207.63

c) \$2208.97

d) \$2209.88

7. a) \$582.39

b) \$1356.04

c) \$18 078.70

8. a) \$3090.10

b) \$190.10

c) \$1856.00

10. a) \$1366.86

b) \$32.73

c) Goh is earning about \$0.01 on his savings and is paying over \$1.00 per day in interest if he does not pay the bill on time. If he were to make minimum payments on his credit card this would result in additional costs of over \$300. During this time he will have earned less than \$2.00 on the portion of his savings that would have been needed to pay off his credit card balance.

13. a) \$370.00

b) \$342.59

c) \$1395.19

d) \$9093.60

e) \$11 201.38

## Compound Interest – Present Value

### Future Value vs. Present Value: What's the Difference?

**Present value** refers to the amount of money needed to invest **today** (the present) so that you will obtain a particular amount in the **future**. In other words, if you know the amount of money you *want* to have in the future, how much principal should you invest *today*?

In the **compound interest formula**,  $A = P(1 + i)^n$ ,  $P$  represents the starting (principal) amount. If we rearrange this formula to isolate  $P$  we obtain the **Present Value formula**...

$$\begin{array}{lcl}
 A = P(1 + i)^n & & \\
 \frac{A}{(1 + i)^n} = P & \nearrow & P = A(1 + i)^{-n} \\
 A(1 + i)^{-n} = P & & \text{or} \\
 & & PV = FV(1 + i)^{-n}
 \end{array}$$

...where  $PV$  stands for **Present Value**

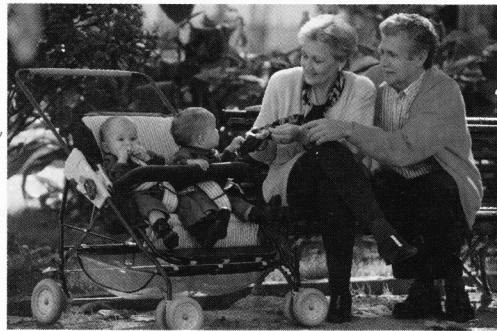
**Ex. 1:** How much would you need to invest **today** into an account that pays 3.6% annual interest compounded **quarterly** if you wanted to have \$3000 in 5 years?

**Ex. 2:** An investment yields an average rate of 9.5% per year. How much would you need to invest so that you are a millionaire in 25 years if the interest is compounded **monthly**?

How much **interest** would you earn?

1. Evaluate. Round each answer to two decimal places.
  - a)  $5000(1.0125)^{-8}$
  - b)  $26\,500(1.04)^{-10}$
2. Calculate the present value of each amount.
  - a) \$500 is required in 2 years. The money can be invested at 6% per year, compounded monthly.
  - b) \$1000 is required in 6 months. The money can be invested at 4.35% per year, compounded semi-annually.
3. Yolanda needs \$18 000 in 4 years to buy a new car.
  - a) How much must she invest now in a GIC paying 5% per year, compounded annually, to have the required amount?
  - b) If the GIC in part a) has interest compounded monthly, how much less will Yolanda need to invest now?
5. Fodil wants to have \$7000 in 3 years to buy a motorcycle. His bank offers a 3-year GIC paying an annual interest rate of 4.32%, compounded annually. How much would Fodil need to invest today in order to have enough money to buy a motorcycle?
7. Michael wants to have \$1000 in 1 year to go on a vacation. His savings account pays an annual interest rate of 3.25%, compounded daily. How much money would Michael need to have in the account today?

9. Rina is 25 years old and has been working full time and saving for 2 years. She wishes to have \$1 million upon retirement at age 65.
  - a) How much would she need to invest today if she could earn an average interest rate of 8% per year, compounded annually?
  - b) How much interest would the investment from part a) earn?
10. The grandparents of twins want to set up an education fund so that each twin has \$30 000 upon her 19th birthday.



How much do the grandparents need to invest when the twins are born if the investment can earn 6.5% interest per year, compounded monthly?

