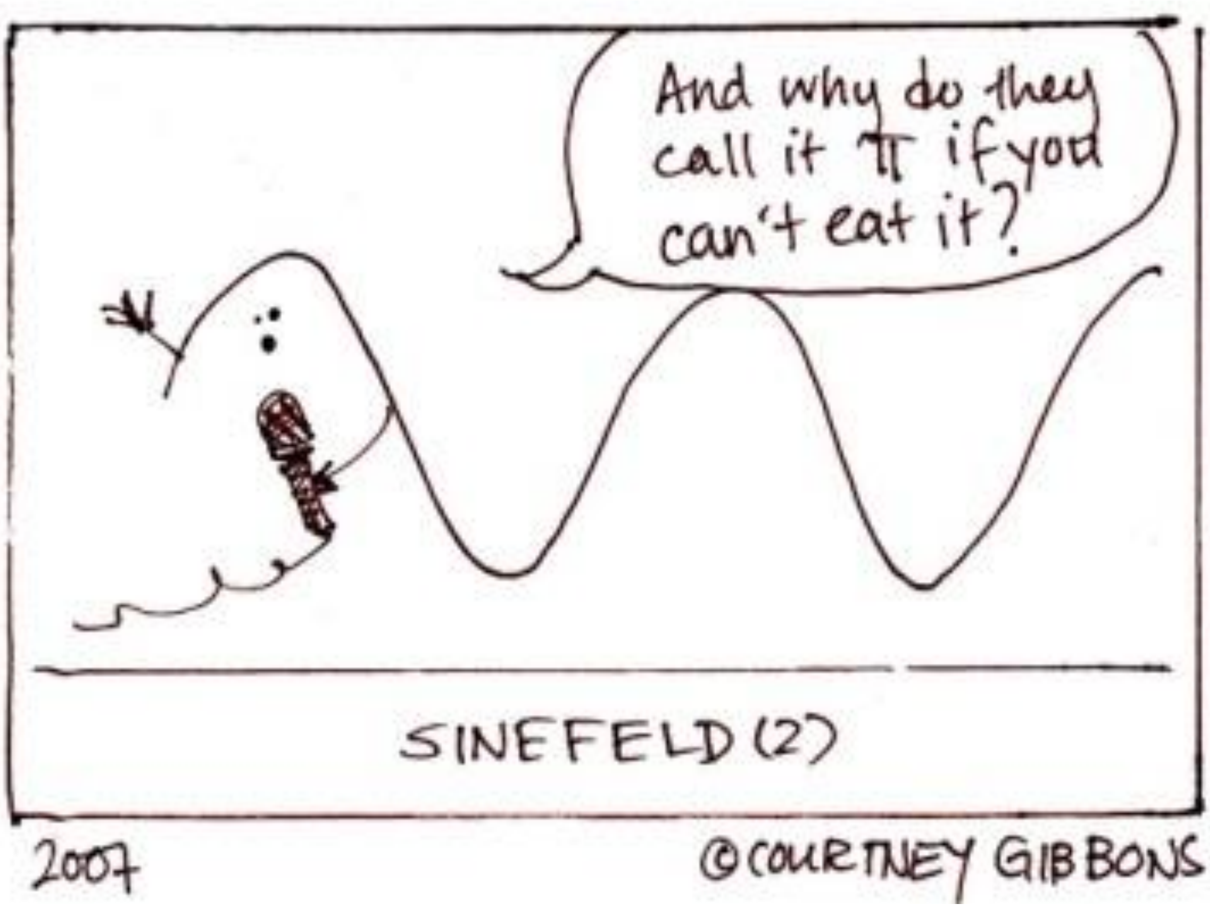

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Unit 5: Sine Functions

UNIT 5: SINE FUNCTIONS – ESSENTIAL LEARNINGS

You will have at least three opportunities to demonstrate the essential learnings in this course. The first opportunity will occur during the unit with formative quizzes and homework checks. Your second opportunity will be on the unit test. Your third opportunity will be on the exam. You need to demonstrate all essential learnings to earn this credit.

Please take the opportunity to keep track of the essential learnings you've demonstrated during this unit using the checklist below.

