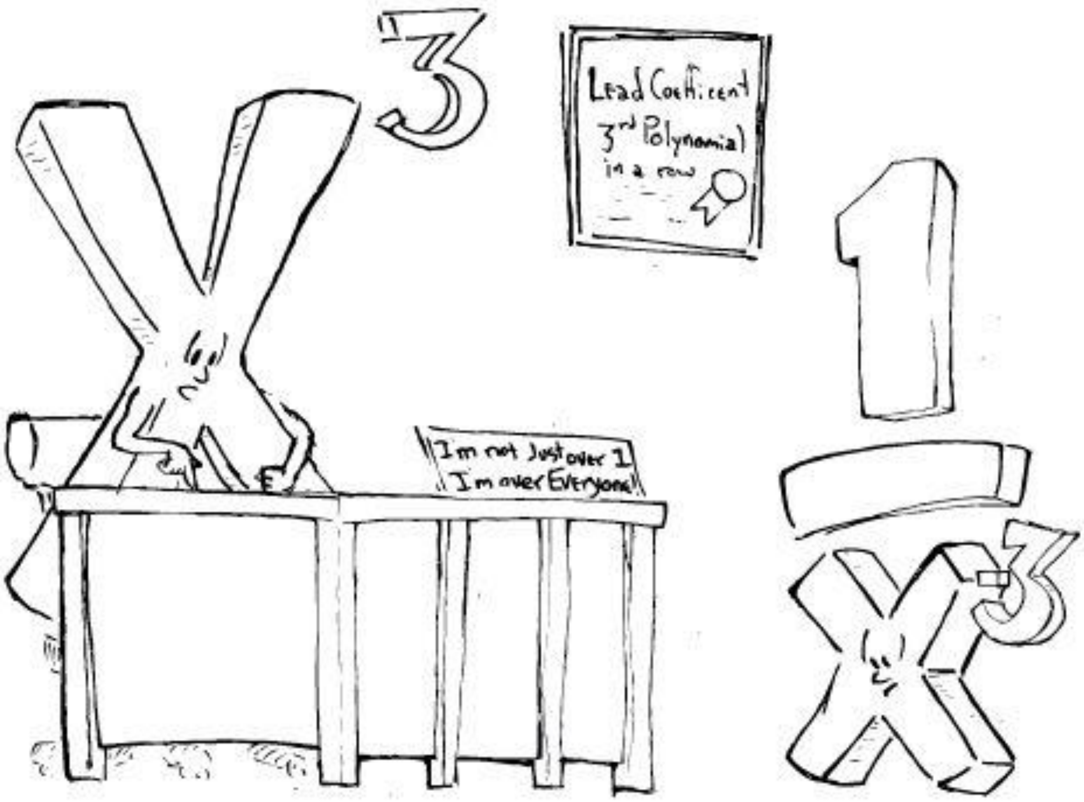

Mark my words! You harness that negative power of yours, and you can make it to the top just like me!

MCF3MI

Unit 6: Exponent Laws

UNIT 6: EXPONENT LAWS – ESSENTIAL LEARNINGS

You will have at least three opportunities to demonstrate the essential learnings in this course. The first opportunity will occur during the unit with formative quizzes and homework checks. Your second opportunity will be on the unit test. Your third opportunity will be on the exam. You need to demonstrate all essential learnings to earn this credit.

Please take the opportunity to keep track of the essential learnings you've demonstrated during this unit using the checklist below.

Essential Learnings: Connecting Graphs and Equations of Exponential Functions	***Homework***	***Associated Lessons***
☐ Use exponent laws of multiplication, division, and powers to simplify and evaluate expressions	*pg. 285 – 286* #1 – 8, 9abc, 11, 13, 14abc	Lesson 6.1
☐ Use exponent laws including integers exponents to simplify and evaluate expressions	*pg. 293 – 295* #1 – 8, 14	Lesson 6.2
☐ Use exponent laws including rational exponents to simplify and evaluate expressions	*pg. 302 - 303* #1aceh, 2abcd, 3abdf, 4abd, 5 – 8	Lesson 6.3
Unit Review:	*assignment*	

REMINDER!

If you are having trouble mastering any of the concepts in this unit, it's important to get extra assistance **quickly**, as each new unit builds on the skills from the unit before it.