- |                          |                        |
|--------------------------|------------------------|
| <b>1. a)</b> 4526.99     | <b>b)</b> 17 902.45    |
| <b>c)</b> 1757.65        | <b>d)</b> 605.01       |
| <b>2. a)</b> \$443.59    | <b>b)</b> \$978.71     |
| <b>c)</b> \$770.69       | <b>d)</b> \$2458.73    |
| <b>3. a)</b> \$14 808.64 | <b>b)</b> \$65.36      |
| <b>4. a)</b> \$198.51    | <b>b)</b> \$429.88     |
| <b>c)</b> \$1813.86      | <b>d)</b> \$4636.14    |
| <b>5.</b> \$6165.88      |                        |
| <b>6.</b> \$2354.76      |                        |
| <b>7.</b> \$968.02       |                        |
| <b>8.</b> \$4574.22      |                        |
| <b>9. a)</b> \$46 030.93 | <b>b)</b> \$953 969.07 |
| <b>10.</b> \$17 508.34   |                        |

## Ordinary Annuities – Future Value

### A. Ordinary Annuities

So far in this unit, we have considered two basic types of investments: those that gather simple interest and those that gather compound interest. In both cases, the investor invests (or borrows) a **single amount** at the beginning, which then has the various applications of interest applied to it.

There are other investment options, however, and it's time to consider a third style of investment: annuities! An **ordinary annuity** is a series of equal payments earning compound interest and made at regular intervals over a fixed period of time. As with most investment types, annuities are often calculated to find future values.

Without further ado, here's the formula for calculating the future value of ordinary annuities:

#### Ordinary Annuities Formula

$$A = \frac{PMT[(1+i)^n - 1]}{i}$$

**Amount, A**, remaining after  $n$  compounding periods; also called **Future Value (FV)**

**Payment, PMT**, is the regular amount invested at each interval

**Total number of regular payments,  $n$** ; calculated by multiplying the number of regular payments made per year by the number of years in the term

**Interest rate,  $i$** , per compounding period (as a decimal); calculated by taking the annual interest rate,  $r$ , and dividing by the number of compounding periods per year,  $N$

$$\therefore i = \frac{r}{N}$$

## **B. Practice Questions for Future Value of Ordinary Annuities**

**Ex. 1:** Carli wants to start saving for a down payment on a condo. She plans to deposit \$1000 per month into an account that pays 3% annual interest, compounded monthly. Determine the amount of money in Carli's account in 2 years.

**Ex. 2:** Bert makes deposits of \$2000 semi-annually into an account that pays 4% interest, compounded semi-annually. How much money will be in the account after a 5-year term?

How much interest will Bert have earned over the 5-year term?

**Ex. 3:** Tim is saving to buy a \$20 000 car in 3 years. He deposits \$550 every month into an account earning interest at 6%, compounded monthly. Will Tim have enough money to buy the car?

4. a) Determine the variables PMT,  $i$ , and  $n$  for each of the annuities in question 1.

- b) Use the future value of an annuity formula to calculate the future value of each annuity in question 1.

6. Bronwyn is in grade 11 and has worked at a restaurant for just over a year. At the end of each month, she deposited \$400 into an account that paid 3% interest per year, compounded monthly.

- a) How much is in Bronwyn's account after 1 year?
- b) If she continues, will the amount in her account after 2 years be double the answer to part a)? Without using a calculator, explain why or why not.
- c) Calculate the amount that will be in Bronwyn's account after 2 years.

7. Jon is 18 and earned over \$15 000 last year. Financial advisors convinced him to invest. Jon bought \$3000 in shares, which, after management fees were paid, averaged 13.5% net growth per year, with interest compounded annually.



- a) Assuming the rate of growth remains the same, determine the future value of Jon's \$3000 investment after one year. (Note that past performance does not necessarily indicate future rates of growth.)
- b) If Jon continues to invest \$3000 annually, calculate the total value of his investments when he makes his 5th investment, assuming the same growth rate.

9. **Chapter Problem Use Technology** When their first child was a month old, and at the end of every month thereafter, Marty and Anya invested a total of \$100 into an RESP (Registered Education Savings Plan) that paid 7.5% annual interest, compounded monthly.

