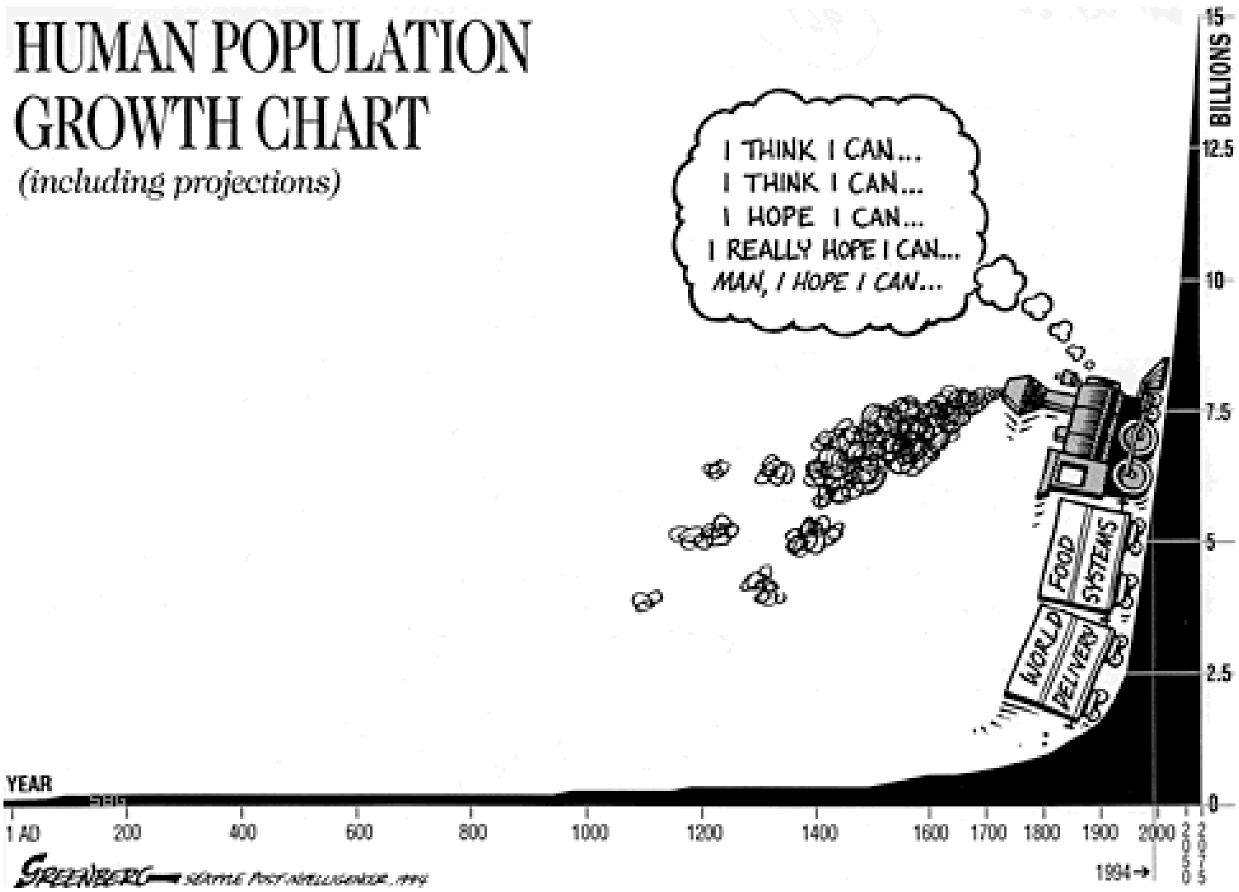


MCF3MI

Unit 7: Exponential Functions

UNIT 7: EXPONENTIAL FUNCTIONS – ESSENTIAL LEARNINGS

You will have at least three opportunities to demonstrate the essential learnings in this course. The first opportunity will occur during the unit with formative quizzes and homework checks. Your second opportunity will be on the unit test. Your third opportunity will be on the exam. You need to demonstrate all essential learnings to earn this credit.

Please take the opportunity to keep track of the essential learnings you've demonstrated during this unit using the checklist below.

Essential Learnings: Connecting Graphs and Equations of Exponential Functions & Solving Problems with Exponential Functions	***Homework***	***Associated Lessons***
☐ Identify the key properties of exponential functions, compared to other functions, and evaluate for specific values **Wednesday, Nov. 27, 2019**	*pg. 310* #6 *pg. 332* #6	Lesson 7.1
☐ Graph an exponential function and explain its key features **Thursday, Nov. 28, 2019**	*pg. 331 – 332* #1 – 4, 7, 11	Lesson 7.2
☐ Solve real world problems involving exponential functions **Friday, Nov. 29, 2019**	*worksheet*	Lesson 7.3
☐ Identify the key properties of exponential functions, compared to other functions **Monday, Dec. 2, 2019**	*pg. 317 – 318* #1, 2 *pg. 323 – 325* #1 – 5, 7 – 10	Lesson 7.4
Summative Task due Tuesday, Dec. 3, 2019 ***Unit Review:*** **Wednesday, Dec. 4, 2019**	*pg. 336 – 337* #1 – 3, 5 – 9 *pg. 338 – 339* #1 – 5, 10 *pg. 344 – 345* #6abcdgh, 8, 9	

REMINDER!

If you are having trouble mastering any of the concepts in this unit, it's important to get extra assistance **quickly**, as each new unit builds on the skills from the unit before it.

!! Unit 7 Test: Thursday December 5, 2019 !!

Exponential Growth

A. Spread of Disease

You and a friend are infected with a particularly nasty and virulent cold, but you are determined to go to a party tonight! This cold is so infectious that when you touch someone they are most likely to get infected, too. Let's assume that both of you sneeze on, cough on, shake hands with, hug, high-five, share a drink with, or smooch someone at an average rate of 2 people per hour. What will the viral transmission look like at this doomed party!?

Number of Hours After Arrival, *t*	**Number of People Infected, *N***	**1st Differences**	**2nd Differences**	**Ratio of "y-values"**

Let's try to model this situation with an equation. Let *N* represent the total number infected after *t* hours.