Reviewing the Exponent Laws

A. Powers

Powers are a convenient way of writing repeated multiplication. A **power** consists of two parts: a base and an exponent. The base tells us which value to repeatedly multiply, and the exponent tells us how many times to perform the multiplication.

For instance, 3^4 is a **power**. The number 3 is the base. The number 4 is the exponent. The power can be written in *exponential form* as 3^4 or in *expanded form* as 3 x 3 x 3 x 3.

Example 1:

5^4	→ base: **5**	exponent: **4**	7^{-2}	→ base: **7**	exponent: **-2**
$(-5)^4$	→ base: _____	exponent: _____	-5^4	→ base: _____	exponent: _____
$2x^3$	→ base: _____	exponent: _____	$(3x)^4$	→ base: _____	exponent: _____

B. Multiplying Powers with the Same Base

When multiplying powers with the same base, we can write the product of each power in expanded form and simplify the expression into a new exponential form. When we do so, a pattern emerges:

Example 2: Simplify the following products. (That means … write as a *single power.*)

a) $5^3 \times 5^4$

b) $2^2 \times 2^3$

c) $4a^2(a^3)$

d) $-2b(b^2)$

Rule for Multiplying Powers

When multiplying powers with the same base, you add the exponents. If you do not see an exponent, it is assumed to be "1". ***Example*:** $x(x^3) = x^4$

$$b^x \times b^y = b^{(x+y)}$$

C. Dividing Powers with the Same Base

When dividing powers with the same base, we can write the quotient of the powers in expanded form and simplify the expression into a new exponential form. When we do so, another pattern emerges:

Example 3: Simplify the following quotients. (That means … write as a *single power.*)

a) $\frac{5^4}{5^2}$

b) $\frac{2^8}{2^5}$

c) $4^5 \div 4^3$

d) $3^7 \div 3^2$

Rule for Dividing Powers

When dividing powers with the <u>same base</u>, you subtract the exponents. ***Example*:** $x^7 \div x^2 = x^5$

$$b^x \div b^y = b^{(x-y)}$$

D. Power of a Power

Raising a power to another exponent is called a “power of a power”. We can write the ***power of a power*** in expanded form and simplify the expression into a new exponential form. When we do so, yet another pattern emerges:

Example 4: Simplify.

a) $(2^3)^4$

b) $(3^2)^3$

c) $(4^2)^5$

d) $(y^3)^3$

Rule for Simplifying a Power of Power

To simplify a power of a power, you multiply the exponents. ***Example*:** $(x^3)^4 = x^{12}$

$$(b^x)^y = b^{(x)(y)}$$

E. Select and Apply the Exponent Rules

1. Simplify: Use the exponent rules to express each of the following as a single power.

a) $\frac{3^2 \times 3^4}{3^3}$

b) $\frac{4^5 \times 4^2}{4^6}$

c) $\frac{5^6 \times 5^7}{\left(5^3\right)^3}$

d) $\frac{\left(2^3\right)^5}{\left(2^4\right)^2}$

e) $\frac{(-3)^7}{(-3)^2 \times (-3)^3}$

f) $\left[\left(\frac{1}{3}\right)^2\right]^3$

2. Simplify and evaluate each of the following without using a calculator.

a) $\frac{\left(\frac{4}{5}\right)^8}{\left(\frac{4}{5}\right)^6}$

b) $\left[\left(\frac{1}{2}\right)^{10}\right]^{\frac{1}{2}}$

HW: p. 285-286 #1-8, 9abc, 11, 13, 14abc

A

For help with questions 1 to 6, refer to Example 1.

1. Use the exponent rules to express each of the following as a single power.
   a) $14^3 \times 14^7$
   b) $9^5 \times 9^4$
   c) $(-3)^5(-3)^2$
   d) $\left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^8$

2. Use the exponent rules to express each of the following as a single power.
   a) $6^{12} \div 6^9$
   b) $11^{10} \div 11^7$
   c) $\frac{7^7}{7^3}$
   d) $\frac{(-2)^5}{(-2)^3}$

6. Use the exponent rules to express each of the following as a single power.
   a) $\frac{\left(\frac{1}{2}\right)^5}{\left(\frac{1}{2}\right)^3}$
   b) $\frac{(0.5)^{15}}{(0.5)^3 \times (0.5)^4}$
   c) $\left[\left(-\frac{3}{4}\right)^3\right]^5$
   d) $(-7)(-7)^5$