- a) Use a TVM Solver or an on-line calculator to determine the amount in the fund on the child's 6th birthday, 12th birthday, and 18th birthday.
- b) By the time their daughter turned 18, how much of their own money had Marty and Anya invested?
- c) How much interest did their RESP earn?

### Achievement Check

10. **Use Technology** Mychal is 40 and has no savings. When he was 20, his twin sister Krystyne began investing \$250 per month into funds that have averaged 8% interest each year, compounded monthly. Mychal believes that he can catch up to his sister by age 60 by investing twice as much per month to make up for the last 20 years.

- a) Calculate the value of each twin's investments at age 60, assuming all of the investments earn the same interest rate.
- b) Why will doubling the investment amount not allow for Mychal's savings to catch up by age 60?



4. a) i)  $PMT = 500, i = 0.04, n = 6$   
 ii)  $PMT = 250, i = 0.002, n = 12$   
 iii)  $PMT = 2000, i = 0.07, n = 6$   
 iv)  $PMT = 1500, i = 0.0105, n = 8$   
 b) i) \$3316.49      ii) \$3033.22  
 iii) \$14 306.58      iv) \$12 450.38
6. a) \$4866.55  
 b) No, it will be more than double because her savings from the first year will be accumulating interest.  
 c) \$9881.13
7. a) \$3405      b) \$19 634.65
9. a) 6th birthday: \$9057.88;  
 12th birthday: \$23 243.58;  
 18th birthday: \$45 460.06  
 b) \$21 600  
 c) \$23 860.06

## Ordinary Annuities – Present Value

### A. Ordinary Annuities – Future Value Vs. Present Value

Recall: An **ordinary annuity** is a series of equal payments earning compound interest and made at regular intervals over a fixed period of time.

When working with annuities, we are often looking at the result of a financial situation in the **future** (“How much money will I have in the future if ...”). Sometimes, however, it will be useful to calculate the amount of money required to invest in the **present**, so that you can adequately provide a series of regular payments over a given period of time. This is a part of smart financial planning and spending.

Let’s look at the formula for calculating the present value of ordinary annuities:

### Ordinary Annuities Formula – Present Value

$$P = \frac{PMT [1 - (1 + i)^{-n}]}{i}$$

**Amount,  $P$ ,** required for investment over  $n$  compounding periods; also called **Present Value (PV)**

**Payment,  $PMT$ ,** is the regular amount withdrawn at each interval

**Total number of regular payments,  $n$ ;** calculated by multiplying the number of regular payments made per year by the number of years in the term

**Interest rate,  $i$ ,** per compounding period (as a decimal); calculated by taking the annual interest rate,  $r$ , and dividing by the number of compounding periods per year,  $N$

$$\therefore i = \frac{r}{N}$$

## B. Practice Questions for Present Value of Ordinary Annuities

- Ex. 1:** Bill sets up a bank account for his long-lost cousin. The cousin can make monthly withdrawals of \$550 dollars from the account, which earns interest at 6.5%, compounded quarterly. If the cousin needs this money for the next 4 years while in college, how much money does Bill have to put in the account to start it?
- Ex. 2:** Shirley has taken a loan to pay for her first car. To repay the loan, her bank is charging her \$327.94 per month for 1 year with interest at 9% per year, compounded monthly. What is the actual cost of the car when Shirley purchased it?
- How much interest will Shirley be paying for the ability to make monthly car payments for 1 year rather than paying the total cost up front?
- Ex. 3:** KCI has budgeted for renovation expenses of \$8 000 every 6 months for the next 8 years. A fund is being set up to finance these renovations. Interest earned on the fund is 5.5%, compounded semi-annually. How much money must be in the fund today so that the school can receive its regular semi-annual renovation income?