Number of Hours After Arrival, *t*	**Number of People Infected, *N***	
	Expanded Form	***Exponential Form***

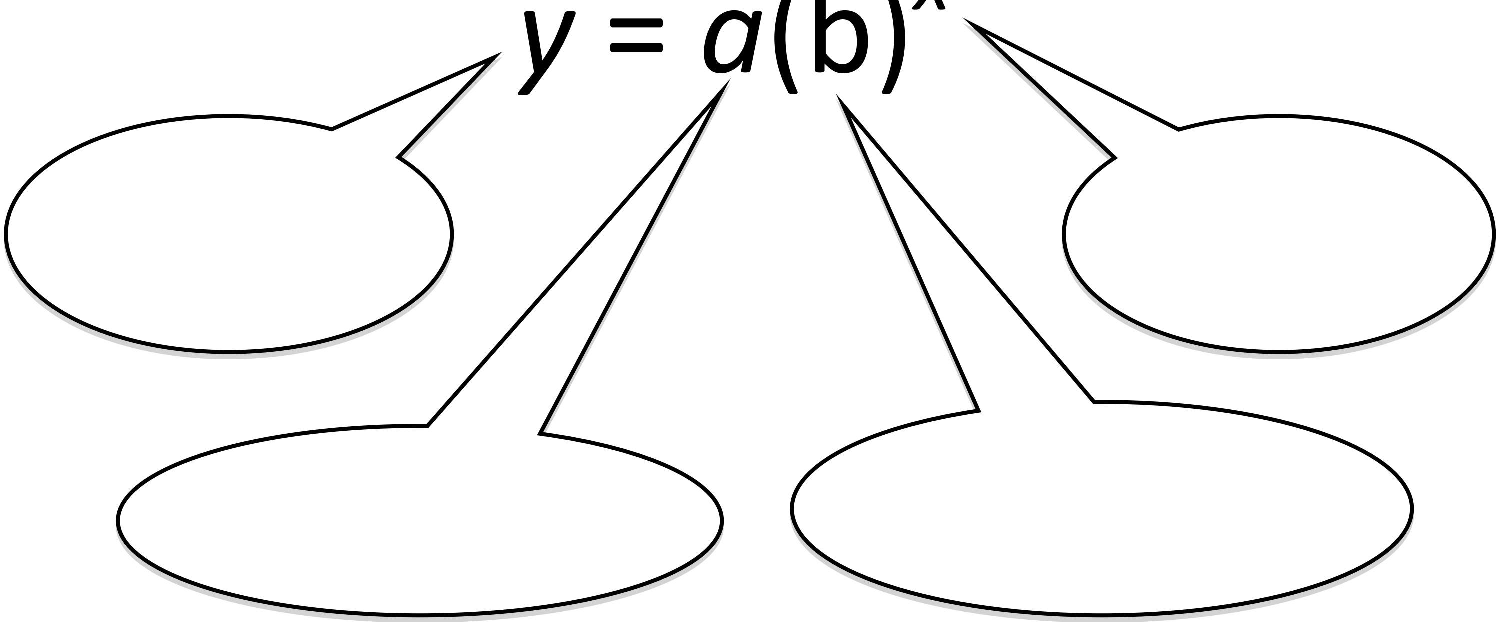
Therefore, the equation that models our sorry party is: ____ , *where*:

- "*N*" represents ____
- "*t*" represents ____
- "2" represents ____
- "3" represents ____

B. Exponential Functions

Here is an example of an **exponential function**. The exponential functions we will investigate consist of four parts:

$$y = a(b)^x$$



<u>Note</u>: Exponential functions are relations that either <u>increase</u> or <u>decrease</u> along their entire domain.

Predict what the graph in each of the following scenarios would look like and then use the online graphing calculator to confirm your hypothesis. Are they all exponential functions?

What if the initial value is:

- 1?
- zero?
- negative?
- a fraction?

What if the base is:

- 1?
- zero?
- negative?
- a fraction?

Which scenarios above yield a relation that is NOT an exponential function?

C. Moore's Law

In 1975, Gordon E. Moore observed that the number of transistors that could fit into a given space on a computer chip doubled every two years. His observation is known as *Moore's Law* and still seems to be correct today. Is this an example of an exponential function? ___________

- Assume that there were approximately 10 000 transistors on a computer chip in 1974.
- Using the year 1974 as year "zero", find the number of doubling periods between then and now. _______
- According to Moore's Law, how many transistors should a computer chip have today? ________________
- Check that with the actual number online (search "Transistor Count"): ____________________________

Was Moore close? ____

On 13 April 2005, Gordon Moore stated in an interview that the law cannot be sustained indefinitely: "It can't continue forever. The nature of exponentials is that you push them out and eventually disaster happens." He also noted that transistors would eventually reach the limits of miniaturization at atomic levels:

> *In terms of size [of transistors] you can see that we're approaching the size of atoms which is a fundamental barrier, but it'll be two or three generations before we get that far—but that's as far out as we've ever been able to see. We have another 10 to 20 years before we reach a fundamental limit. By then they'll be able to make bigger chips and have transistor budgets in the billions.*

D. Model Exponential Growth with Equations

Let's write a generalized equation for Moore's Law, with 10 000 as our initial value for the number of transistors, 2 as our growth factor, *N* as the total number of transistors, and *t* as the amount of time that has passed, in years:

Ex. 1: Power consumption. The power consumption of computer nodes doubles every 18 months. In 1991, a Cray C90 vector supercomputer required 500 kilowatts (kW) of power.

- Using 1991 as time zero, create an exponential function to model the relation.
- Use the equation to figure out how much power a supercomputer should use today.
- According to this model, how much power will be necessary in 5 years?

HW: p. 310 #6; p. 327 Example #1; p. 330 C1 and C3; p. 332 #6

Microprocessors (sourced from Wikipedia)

A microprocessor incorporates the functions of a computer's central processing unit on a single integrated circuit. It is a multipurpose, programmable device that accepts digital data as input, processes it according to instructions stored in its memory, and provides results as output.