7. Use the exponent rules to simplify each algebraic expression.
   a) $(s^3)^4$
   b) $(\pi r^2)^3$
   c) $(2x^2y^3)(3xy^4)$
   d) $\frac{45x^7y^4}{9xy^3}$
   e) $\frac{(4x^4y^5)^2}{8xy^8}$

8. Consider the expression $\frac{5^4 \times 5^7}{5^6 \times 5^3}$.
   a) Evaluate each power using a calculator and then evaluate the whole expression.
   b) Simplify the expression first using the exponent rules and then evaluate the simplified form.
   c) Which method, a) or b), do you prefer? Why?

3. Use the exponent rules to express each of the following as a single power.
   a) $(3^3)^6$
   b) $(6^2)^7$
   c) $(10^5)^2$
   d) $(-5^2)^4$

4. Use the exponent rules to express each of the following as a single power.
   a) $4^8 \times 4^5$
   b) $7^2 \times 7^7$
   c) $\frac{5^8}{5^3}$
   d) $\frac{6^4}{6}$

5. Use the exponent rules to express each of the following as a single power.
   a) $\frac{8^3 \times 8}{8^2}$
   b) $\frac{4^7}{(4^2)^3}$
   c) $\frac{(-3)^4 \times (-3)^5}{[(-3)^3]^2}$
   d) $(9^4)^3$

Connect and Apply

9. Consider the power 2^{12}.
   a) Write this power as the product of two powers in three different ways.
   b) Write this power as the quotient of two powers in three different ways.
   c) Write this power as the power of a power in three different ways.

11. The Richter scale measures the magnitude of an earthquake. Each number on the scale is the exponent of the magnitude, using a base of 10. For example, an earthquake measuring 3.0 on the Richter scale has a magnitude 10 times greater than an earthquake measuring 2.0 on the scale. An earthquake measuring 4.0 on the Richter scale has a magnitude 100 times (or 10^2) greater than the magnitude of an earthquake measuring 2.0 on the scale.

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a) Compare the magnitude of an earthquake measuring 6.0 on the Richter scale with the magnitude of an earthquake measuring 3.0 on the Richter scale.

b) In the last 250 years, southern Ontario has experienced three earthquakes measuring approximately 5.0 on the Richter scale. The most powerful recorded earthquake in Canadian history was the Cascadia earthquake on January 26, 1700, which occurred off the coast from Vancouver Island to northern California. It has been estimated at 9 on the Richter scale. How does the magnitude of the Cascadia earthquake compare with the magnitude of the Ontario earthquakes?

13. a) Determine two positive integers x and y such that $8^x = 4^y$.

b) Are these the only values of x and y that will work? If so, explain why. If not, find another set of values that will work.

14. Computers use the binary number system, which represents numbers as sums of powers of 2. For example, $12 = 2^3 + 2^2$.

a) Express 42 as a sum of powers of 2.

b) Express 192 as a sum of powers of 2.

c) What number is represented as $2^9 + 2^7 + 2^5 + 2^3$?

1. a) 14^{10} **b)** 9^9 **c)** $(-3)^7$ **d)** $\left(\frac{1}{2}\right)^{14}$

2. a) 6^3 **b)** 11^3 **c)** 7^4 **d)** $(-2)^2$

3. a) 3^{18} **b)** 6^{14} **c)** 10^{10} **d)** $(-5)^8$

4. a) 4^{13} **b)** 7^9 **c)** 5^5 **d)** 6^3

5. a) 8^2 **b)** 4 **c)** $(-3)^3$ **d)** 9^{12}

6. a) $\left(\frac{1}{2}\right)^2$ **b)** $(0.5)^8$ **c)** $\left(-\frac{3}{4}\right)^{15}$ **d)** $(-7)^6$

7. a) s^{12} **b)** π^3r^6 **c)** $6x^3y^7$

d) $5x^6y$ **e)** $2x^7y^2$

8. a) 25 **b)** 25

c) Answers may vary.