4. a) Determine the values of the variables PMT,  $i$ , and  $n$  for each of the annuities in question 1.
- b) Use the present value of an annuity formula to calculate the present values in question 1.
6. Determine the lump-sum amount needed to generate a retirement income of \$30 000 per year for 20 years, assuming interest at 5% per year, compounded annually.
7. A furniture retailer offers a payment plan at 8.8% annual interest, compounded monthly. Customers can pay for a living room set by monthly payments of \$288.15 for 2 years.
- a) Determine the total cost for a customer who accepts the payment plan.
- b) Determine the cost of the furniture for a customer paying cash at the time of purchase.
- c) How much total interest would be paid by a customer who accepts the payment plan?
8. A local business has created a scholarship, starting next year. The scholarship will give \$2000 to one eligible high school student for each of the following 10 years. If the business can invest money at 12% per year, compounded annually, how much money must it invest today in order to provide the scholarship fund?
9. Marko is 28 and has 2 years' worth of payments remaining on his student loan. He pays \$367 per month.
- a) If Marko's bank charges 7.5% interest per year, compounded monthly, determine the present value of the remainder of the loan.
- b) If the bank is willing to accept early payment, how much interest would Marko save by paying back the rest of the loan today?

4. a) i)  $\text{PMT} = 3000, i = 0.08, n = 6$   
ii)  $\text{PMT} = 399, i = 0.024, n = 9$   
iii)  $\text{PMT} = 2000, i = 0.0361, n = 8$   
iv)  $\text{PMT} = 750, i = 0.0225, n = 8$   
b) i) \$13 868.64      ii) \$3195.45  
iii) \$13 685.01      iv) \$5435.39
5. a) \$8935.66  
b) \$23 475.87  
c) \$45 149.13
6. \$373 866.31
7. a) \$6915.60  
b) \$6320.04  
c) \$595.56
8. \$11 300.45
9. a) \$8155.63      b) \$652.37

Ordinary Annuities – Payments

A. Considering Payments of Ordinary Annuities

Inevitably, you will reach a point in your life where you will want to purchase something that you won’t have enough money to buy outright. At those times, most people will take out a loan. Whenever people take out loans to pay for larger or more expensive items, however, there are important factors to consider regarding the financial arrangements. What are some of the factors to consider?

Making payments on a big-ticket item almost always results in the consumer **paying more** than the original price of the big-ticket item. This is not always a bad thing, however. What considerations would impact whether paying more for a big-ticket item is a good or a bad thing?

With a range of different factors to consider when taking out a loan, it’s helpful to know how to calculate the payments required. Let’s look at the formulas for calculating payments in both future value and present value scenarios.

<i>Calculating Annuities Payments (PV)</i>	<i>Calculating Annuities Payments (FV)</i>

B. Practice Questions for Payments of Ordinary Annuities

**Ex. 1:** Ryan wants to get together \$40 000 to purchase the first house in his housing empire. He does the math, and he begins making regular quarterly deposits into a savings account that earns 8.5%, compounded quarterly, for 5 years. What must his regular deposits be in order to achieve his goal of saving \$40 000 in 5 years?

**Ex. 2:** Kelly purchased her first new car last week for \$20 406, taxes included. The loan representative at Kelly's bank quoted her a monthly payment of \$423.59 for 5 years. How much will Kelly have paid for the car in 5 years' time?

How much interest did Kelly end up paying?

Calculate the approximate value of the car after 5 years, if it depreciates at a rate of 20% per year.

**Ex. 3:** Beth has a \$12 000 student loan that she needs to repay. Payments are made monthly for the next 2 years, with interest calculated at 9% per year, compounded monthly. How much are Beth's monthly payments?

**Ex. 4:** Ken and Barbie recently bought their dream home for \$289 000. They were quoted a payment of \$429.34 per week, based on an amortization period of 25 years. How much do Ken and Barbie pay for the house, assuming that the payment remains the same for the duration of the mortgage?

How much interest did Ken and Barbie end up paying?

Calculate the approximate value of the house after 25 years, if it appreciates at a rate of 5% per year.

3. Substitute the values for  $PV$ ,  $i$ , and  $n$  from each part of question 1 into the formula  $PMT = PV \left[ \frac{i}{1 - (1 + i)^{-n}} \right]$  to solve for the payment.

4. a) Rearrange the future value of an ordinary simple annuity formula  $FV = PMT \left[ \frac{(1 + i)^n - 1}{i} \right]$  to solve for the payment.
- b) Use the formula determined in part a) to calculate the annual payment needed to generate \$3000 in 4 years if interest is earned at 5% per year, compounded annually.