Six-Core Opteron 2400	904,000,000	2009	AMD	45 nm	346 mm²
16-Core SPARC T3	1,000,000,000[10]	2010	Sun/Oracle	40 nm	377 mm²
Quad-Core + GPU Core i7	1,160,000,000	2011	Intel	32 nm	216 mm²
Six-Core Core i7 (Gulftown)	1,170,000,000	2010	Intel	32 nm	240 mm²
8-core POWER7 32M L3	1,200,000,000	2010	IBM	45 nm	567 mm²
8-Core AMD Bulldozer	1,200,000,000[11]	2012	AMD	32nm	315 mm²
Quad-Core + GPU AMD Trinity	1,303,000,000	2012	AMD	32 nm	246 mm²
Quad-core z196[12]	1,400,000,000	2010	IBM	45 nm	512 mm²
Quad-Core + GPU Core i7	1,400,000,000	2012	Intel	22 nm	160 mm²
Dual-Core Itanium 2	1,700,000,000[13]	2006	Intel	90 nm	596 mm²
Six-Core Xeon 7400	1,900,000,000	2008	Intel	45 nm	503 mm²
Quad-Core Itanium Tukwila	2,000,000,000[14]	2010	Intel	65 nm	699 mm²
8-core POWER7+ 80M L3	2,100,000,000	2012	IBM	32 nm	567 mm²
Six-Core Core i7/8-Core Xeon E5 (Sandy Bridge-E/EP)	2,270,000,000 [15]	2011	Intel	32 nm	434 mm²
8-Core Xeon Nehalem-EX	2,300,000,000[16]	2010	Intel	45 nm	684 mm²
10-Core Xeon Westmere-EX	2,600,000,000	2011	Intel	32 nm	512 mm²
Six-core zEC12	2,750,000,000	2012	IBM	32 nm	597 mm²
8-Core Itanium Poulson	3,100,000,000	2012	Intel	32 nm	544 mm²
62-Core Xeon Phi	5,000,000,000	2012	Intel	22 nm	
Xbox One Main SoC	5,000,000,000	2013	Microsoft/AMD	28 nm	363 mm²

6. Laura's grandfather put 1¢ into a piggy bank and gave it to her on her first birthday. Every year, he doubled the amount that he put into the piggy bank.

a) Make a table showing the amounts for Laura's first 10 birthdays.

b) Is this an exponential relation? Explain why or why not.

c) Suppose that he had decided to use the square of her age to determine the gift. Add a column to your table in part a) to show the amounts under a quadratic plan. Which is better for Laura?

d) Use a calculator to find how much he should put in on Laura's 30th birthday under each plan.

6. Unless kept frozen (below 0°C), fresh seafood will spoil due to bacterial growth. The rate of spoilage increases with temperature according to the model $R = 100(2.7)^{\frac{T}{8}}$, where 100 is the rate at 0°C, T is the temperature in degrees Celsius, and R is the spoilage rate at temperature T. The model is considered accurate to a temperature of 25°C.

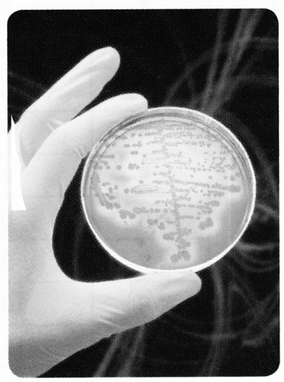
a) Sketch a graph of spoilage rate R versus temperature T from 0°C to 25°C.

b) Use your graph to predict the temperature at which the spoilage rate doubles to 200.

c) What is the spoilage rate at 20°C?

d) If the maximum acceptable spoilage rate is 500, what is the maximum storage temperature?

Example 1 Growth of Bacteria



For his science project, Ranjit placed agar, a gel made from seaweed, in a petri dish and infected it with bacteria. The measurement of the growth ring was used to estimate the number of bacteria present. Ranjit measured the growth ring for two weeks. He constructed an exponential growth model using the function $N(t) = 5(1.2)^{\frac{t}{2}}$, where $N(t)$ represents the number of bacteria present, in thousands, and t represents the time, in days.

a) Calculate the number of bacteria present at the start of the experiment.

b) How many bacteria were present after one day? two days? seven days?

c) Sketch a graph showing the number of bacteria versus time for the two-week experiment.

Communicate Your Understanding

C1 The media sometimes use the term *exponential growth* to describe anything that is increasing quickly. For example, house prices in a town might be increasing "exponentially" based on average prices (in thousands of dollars) over the past four years of 200, 212, 228, and 248. Is this correctly identified as an exponential relation? Explain.

C3 The exponential growth model for bacteria in a petri dish predicts no limit to the number of bacteria present. Discuss some factors that might limit the growth in a real culture.

6. a), c)

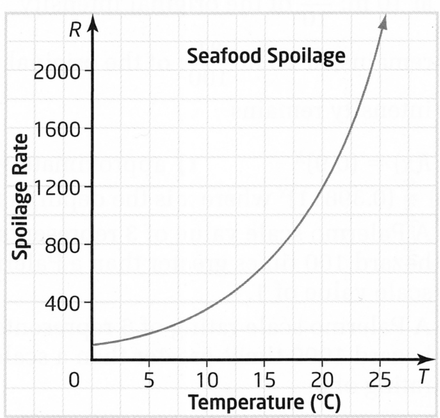
Birthday	Doubling Value ($)	Quadratic Value ($)
1	0.01	0.01
2	0.02	0.04
3	0.04	0.09
4	0.08	0.16
5	0.16	0.25
6	0.32	0.36
7	0.64	0.49
8	1.28	0.64
9	2.56	0.81
10	5.12	1.00

b) yes

c) For the first six years the quadratic plan will be better for Laura, but for all subsequent birthdays the annual gift will be larger. By the eighth year the total of all gifts will be greater for the first plan.

d) doubling plan: $5 368 709.12; quadratic plan: $9.00

6. a)



b) 5.5°C **c)** 1200 **d)** 13°C

Exponential Decay

A. Dehydration

Biological Half-Life: "*The time required for half the quantity of a drug or other substance deposited in a living organism to be metabolized or eliminated by normal biological processes*." The biological half-life of water in a human is 7 to 14 days. This means that it takes one to two weeks to lose half of the water in your body. Why such a broad range? Because the half-life of water can be dramatically altered by a person's environment and behaviour. What are some factors that would affect your rate of dehydration?

The human body is composed of approximately 60% water. Use your weight to estimate how many pounds of water you consist of right now: _________________ . If you were to lose more than 15% of your body weight as water, it would most likely be the end of you! How many pounds of water does this represent? ________________ What would be the amount of water remaining in your body at the time of death? ___________________________

Imagine you are lost in the wilderness without food or water. Let's assume that the maximum daily temperature is 21 degrees, and we will run two simulations: in the first simulation, you have found or created shelter and have thus protected yourself from the elements, so the biological half-life of your water will be 14 days. In the second simulation, you have not found shelter and are thus exposed to the elements, reducing the biological half-life of your water to 7 days. How many days could you survive in each simulation?

Number of days without water, *d*	**Amount of water remaining in your body, *W***	
	Simulation #1 – 14-day half-life	***Simulation #2 – 7-day half-life***

To get more specific, we can graph the exponential function using the graphing calculator and use the TRACE function or the INTERSECT method to find the exact number of days we would survive.