9. Answers may vary. Samples are provided.

a) $2^3 \times 2^9$; $2^4 \times 2^8$; 2×2^{11}

b) $\frac{2^{15}}{2^3}$; $\frac{2^{16}}{2^4}$; $\frac{2^{150}}{2^{138}}$

c) $(2^2)^6$; $(2^3)^4$; $(2^4)^3$

11. a) 1000

b) The magnitude of the Cascadia earthquake was 10 000 times greater than that of the Ontario earthquakes.

13. a) Answers may vary. Likely combinations include $x = 2$, $y = 3$; $x = 4$, $y = 6$

b) Answers may vary. It should be noted that $8^x = 2^{3x}$ and $4^y = 2^{2y}$.

By equating the right sides of these equations, any pair of integers that satisfy the equation $3x = 2y$ will satisfy the original equation.

14. a) Answers may vary. $2^5 + 2^3 + 2^1$

b) Answers may vary. $2^7 + 2^6$ **c)** 680

Powers with Integer Exponents

A. Powers with Zero Exponents

For the expression, complete the following:
(i) use the exponent law to subtract the exponents
(ii) expand the powers and reduce the expression
(iii) use a calculator to evaluate the result

$\frac{5^4}{5^4}$ (i) (ii) (iii)

Powers with Zero Exponents

When dividing two powers with the <u>same base</u> and the <u>same exponent</u> the result is 1.
When applying the exponent law for dividing powers, you get an exponent equal to zero.
Thus, any base (with the exception of 0) raised to an exponent of zero equals 1.

Since: $b^x \div b^x = 1$

And: $b^x \div b^x = b^0$

Then: $\mathbf{b^0 = 1,\ b \neq 0}$

Ex. 1: Simplify the following, then evaluate:

a) $5^3 \times 5^{-3}$

b) $4^7 \div 4^7$

c) $(2^3)^0$

d) $\left[\left(\frac{1}{3}\right)^0\right]^3$

e) $\frac{(2^3)^5}{(2^5)^3}$

f) $\frac{(-3)^8}{(-3)^5 \times (-3)^3}$

B. Powers with Negative Exponents

For the expression, complete the following: (i) use the exponent law to subtract the exponents
(ii) expand the powers and reduce the expression
(iii) use a calculator to evaluate the result

$\frac{2^2}{2^5}$ (i) (ii) (iii)

Powers with Negative Exponents

An integer base raised to a negative exponent is equal to the reciprocal of the base raised to the corresponding positive exponent:

$$b^{-n} = \frac{1}{b^n}$$

A fractional base raised to a negative exponent is equal to the reciprocal of the fraction raised to the corresponding positive exponent:

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Ex. 2: Simplify the following, then evaluate:

a) $5^4 \times 5^{-5}$ b) $3^4 \div 3^7$ c) $\frac{(4^{-2})^3 \times 4^5}{4^2}$ d) $\left(\frac{3}{5}\right)^{-2}$

Ex. 3: Represent the following as a power with a negative exponent:

a) $\frac{1}{25}$ b) $\frac{1}{16}$ c) $\frac{1}{1000}$ d) $0.000\ 01$

HW: p. 293-295 #1-8, 14 (for #4 you must convert each power with a negative exponent into a fraction, then find a common denominator to add or subtract the values remaining)

A

For help with questions 1 and 2, refer to Example 1.

1. Evaluate each power without a calculator. Express each answer as a fraction.

a) 11^{-2} **b)** 15^{-1} **c)** 4^{-4}

d) 3^{-5} **e)** 2^{-8} **f)** 1^{-12}

2. Evaluate each power using a calculator. Express each answer as a fraction.

a) 16^{-3} **b)** 8^{-4}

c) 5^{-5} **d)** 7^{-3}

3. Express each fraction or decimal as a power with a negative exponent. For the base, use the smallest positive integer possible.

a) $\frac{1}{49}$ **b)** $\frac{1}{125}$

c) $\frac{1}{10\ 000}$ **d)** 0.000 000 1

For help with question 4, refer to Examples 1 and 3.

4. Evaluate. Simplify answers as much as possible while leaving them as fractions.

a) $2^2 + 2^{-2}$

b) $3^0 - 3^{-1}$

c) $4^0 + (-4)^0$

d) $5^{-2} + 5^{-1}$

5. Evaluate. Where fractions are used, leave answers in fraction form.

a) $\left(\frac{1}{4}\right)^{-2}$ **b)** $\left(\frac{2}{3}\right)^{-3}$

c) $(0.01)^{-4}$ **d)** $(0.2)^{-5}$

6. Use two different positive bases to express each fraction as a power with a negative exponent.

a) $\frac{1}{16}$ **b)** $\frac{1}{81}$

For help with questions 7 and 8, refer to Example 3.