6. Calculate the total amount of interest paid on each of the following loans.

- a) monthly payments of \$149 for 36 months on a new home-theatre system with a pre-tax price of \$2999 (taxes paid at time of purchase)
- b) weekly payments of \$212.47 for 25 years on a \$149 900 home mortgage

7. a) Determine the value of the home-theatre system in question 6, part a) at the time the loan is fully repaid, assuming the system depreciates 40% per year.
- b) Determine the value of the home in question 6, part b) after 25 years if it appreciates by 3% per year.

8. Mori has 2 years to repay a \$4000 personal loan at 8.5% per year, compounded monthly.

- a) Calculate her monthly payment using the appropriate formula.
- b) Calculate the total amount Mori ends up paying.
- c) Calculate the amount of interest Mori will pay over the life of the loan.

10. A couple with two small children is expecting a third child. They are deciding between two new minivans. One has a pre-tax cost of \$28 000 and is financed at 0.9% per year, compounded monthly, for 4 years. The other has a pre-tax cost of \$24 500 with financing available at a bank for 7% per year, also compounded monthly for 4 years. Taxes will be paid at the time of purchase in either case.

- a) Determine the monthly payment for each minivan.
- b) Determine the total cost of each loan. If a loan is used to pay for the van, which one ends up costing less?
- c) Determine the total interest that would be paid for each loan.
- d) What factors other than cost should influence the couple's decision?





4. a)  $PMT = FV \left[ \frac{i}{(1 + i)^n - 1} \right]$   
 b) \$696.04
5. a) \$11 856                      b) \$5982  
 c) \$21 960                      d) \$276 211
6. a) \$2365                      b) \$126 311  
 c) \$1240                      d) \$389
7. a) \$647.78              b) \$313 857.31              c) \$87.44
8. a) \$181.82              b) \$4363.74              c) \$363.74
9. a) \$137.05                      b) \$289.30
10. a) first van: \$594.12  
 second van: \$586.68  
 b) first van: \$28 517.52  
 second van: \$28 160.78  
 The second van costs slightly less.  
 c) first van: \$517.52  
 second van: \$3660.78  
 d) Answers may vary; The couple should consider safety ratings, fuel consumption, predicted reliability, depreciation rates, performance, and comparative features.

**Compound Interest – Future Value**

1. Sarah receives a gift of \$5000 from her great aunt, which she wants to invest. She puts the money in an account at 6.5% interest, compounded quarterly, for 3 years. How much money does Sarah have after 3 years?
2. A king needs to borrow some money to finish his moat. He borrows \$2500, at 8.25% interest, compounded semi-annually, for 2 years. How much money does the king need to pay back at the end of the loan?
3. Will finds \$200 on the street. He puts the money in the bank and vows not to touch for 20 years. He invests his \$200 at 3.75%, compounded monthly. How much money does Will have after 20 years have passed?
4. Jason sells his hockey cards for \$1200, so he can have enough money to make a down payment on a used car. The down payment will cost \$1500 and the used car lot will hold the car for 2 years for Jason. If Jason invests at 10.4%, compounded bi-weekly, for 2 years, will Jason have enough money for the down payment?

**Compound Interest – Present Value**

1. Melina wants to take a trip to Vancouver for graduation. She will need \$5000 for tickets, accommodations, and entertainment. How much does she have to invest today, at 9% interest, compounded monthly, for 2 years in order to have enough money for her trip?
2. Mr. and Mrs. Moneybags want their granddaughter to have \$24 000 dollars on her 16<sup>th</sup> birthday. How much money do they need to invest today at 5% interest, compounded semi-annually, to provide this gift to their granddaughter on her sweet sixteen?
3. Amanda needs to start repaying her student loan of \$9000 in 8 years. The bank is willing to accept full payment now, discounted at an interest rate of 7.5%, compounded monthly. How much is bank willing to accept today?
4. Todd wants to buy a condo for \$350 000, whose construction will be finished in 4 years. In 4 years, he will need to make a down payment of 10% of the condo's price. How much money does Todd have to invest today, at an interest rate of 5.2%, and compounded weekly, in order to afford the down payment in 4 years?