If a person loses only 2.5% of their body weight in water, they lose approximately 25% of their normal functioning capacity (balance, coordination, "clear-headedness"). This loss of normal functioning can be as terminal as the total dehydration! Use the online graphing calculator for each of the simulations to find out how long it would take to lose 2.5% of your water weight.

B. Half-Life Equations

Here is an **exponential function** being modeled as a **half-life equation.** Notice the rational exponent!

$$y = a\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

Amount remaining after "*t*" time has passed

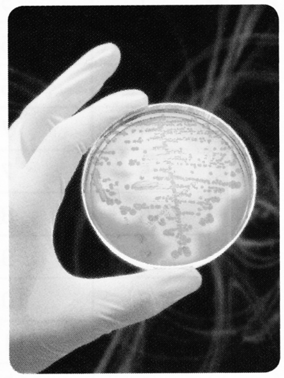
Initial amount

Half-life: the length of time for half of the substance to break down/decay

Time, "t"

HW: <u>Read</u> Examples 1 and 2 on p. 327-330; <u>Complete</u> p. 331-332 #1-4, 7, 11

Example 1 Growth of Bacteria



For his science project, Ranjit placed agar, a gel made from seaweed, in a petri dish and infected it with bacteria. The measurement of the growth ring was used to estimate the number of bacteria present. Ranjit measured the growth ring for two weeks. He constructed an exponential growth model using the function $N(t) = 5(1.2)^{\frac{t}{2}}$, where $N(t)$ represents the number of bacteria present, in thousands, and t represents the time, in days.

a) Calculate the number of bacteria present at the start of the experiment.

b) How many bacteria were present after one day? two days? seven days?

c) Sketch a graph showing the number of bacteria versus time for the two-week experiment.

d) Use the graph to estimate the time needed for the number of bacterial culture to double from the number present at the start of the experiment. Round your estimate to the nearest day.

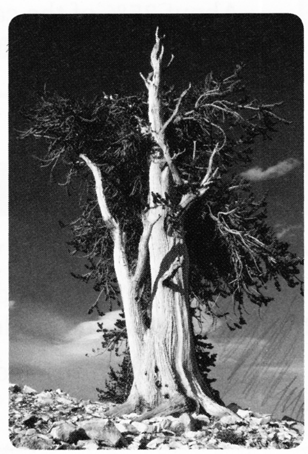
e) Write any restrictions on the domain and range of the function. Explain the reason for these restrictions.

Example 2 Connecting Carbon Dating to Exponential Functions

Living organisms contain both carbon-12, which does not decay, and radioactive carbon-14. When an organism dies, the amount of carbon-14 decreases exponentially with a half-life of about 5730 years. One can measure the amount of carbon-14 left in an organism to estimate how long ago it died. The decay process can be modelled using the function $P = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$, where P represents the percent of carbon-14 remaining and t represents the time, in years.

a) Bristlecone pine fossils found in the western United States show an age of about 6000 years using carbon dating. What percent of the original carbon-14 remains in these fossils?

b) Sediments deposited by glaciers during the last ice age covered and buried layers of peat. Carbon dating shows about 25% of the original carbon-14 remaining in the peat. Estimate the age of the glacial deposits to the nearest hundred years.



Making Connections

Carbon dating allows scientists to estimate how long ago an organism died by measuring the amount of carbon-14 in its remains and working with an exponential function. This technique is considered reliable to within 50 000 years using standard counting techniques, and the reliability can be extended to 100 000 years using more sophisticated techniques. To learn more about carbon dating, go to *www.mcgrawhill.ca/functionsapplications11* and follow the links.

For help with questions 1 and 2, refer to Example 2.

1. A fossil was found in a rock layer believed to have been deposited 15 000 years ago. What percent of carbon-14 is expected to remain in the fossil?

2. The fossil of a tree was found to contain 10% of its original carbon-14. To the nearest hundred years, estimate the age of the fossil. Use a calculator to check your estimate. If a graphing calculator is available, use the intersect method from Example 2. How close was your estimate?

3. Suppose that a bacterial culture was known to double every day. After 30 days, it covered the entire surface area of the agar in the petri dish. When did it cover half the area? Explain.

Connect and Apply

B

4. The persistence of drugs in the human body often follows an exponential model. Suppose that a new drug follows the model $M = M_0(0.79)^{\frac{h}{3}}$, where M is the mass of drug remaining in the body, M_0 is the mass of the dose taken, in milligrams, and h is the time in hours since the dose was taken.

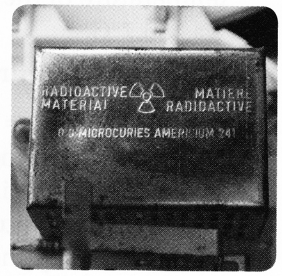
a) A standard dose is 100 mg. Sketch a graph showing the mass remaining in the body up to 48 h.

b) Use your graph to estimate the half-life of the drug in the body.

c) Check your estimate in part b) using the equation.

d) Once the mass remaining drops is less than 1 mg, the standard test can no longer detect the drug. How long will it take before the drug is no longer detectable? Use a graphing calculator.

7. Household smoke detectors use the radioactive element americium (Am).



The isotope Am-241 has a half-life of approximately 432 years, and the average smoke alarm contains about 200 μg (micrograms) of this isotope.

a) Model the decay curve for 200 μg of Am-241 using an exponential function.

b) Smoke alarms are typically replaced every 20 years. How much Am-241 remains after 20 years?

c) Estimate how long it would take for the amount of Am-241 to decay to less than 10 μg.

11. Tritium is a radioactive isotope of hydrogen that occurs as a by-product in CANDU nuclear reactors. If ingested into the human body, it has a biological half-life of about 10 days–that is, the body eliminates half of the tritium every 10 days.

a) If a person ingested 20 g of tritium, how much would be left after 10 days? after 20 days? after 30 days?

b) Write an exponential decay model that predicts the mass of tritium left after d days.

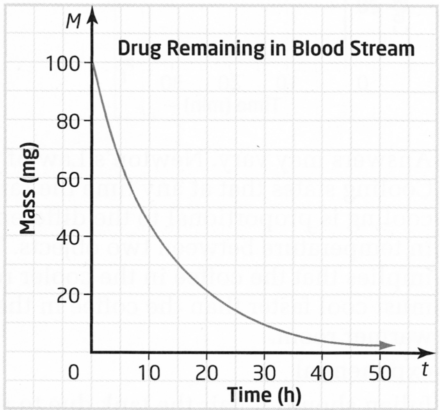
c) Use your model to predict how long it would take for the amount of tritium to drop to less than 0.1 g.