7. Use the exponent rules to express each of the following as a single power.

a) $\frac{7^3 \times 7^{-1}}{7^2}$

b) $\frac{3^4}{(3^2)^3}$

c) $\frac{(-4)^{-2} \times (-4)^5}{[(-4)^3]^2}$

d) $(12^0)^3$

8. Use the exponent rules to express each of the following as a single power.

a) $\frac{\left(\frac{1}{2}\right)^3}{\left(\frac{1}{2}\right)^5}$

Achievement Check

14. A magic square is a grid of numbers where the sum of each row is equal to the sum of each column. A Magic Product Square is a grid where the product of each row is equal to the product of each column.
Inspect the Magic Product Square shown.

16	$\frac{1}{8}$	4
$\frac{1}{2}$	2	8
1	32	$\frac{1}{4}$

a) Why is this a Magic Product Square? (Hint: consider the products of the first column and the middle row.)

b) Rewrite each number in the Magic Product Square as a power of 2.

c) How are the exponents of the numbers in any row, column, or diagonal of the Magic Product Square related?

d) Increase each exponent by 3. Does the square remain a Magic Product Square? Justify your answer.

e) The Magic Product is the cube of the central square. Verify that this is the case for the squares in this problem.

1. a) $\frac{1}{121}$ **b)** $\frac{1}{15}$ **c)** $\frac{1}{256}$

d) $\frac{1}{243}$ **e)** $\frac{1}{256}$ **f)** 1

2. a) $\frac{1}{4096}$ **b)** $\frac{1}{4096}$ **c)** $\frac{1}{3125}$ **d)** $\frac{1}{343}$

3. a) 7^{-2} **b)** 5^{-3} **c)** 10^{-4} **d)** 10^{-7}

4. a) $4\frac{1}{4}$ **b)** $\frac{2}{3}$ **c)** 2 **d)** $\frac{6}{25}$

5. a) 16 **b)** $\frac{27}{8}$

c) $100\ 000\ 000$ **d)** 3125

6. a) $2^{-4}; 4^{-2}$ **b)** $3^{-4}; 9^{-2}$

7. a) 1 **b)** 3^{-2} **c)** $(-4)^{-3}$ **d)** 1

8. a) 2^2 **b)** 4^{-2} **c)** $\left(-\frac{3}{4}\right)^{10}$ **d)** $(-9)^{-2}$

Powers with Rational Exponents

Use a calculator to complete the following.

i) $9^{\frac{1}{2}} =$ ______ *and* $\sqrt{9} =$ ______ $\therefore$

ii) $64^{\frac{1}{3}} =$ ______ *and* $\sqrt[3]{64} =$ ______ $\therefore$

iii) $16^{\frac{1}{4}} =$ ______ *and* $\sqrt[4]{16} =$ ______ $\therefore$

Rules: **i)** $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $\sqrt[n]{a}$ is called a **radical** and means the n^{th} root of *a.*

ii) $a^{\frac{m}{n}} =$ *or* $a^{\frac{m}{n}} =$

Ex. 1: Write each expression in radical form.

a) $x^{\frac{1}{2}}$ **b)** $a^{\frac{4}{5}}$ **c)** $125^{-\frac{2}{3}}$

Ex. 2: Write each expression as a power using rational exponents. Do not evaluate.

a) $\sqrt[3]{12}$ **b)** $\left(\sqrt[4]{13}\right)^5$ **c)** $\sqrt[7]{\left(17^3\right)}$

Ex. 3: Evaluate, without using a calculator.

a) $16^{\frac{1}{4}}$ **b)** $(-64)^{\frac{1}{3}}$ **c)** $36^{\frac{3}{2}}$

d) $\left(\frac{49}{25}\right)^{\frac{1}{2}}$ **e)** $125^{-\frac{2}{3}}$ **f)** $(-32)^{-\frac{2}{5}}$

g) $1000^{\frac{1}{3}}$ **h)** $16^{\frac{3}{4}}$ **i)** $\left(\frac{64}{27}\right)^{-\frac{2}{3}}$

Ex. 4: Use a calculator to evaluate each of the following.

a) $0.008^{\frac{4}{3}}$ **b)** $22^{\frac{1}{3}}$ **c)** $54^{-\frac{2}{3}}$

HW: p. 302-303 #1aceh, 2abcd, 3abdf, 4abd, 5, 6, 7, 8

For help with questions 1 and 2, refer to Example 1.