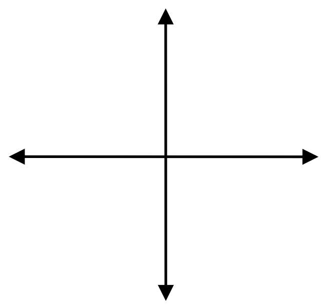
Essential Learnings: Connecting Graphs and Equations of Sine Functions & Solving Problems with Sine Functions	***Homework***	***Associated Lessons***
☐ Describe key properties of periodic functions given graphs or equations	*pg. 245 – 246* #1, 2, 9, 10	Lesson 5.1
	worksheet	Lesson 5.2
☐ Sketch y = sin x, determine its key properties, and sketch transformations with all appropriate labels	*pg. 261 – 263* #1, 5b, 6b, 7, 9ab, 11ac, 13a, 14	Lesson 5.3
	pg. 261 – 262 #2a, 3b, 5, 6, 9cdef, 10	Lesson 5.4
☐ Solve real world problems involving periodic functions by gathering data or generating equations	*pg. 267* #5 *On-Line Calculator:* *pg. 266 – 267* #1, 2, 6, 8	Lesson 5.5
☐ Determine the roles of a, c, and d and explain their transformations to y = sin x	*pg. 268 – 269* #6, 7 *pg. 275* #9, 13 *On-Line Calculator:* *pg. 268 – 269* #8, 9	Lesson 5.6
Unit Review:	*pg. 270 – 271* #4, 5, 8, 9, 10 *On-Line Calculator:* *pg. 270 – 271* #13, 14	

Angles in Standard Position

Up until now, we have been looking at right and acute angles confined inside triangles. Well, today we begin looking at angles in standard position, which means that they can take on any value from negative infinity to positive infinity!

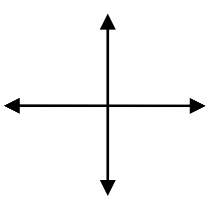
I. Angles in Standard Position

An **angle in standard position** has its **vertex** is at the origin and its **initial arm** on the positive x-axis of a coordinate grid. The **terminal arm** rotates about the origin and stops after a specified number of degrees. If the **angle of rotation is positive**, the direction of the rotation is counterclockwise (up). If the **angle of rotation is negative**, the direction of the rotation is clockwise (down). The coordinate grid is broken into 4 quadrants.

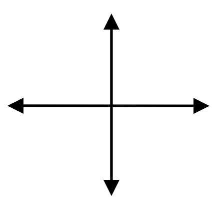


Ex. 1: Sketch each angle in standard position and label the quadrant it is in.

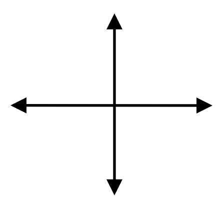
a) 140°



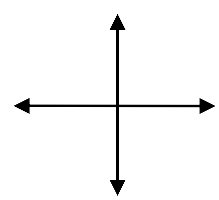
b) 240°



c) – 60°



d) – 210°

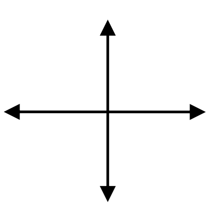


II. Co-terminal Angles

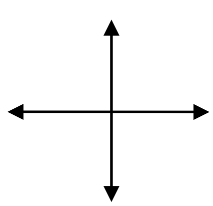
Co-terminal angles are angles in standard position that have the *same terminal arm*. There are <u>many</u> ways to arrive at the same terminal arm! You can use positive and negative angles of rotation, as well as moving through multiples of 360°!

Ex. 2: Sketch each angle in standard position and label its quadrant. Then, sketch an angle that is co-terminal.

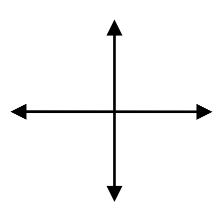
b) 60°



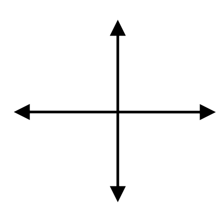
b) 200°



c) – 30°



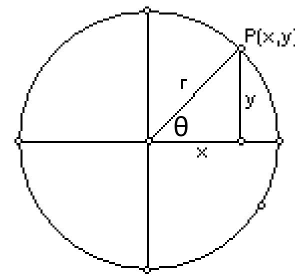
d) – 190°



III. Circles and the Sine Ratio

Let's consider a circle with centre (0, 0) on a coordinate grid …

- the point P(*x*, *y*) is any point on the **terminal arm of the circle** as θ goes from 0° to 360°
- the letter *r* is the radius of the circle. In a **"UNIT CIRCLE"** the radius is set at 1 unit, therefore *r* = 1.