1. 16.3%

2. 19 000 years

3. day 29

4. a)



b) Estimates may vary. 9 h

c) 8.8 h

d) approximately 58.6 h

7. a) $M = 200\left(\frac{1}{2}\right)^{\frac{t}{432}}$

b) 193.7 μg **c)** 1867 years

11. a) After 10 days the mass will be 10 g;
After 20 days the mass will be 5 g;
After 30 days the mass will be 2.5 g.

b) $M(d) = 20\left(\frac{1}{2}\right)^{\frac{d}{10}}$ **c)** 77 days

Applications of Exponential Functions

A. Recall Exponential Functions

As we have learned, exponential functions can model growth or decay, where the function is either increasing or decreasing at a changing rate over its entire domain.

Exponential growth and decay problems can be modelled using the formula: $y = a(b)^x$.

For exponential growth and decay problems, we multiply an **initial value** (“*a*”) by a **growth/decay factor** (“b”) over a number of growth/decay **intervals** (“x”):

- the base is called the “**growth factor**” when $b > 1$
- the base is called the “**decay factor**” when $0 < b < 1$
- the function **neither grows nor decays** when $b = 1$

Note: If a function doubles, the base is 2; if it triples, the base is 3; if it halves (as with half-lives), the base is (½).

B. Growth/Decay *Factors* vs. Growth/Decay *Rates*

The difference between the growth/decay factor and 1 is called the **rate** of growth/decay. Often, rates of growth and decay are given as a percentage.

Ex. 1: To model growth, we add the growth rate to 1. This gives us the **growth factor.**

Ex. 2: To model decay, we subtract the decay rate from 1. This gives us the **decay factor**.

C. Exponential Growth and Decay Problems:

1) Given the equation $y = 35(0.57)^x$

   a) Does this equation represent growth or decay? ________________

   b) What is the rate of growth or decay? ________________

   c) What is the initial value? ________________

   d) Evaluate for $x = 5$ ________________

2) Given the equation $y = 225(1.23)^x$

   a) Does this equation represent growth or decay? ________________

   b) What is the rate of growth or decay? ________________

   c) What is the initial value? ________________

   d) Evaluate for $x = 2$ ________________

3) Given the equation $y = 154(1.06)^x$

   a) Does this equation represent growth or decay? ________________

   b) What is the rate of growth or decay? ________________

   c) What is the initial value? ________________

   d) Evaluate for $x = 7$ ________________

4) A typical cup of coffee contains 100 mg of caffeine. Every hour, approximately 16% of the amount of caffeine in the body is metabolized and eliminated. Let *C* represent the amount of caffeine in the body in mg and let *t* represent the number of hours since a typical cup of coffee was consumed.

(a) Write an equation where *C* is a function of *t* for a typical cup of coffee. ________________

(b) How much caffeine is left in the body after 3 hours? ________________

(c) Suppose an individual cannot sleep unless she has less than 50 mg of caffeine in her blood. How many hours after drinking coffee must she wait until she is able to fall asleep?

____

5) Ryan is saving for his college tuition. He has $2,550 in a savings account that pays 6.25% annual interest.

   a) Write an exponential equation describing this situation. ________________

   b) How much money will Ryan have in his account 6 years from now? ________________

6) A used car was purchased for $12,329 this year. Each year the car's value decreases 8.5%.

   a) Write an exponential equation describing this situation. ________________

   b) What will the car be worth in ten years? ________________

7) Jeremiah owns a business. His first year he made $11,212, each of the following years his profit increased 12%.

a) Write an exponential equation describing the situation. _______________

b) What will he make in 20 years? _______________

8) Dianna just bought a home. She paid $240,000. She is able to pay 20% of the loan off each year.

a) Write an exponential equation describing the situation. _______________

b) What will she owe in 10 years? _______________

9) You just won the lotto! You have two options, you can take the full $1,000,000 now or they will pay you 1000 dollars this year and then double the amount they pay you every year for 10 years. Which way will get you more money?

____

10) A biologist is researching a newly discovered species of bacteria. At time $t = 0$ hours, he puts one hundred bacteria into what he has determined to be a favourable growth medium. The population of bacteria doubles every 3 hours. How many bacteria are there in six hours?

____

11) A certain type of bacterium, given a favourable growth medium, doubles in population every 6.5 hours. Given that there were approximately 100 bacteria to start with, how many bacteria will there be in a day and a half?

____

12) A new 2000 Volkswagon Jetta GL-Sedan has a base price of $15,230 and will depreciate 30% each year. If you owned the Jetta and wished to trade it in after 4 years what would be the value of the car?

____

13) Mike is considering selling his 2004 Euclid which is in perfect condition. Its initial value was $26,000. A local car dealer, Sam, tells him that a two year old car is worth $20,000 and a five year old car is worth $11,000. With further research, Mike learns from the local car guide that his car depreciates 14% each year, and he decides to use this method for his calculations.

(a) Calculate the price of the car after 5 years. _______________

(b) Does Sam offer a fair deal to Mike? _______________

14) Amy bought a diamond ring for $6,000. If the value of the ring increases at a constant rate of 3.83% per year, how much will the ring be worth in twenty-one years?

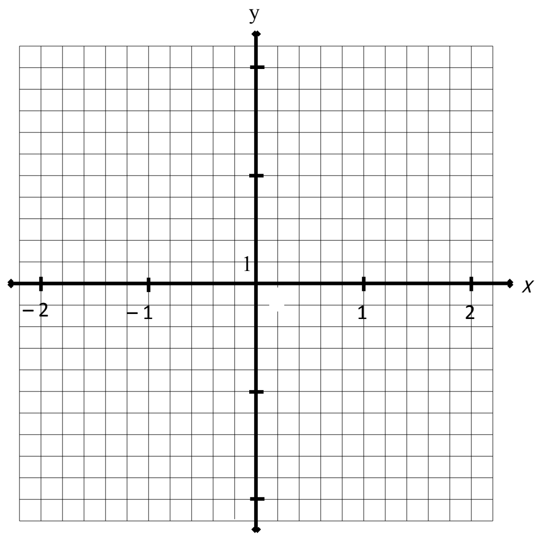
____

Properties of Exponential Functions

Recall that **exponential functions** are curves that ***increase*** or ***decrease*** throughout their entire domains. They have the basic form $y = a(b)^x$, where *growth* is modeled when b > 1 and *decay* is modeled when 0 < b < 1. They describe many different phenomena, as we have seen, including population growth and radioactive decay.