1. Evaluate, without using a calculator.

a) $64^{\frac{1}{2}}$ b) $121^{\frac{1}{2}}$ c) $(-125)^{\frac{1}{3}}$

d) $216^{\frac{1}{3}}$ e) $0.04^{\frac{1}{2}}$ f) $(-0.027)^{\frac{1}{3}}$

g) $\sqrt{144}$ h) $\sqrt[4]{81}$ i) $\sqrt[5]{243}$

2. Evaluate, without using a calculator.

a) $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ b) $\sqrt[3]{\frac{8}{27}}$ c) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

d) $\left(-\frac{32}{243}\right)^{\frac{1}{5}}$ e) $\sqrt[5]{0.002\ 43}$ f) $(0.064)^{\frac{1}{3}}$

For help with questions 3 and 4, refer to Example 2.

3. Evaluate, without using a calculator.

a) $64^{\frac{2}{3}}$ b) $(-125)^{\frac{4}{3}}$ c) $16^{\frac{3}{2}}$

d) $32^{\frac{4}{5}}$ e) $0.0001^{\frac{3}{4}}$ f) $0.09^{\frac{3}{2}}$

4. Evaluate, without using a calculator.

a) $\left(\frac{4}{9}\right)^{\frac{3}{2}}$ b) $\left(\frac{27}{64}\right)^{\frac{2}{3}}$ c) $\left(-\frac{32}{243}\right)^{\frac{2}{5}}$

d) $\left(\frac{64}{125}\right)^{\frac{2}{3}}$ e) $0.00032^{\frac{4}{5}}$ f) $(0.125)^{\frac{2}{3}}$

5. Evaluate, without using a calculator.

a) $\left(\frac{1}{16}\right)^{-\frac{1}{2}}$ b) $\left(\frac{9}{4}\right)^{-\frac{3}{2}}$

c) $\left(-\frac{27}{8}\right)^{-\frac{2}{3}}$ d) $\left(\frac{32}{243}\right)^{-\frac{2}{5}}$

6. Use a calculator to evaluate each of the following. Round answers to 3 decimal places.

a) $29^{\frac{1}{2}}$ b) $32^{\frac{3}{4}}$

c) $(-12)^{-\frac{1}{3}}$ d) $18^{-\frac{2}{5}}$

7. Write each expression as a power. Do not evaluate.

a) $\sqrt[4]{29}$ b) $(\sqrt[4]{29})^3$

Connect and Apply

8. Which roots of negative numbers can be evaluated? Which cannot be evaluated? Explain.

1. a) 8 **b)** 11 **c)** -5
d) 6 **e)** 0.2 **f)** -0.3
g) 12 **h)** 3 **i)** 3

2. a) $\frac{1}{2}$ **b)** $\frac{2}{3}$ **c)** $\frac{2}{3}$
d) $-\frac{2}{3}$ **e)** 0.3 **f)** 0.4

3. a) 16 **b)** 625 **c)** 64
d) 16 **e)** 0.001 **f)** 0.027

4. a) $\frac{8}{27}$ **b)** $\frac{9}{16}$ **c)** $\frac{4}{9}$
d) $\frac{16}{25}$ **e)** 0.0016 **f)** 0.25

5. a) 4 **b)** $\frac{8}{27}$ **c)** $\frac{4}{9}$ **d)** $\frac{9}{4}$

6. a) 5.385 **b)** 13.454 **c)** -0.437 **d)** 0.315

7. a) $29^{\frac{1}{4}}$ **b)** $29^{\frac{3}{4}}$

8. A negative base raised to an odd exponent will have a negative value. As a result, an odd root of a negative value can be found. A negative base raised to an even exponent will have a positive value. As a result, it is not possible to find an even root of a negative value as this would require the product of an even number of factors to take on a negative value.