**Ordinary Annuities – Future Value**

1. Lauren is saving money from what she earns at her part-time job. She deposits \$115 at the end of each month into a savings account that pays interest at 3%, compounded monthly. What will her savings be at the end of 2 years?
2. Ben makes deposits of \$3000 semi-annually into an account that pays 4.6% interest, compounded semi-annually. How much money will be in the account after a 6-year term?
3. Jon is saving to buy a \$50 000 sports car in 5 years. He deposits \$650 every month into an account earning interest at 6%, compounded monthly. Will he have enough money to buy the sports car?

4. How much should Jon have been depositing monthly in the same conditions in order to buy the sports car?

### **Ordinary Annuities – Present Value**

1. After college, Tom has to start repaying his student loan. His monthly loan payments of \$260 will be withdrawn from an account earning 5.25% interest, compounded monthly. How much must he deposit in the account today so that the loan payments can be withdrawn for 1 year?
2. Gareth has purchased a new super computer. The store has offered him a payment plan of \$250 payments made at the end of every 3 months, for 3 years. The plan is really a loan that involves interest calculated at 9%, compounded quarterly. What is the actual cost of the super computer if Gareth pays for it now?
3. The Grand River Hospital has budgeted for renovation expenses of \$8 000 every 6 months for the next 8 years. A fund is being set up to finance these renovations. Interest earned on the fund is 5.5%, compounded semi-annually. How much money must be in the fund today so that the hospital can receive its regular semi-annual renovation income?
4. Mr. Smith helps his sister set up a bank account for her mother-in-law. The mother-in-law will make monthly withdrawals of \$550 dollars from the account, which earns interest at 6.5%, compounded quarterly. If her mother-in-law needs this money for 4 years while she's travelling Europe, how much money does Mr. Smith's sister need to put in the account to start it?

### **Ordinary Annuities – Payments**

1. Zak plans to begin making regular quarterly deposits into a savings account that earns 7.25%, compounded quarterly, for 4 years. What must his regular deposits be in order to achieve his goal of saving \$4000 in 4 years?
2. Nick's small business borrowed \$15 000 for start-up and must begin to repay it today. The payments are to be made at the end of every 6 months over a 6-year term. Interest on the loan is 9.75%, compounded semi-annually. What will the size of each regular payment be?
3. Taylor's bank account balance is \$6200. The account earns interest at 3.5%, compounded semi-annually. What is the maximum amount she could withdraw every 6 months for 3 years?
4. Beth wants to get together \$40 000 to purchase a double-decker bus for country tours. She does the math, and she begins making regular quarterly deposits into a savings account that earns 8.5%, compounded quarterly, for 5 years. What must her regular deposits be in order to achieve her goal of buying the bus in 5 years?

**Answers:**

Compound Interest – Future Value

- |              |              |             |                   |
|--------------|--------------|-------------|-------------------|
| 1) \$6067.04 | 2) \$2938.73 | 3) \$422.91 | 4) \$1476.84 (no) |
|--------------|--------------|-------------|-------------------|

Compound Interest – Present Value

- |              |                |              |                |
|--------------|----------------|--------------|----------------|
| 1) \$4179.16 | 2) \$10 890.49 | 3) \$4948.54 | 4) \$28 430.20 |
|--------------|----------------|--------------|----------------|

Annuities – Future Value

- |              |                |                |             |
|--------------|----------------|----------------|-------------|
| 1) \$2840.82 | 2) \$40 921.89 | 3) \$45 350.52 | 4) \$716.64 |
|--------------|----------------|----------------|-------------|

Annuities – Present Value

- |              |              |                 |                |
|--------------|--------------|-----------------|----------------|
| 1) \$3033.06 | 2) \$2603.69 | 3) \$102 436.59 | 4) \$18 233.29 |
|--------------|--------------|-----------------|----------------|

Annuities – Payments

- |             |              |              |              |
|-------------|--------------|--------------|--------------|
| 1) \$217.72 | 2) \$1680.47 | 3) \$1097.54 | 4) \$1625.88 |
|-------------|--------------|--------------|--------------|