I. Comparing the Graphs of Three Exponential Functions

Create a table of values for each function, and graph them all on the same set of axes.



$f(x) = 2^x$

x	y
-2	
-1	
0	
1	
2	

$g(x) = 3^x$

x	y
-2	
-1	
0	
1	
2	

$h(x) = \left(\frac{1}{2}\right)^x$

x	y
-2	
-1	
0	
1	
2	

Property	*f(x)*	*g(x)*	*h(x)*
Domain			
Range			
x-intercept			
y-intercept			
Growth/decay			
As $x \to +\infty$			
As $x \to -\infty$			
Asymptote			

II. Comparing Linear, Quadratic, and Exponential Functions

A. Equations

Use the *equations* of the following relations to determine whether they are linear, quadratic, or exponential.

i) $f(x) = 2x + 3$ ii) $g(x) = x^2 - 2$ iii) $h(x) = 2^x$

B. Tables of Values

Use the *table of values* to confirm whether the relations are linear, quadratic, or exponential.

i) $f(x) = 2x + 3$

x	*y*	*1st diff.*
-3		
-2		
-1		
0		
1		
2		
3		

ii) $g(x) = x^2 - 2$

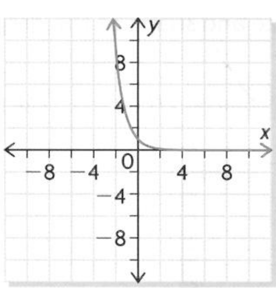
x	*y*	*1st diff.*	*2nd diff.*
-3			
-2			
-1			
0			
1			
2			
3			

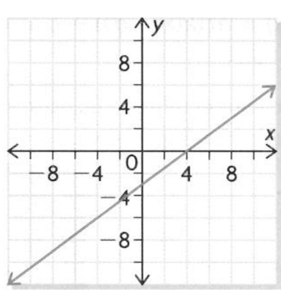
iii) $h(x) = 2^x$

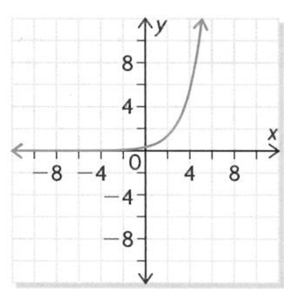
x	*y*	*1st diff.*	*2nd diff.*	*Ratio*
-3				
-2				
-1				
0				
1				
2				
3				

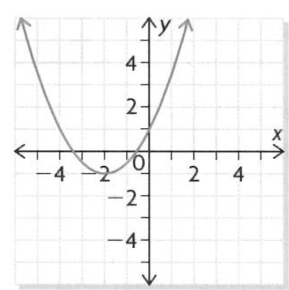
C. Graphs

Use the *graphs* to recognize whether the relations are linear, quadratic, or exponential (growth or decay).









HW: p. 317-318 # 1, 2; p. 323-325 # 1-5, 7-10

For help with question 1, refer to Example 1.

1. Match each item in Column A with its description from Column B.

Column A		Column B	
a)	domain	**i)**	domain for the function $f(x) = a^x$
b)	range	**ii)**	the set of all possible x-values for a function
c)	$x \in \mathbf{R}$	**iii)**	y-intercept for the function $f(x) = a^x$
d)	$y > 0, y \in \mathbf{R}$	**iv)**	the set of all possible y-values for a function
e)	asymptote	**v)**	forbidden value of range $f(x) > 0$
f)	1	**vi)**	range for the function $f(x) = a^x$, where $a > 1$
g)	-2	**vii)**	a straight line that a curve approaches

For help with questions 2 and 3, refer to Example 2.

2. Consider the exponential functions.
   - $f(x) = 4^x$
   - $g(x) = 2^x$
   - $h(x) = (0.25)^x$

   a) Which is greatest when $x = 5$?

   b) Which is greatest when $x = -5$?

   c) For what value of x do all three functions have the same value? What is this value?

A

For help with questions 1 and 2, refer to the Example.

1. Classify each of the following relations as linear, quadratic, or exponential.

   a) $y = -3x + 2$

   b) $y = 3x$

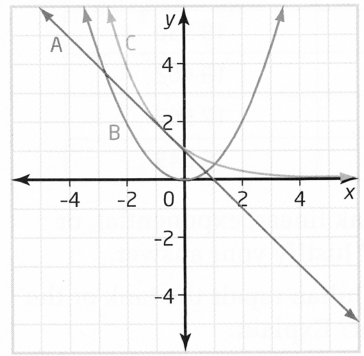
   c) $y = (x - 1)^2$

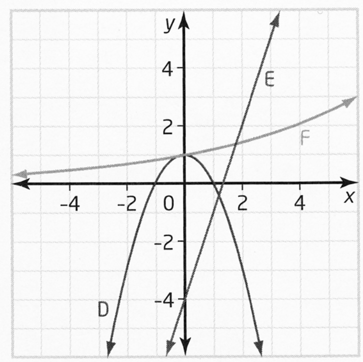
   d) $y = 2x^2$

   e) $y = (0.1)^x$

   f) $y = x$

2. Classify each of the relations shown as linear, quadratic, or exponential.





3. Predict whether a linear, quadratic, or exponential model would best fit each situation.

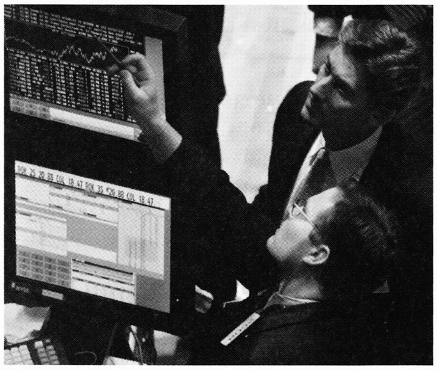
   a) the height of a skydiver in the first few seconds after jumping from an aircraft

   b) the position of a train moving down a straight track at a constant speed of 120 km/h

   c) the number of yeast cells in a loaf of bread dough, given that the cells divide every 10 min

   d) the value of shares that have been increasing at a rate of 10% every year

5. Elspeth purchased shares in Northern Gold Mines for $20.32 per share. She kept track of the share value each week for 10 weeks, as shown. Is its growth best modelled by a linear, a quadratic, or an exponential function? Justify your answer.



Time (weeks)	Share Value ($)
0	20.32
1	20.52
2	20.73
3	20.94
4	21.15
5	21.36
6	21.57
7	21.79
8	22.00
9	22.22
10	22.45

7. Julian suspects that there is a leak somewhere in his scuba gear. If the leak is in the tank itself, the pressure in the tank should decrease exponentially. If the leak is in the regulator, the pressure should decrease linearly. Julian filled the tank to its maximum rating of 22 500 KPa and kept a record of time and pressure, as shown.

Time (min)	Pressure (KPa)
0	22 500
5	21 395
10	20 345
15	19 350
20	18 402
25	17 502
30	16 643

a) Is the leak linear, exponential, or neither? Justify your answer.

b) Should Julian repair the tank or the regulator? Explain.

8. The cost of the paint required to cover a spherical gas tank is shown. Is the relation between cost and radius best modelled using a linear, quadratic, or exponential relation? Justify your answer.

Radius (m)	Cost of Paint ($)
1.2	94.91
1.8	213.55
2.4	379.63
3.0	593.19

9. The loudness of a sound is measured in units called decibels (dB). Sound waves exert pressure on your eardrums. Your brain interprets this pressure as the sense of hearing. The pressure exerted by various sounds is shown, measured in milliPascals (mPa). Is the relation between pressure and decibel loudness best modelled with a linear, quadratic, or exponential function? Justify your answer.

Sound	Loudness (dB)	Pressure (mPa)
telephone dial tone	80	200
city traffic	90	632
train	100	2 000
gas lawnmower	110	6 325
jet engine	120	20 000

10. Some vehicles now use compressed natural gas as a fuel. Jessica filled the natural gas tank on her van to a pressure of 30 atmospheres (atm). As she drove the car, she burned the natural gas at a constant rate.

Time (min)	Pressure (atm)
0	30.0
5	27.5
10	25.0
15	22.5
20	20.0

a) Predict whether the relationship between distance driven and tank pressure should be modelled using a linear, quadratic, or exponential relation. Explain your choice.

b) The pressure in the tank as Jessica drove the car is shown. Does the relation match your prediction? Explain.

c) Explain why the situation modelled in this problem is not the same as a leak in a system under pressure.

1. a) ii) **b)** iv) **c)** i) **d)** vi)

e) vii) **f)** iii) **g)** v)

2. a) $f(x)$ **b)** $h(x)$

c) value of 1, when $x = 0$

1. a) linear **b)** linear

c) quadratic **d)** quadratic

e) exponential **f)** linear

2. A: linear; B: quadratic; C: exponential; D: quadratic; E: linear; F: exponential

3. a) quadratic **b)** linear

c) exponential **d)** exponential

4. a) quadratic **b)** exponential

c) linear **d)** none

5. Either linear, quadratic, or exponential, when performing a linear, quadratic, and exponential regression on the data.

7. a) exponential

b) Julian should repair the tank due to a leak.

8. quadratic; The amount of paint required is dependent upon the surface area of the spherical gas tank. The surface area of a sphere increases as a quadratic function.

9. exponential

10. a), **b)** linear

c) Explanations may vary. In the case of the leak under pressure, there was an uncontrolled loss of gas through a hole of fixed size. As gas escaped from the tank, the pressure in the tank fell, causing the rate of loss to decrease. This situation fit an exponential model.

In the case of the compressed natural gas, the gas is being removed through a regulator at a constant rate. To make this happen, the regulator measures the flow and decreases its resistance as the pressure in the tank drops to ensure a constant rate of mass leaving the tank. This results in a linear decrease in pressure.

Review for Unit 7 Test

Representative Questions for Review:

1) How can you determine an exponential function from its table of values? Its equation? Its graph?

2) Is the following exponential function for growth or decay: $y = 1.08^x$?

3) Growth of your gut bacteria doubles every week. If you started with 18 bacteria, how many bacteria will be in your gut in 100 days?

4) Skin cells have a half-life of approximately 2 weeks. If you have 1.6 trillion skin cells today, how many of those will still be around in 8 weeks? Give your answer in billions of skin cells.

5) In 2010, Antarctica had 595 000 Emperor Penguins, but their population numbers have been shrinking since 2000. If their population is decreasing by 2.5% every year, how many Emperor Penguins will there be in 2020? Looking back in time, how many were there in 2000?

Textbook review questions:

p. 336-337 # 1, 2, 3, 5, 6, 7, 8, 9

p. 338-339 # 1, 2, 3, 4, 5, 10

p. 344-345 # 6abcdgh, 8, 9

1. Simplify each of the following. Leave in exponential form.

a) $\frac{5^4 \times 5^3}{5^2}$

b) $[(-2)^3]^2$

c) $\frac{y^9}{y^2 \times y^4}$

d) $(k^3)^2 \div k$

6.2 Evaluate Powers with Integer Exponents, pages 288-295

2. Evaluate each of the following. When appropriate, express answers as fractions.

a) 3^{-4}

b) $\left(\frac{1}{9}\right)^{-2}$

c) $2^0 - 2^{-1}$

6.3 Investigate Rational Exponents, pages 296-304

3. Evaluate each of the following. Where appropriate, use a calculator.

a) $243^{\frac{1}{5}}$

b) $0.001^{\frac{2}{3}}$

c) $2^{\frac{11}{12}}$

5. The population growth rate of the Golden Horseshoe area around Lake Ontario is among the fastest in Canada, an estimated 1.84% per year.

a) The population of the area according to the 2001 census was 11.5 million. What is its estimated population for 2011?

b) What was its population in 1996?

6. Sixteen contestants begin a game of Odd Man Out. Each person rolls a number cube. If the number on the cube is odd, the contestant must leave the game. The last one left wins the prize.

a) Write an exponential relation to model the number of contestants N left after n rolls of the number cubes.

b) How many rolls are expected before a winner is declared?

6.5 Exponential Functions and Their Properties, pages 312-317

7. Consider the exponential function $y = 0.2^x$.

a) Make a table of values and sketch a graph of the function.

b) Find the domain, range, intercepts, and intervals of increase and decrease, as well as any asymptotes.

8. Use first differences, second differences, and/or ratios to classify each relation as linear, quadratic, exponential, or none of these.

a)

x	y
-3	6
-2	1
-1	-2
0	-3
1	-2
2	1
3	6

b)

x	y
-3	-3.737
-2	-3.446
-1	-3.000
0	0.000
1	3.000
2	3.446
3	3.737

c)

x	y
-3	16
-2	8
-1	4
0	2
1	1
2	0.5
3	0.25

6.7 Exponential Growth and Decay, pages 326-333

9. A new drug persists in the human body following the model $M = M_0(0.55)^{\frac{h}{2}}$, where M is the mass of drug remaining in the body, in milligrams; M_0 is the mass of the dose taken, also in milligrams; and h is the time in hours since the dose was taken.

a) A standard dose is 200 mg. Sketch a graph showing the mass remaining in the body for up to 12 h.

b) Use your graph to estimate the half-life of the drug in the body.

c) Check your estimate in part b) using the equation.

1. Simplify $\left(\frac{1}{3}\right)^2 + 5^0$.

A $1\frac{1}{9}$

B $5\frac{1}{9}$

C $5\frac{1}{3}$

D $8\frac{1}{3}$

2. An antique automobile was found to double in value every 10 years. If the current value is \$100 000, what was the value of the vehicle 20 years ago?

A \$50 000

B \$25 000

C \$12 500

D \$5 000

3. Evaluate $256^{\frac{3}{4}}$.

A 192

B 64

C 16

D 4

4. Simplify $\frac{2^9}{(2^3)^4}$.

A 2^3

B 2^2

C 2^{-2}

D 2^{-3}

5. Consider the exponential functions $y = 0.5^x$, $y = 2^x$, and $y = 5^x$. What value of x results in the same y-value for each?

A -1

B 0

C 1

D There is none.

10. A large rubber sac is used to maintain the pressure in a jet pump that provides water for a barn. Brandon suspects a leak in the pump assembly. If the leak is in the sac, it should follow an exponential relation. If the leak is in the water tank, it should follow a linear relation. He pumps it to a pressure of 175 KPa and then measures the pressure every minute. The table shows his results.

Time (min)	Pressure (KPa)
0	175.0
1	161.8
2	148.3
3	134.7
4	121.3
5	107.8
6	94.4

a) Is the leak best modelled by a linear or exponential function? Justify your answer.

b) Should Brandon replace the sac? Explain.

6. Evaluate each power. Round answers to four decimal places where necessary.

a) 2^4 **b)** 3^3 **c)** 0.5^2

d) 1^{12} **e)** $(1 + 0.2)^{-4}$ **f)** $\left(1 + \frac{0.043}{6}\right)^{24}$

g) 1.2^{-2} **h)** 1.03^{28} **i)** $\left(1 + \frac{0.035}{12}\right)^{-18}$

8. Create a table of values and sketch the function $y = 1.5^x$ for $0 \leq x \leq 8$.

9. Determine whether each table of values represents an exponential function. Justify your reasoning using finite differences, a graph, or common ratios.

a)

x	y
0	7
1	9
2	15
3	25
4	39

b)

x	y
-2	0
-1	3.5
0	7.0
1	10.5
2	14.0

c)

x	y
0	200
1	600
2	1 800
3	5 400
4	18 200

d)

x	y
0	100
1	80
2	64
3	51.2
4	40.96

1. a) 5^5 **b)** $(-2)^6$ **c)** y^3 **d)** k^5

2. a) $\frac{1}{81}$ **b)** 81 **c)** $\frac{1}{2}$

3. a) 3 **b)** 0.01 **c)** 1.8877

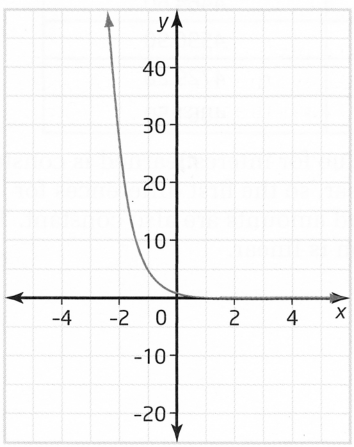
4. 0.241 years

5. a) 13.8 million **b)** 10.5 million

6. a) $N = 16(0.5)^n$ **b)** 4

7. a)

x	$y = 0.2^x$
-3	125
-2	25
-1	5
0	1
1	0.2
2	0.04
3	0.008

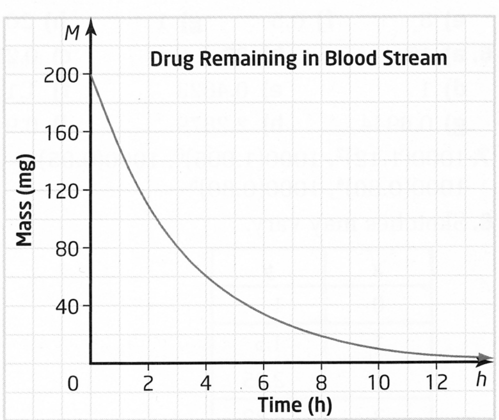


b) Domain: $x \in \mathbf{R}$; Range: $y > 0$, $y \in \mathbf{R}$; There is no x-intercept; y-intercept $= 1$; The function decreases for its entire domain. There is no increasing interval. The function has a horizontal asymptote with equation $y = 0$ which is approached on the right side of the graph.

8. a) quadratic **b)** neither

c) exponential

9. a)



b) 2.3 h **c)** Answers may vary.

1. A

2. B

3. B

4. D

5. B

6. 4 m

7. $P(2, 2, 12, 12, 12, 2, 2, 2, 2) = \left(\frac{1}{36}\right)^9$

8. a) $f(t) = 10 \times 1.5^t$ **b)** 76 fish

9. $f(t) = 5^x$

10. a) linear

b) Brandon should not replace the sac. Explanations may vary.

6. a) 16 **b)** 27 **c)** 0.25

d) 1 **e)** 0.4823 **f)** 1.1869

g) 0.6944 **h)** 2.2879 **i)** 0.9489

7. $1000(1.12)^2$, $1000(1.005)^6$, $1000(0.95)^4$, $1000(0.80)^4$, $1000(0.80)^5$

8. Sketches may vary.

x	y
0	1
1	1.5
2	2.25
3	3.38
4	5.06
5	7.59
6	11.39
7	17.09
8	25.63

9. a) first differences: 2, 6, 10, and 14
second differences: 4; quadratic function

b) first differences: 3.5; linear function

c) first differences: 400, 1200, 3600, and 12 800
second differences: 800, 2400, 9200 third differences: 1600, 6800; Since the third differences are not the same, the table of values does not represent an exponential function.

d) By graphing the table of values, the function is exponential; it decays as the x-values increase.