We then define the **sine ratio** as follows…

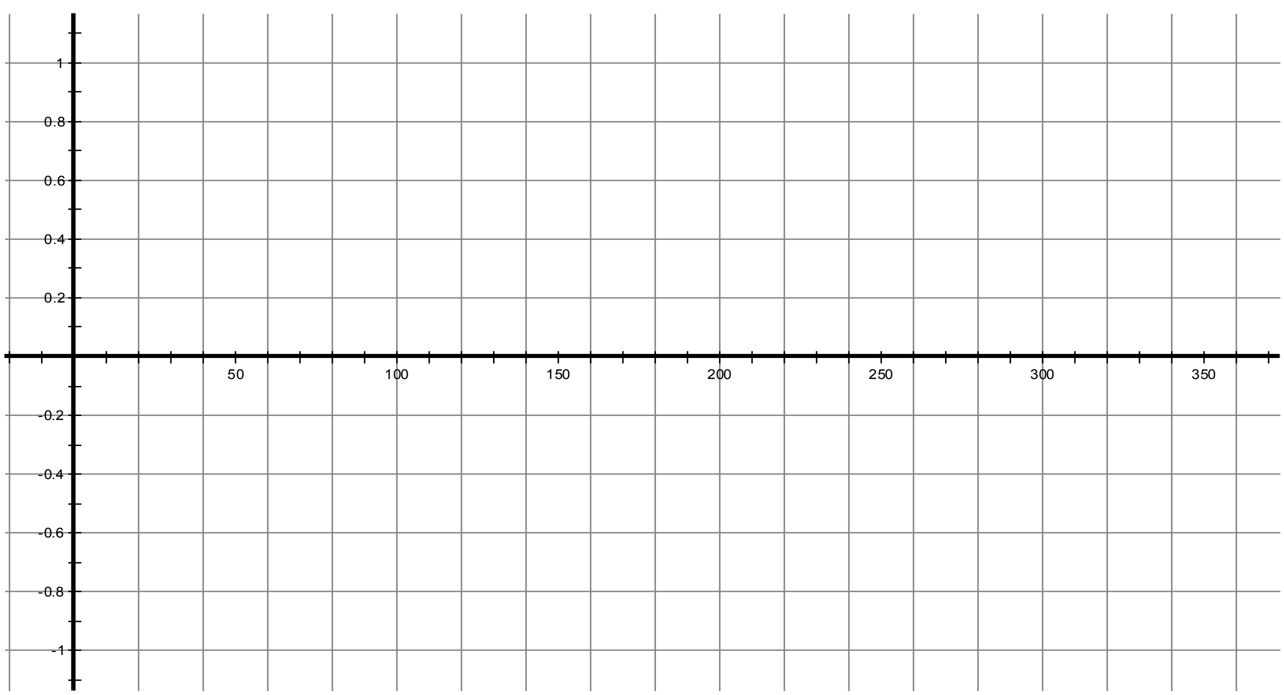
Complete the table using a calculator. Round your answers to 2 decimal places.

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = \sin\theta$													

Summary,

	1st Quadrant (0° – 90°)	**2nd Quadrant** (90° – 180°)	**3rd Quadrant** (180° – 270°)	**4th Quadrant** (270° – 360°)
$\sin\theta$ (+ve/-ve)				

We'll now graph the above table of values for the ***sine*** ratio using the relation $y = \sin\theta$.



Is the above curve a ***function***? ____________ Why or why not? _________________________________

HW: p. 245-246 #1, 2, 9, 10 [For #10, use the graph above to locate other angles that give the same y-value]

1. Find two coterminal angles for each given angle. Draw each set of coterminal angles on the same set of axes in standard position.

a) 90° **b)** 45° **c)** 83°

d) 0° **e)** 130° **f)** 180°

g) 205° **h)** 294° **i)** 310°

2. Which pairs of angles are coterminal? Justify your answer.

a) 30° and 210°

b) 70° and 430°

c) 100° and 820°

d) 380° and 680°

e) 40° and −320°

f) −50° and −400°

9. Find the measure of angle θ. Then, find one coterminal angle with the same value of $\sin \theta$. Round your answers to the nearest degree.

a) $\sin \theta = 0.8660$ 60 120

b) $\sin \theta = 0.7071$ 45 135

c) $\sin \theta = 0.5$ 30 150

d) $\sin \theta = 0.2588$

e) $\sin \theta = 0.9848$

f) $\sin \theta = 0.8910$

g) $\sin \theta = 0$

10. Find the measure of angle θ. Find a coterminal angle that is positive. Then, find one non-coterminal angle, if possible, that gives the same value of $\sin \theta$. Round your answers to the nearest degree.

a) $\sin \theta = -0.8660$

b) $\sin \theta = -0.7071$

c) $\sin \theta = -0.5$

d) $\sin \theta = -0.2588$

e) $\sin \theta = -0.9848$

f) $\sin \theta = -0.8910$

g) $\sin \theta = -1$

1. a) 450° and −270° **b)** 405° and −315°
c) 443° and −277° **d)** 360° and −360°
e) 490° and −230° **f)** 540° and −180°
g) 565° and −155° **h)** 654° and −66°
i) 670° and –50°

2. a) not coterminal; Their difference is not a multiple of 360°.
b) coterminal; Their difference is 360°.
c) coterminal; Their difference is 720°.
d) not coterminal; Their difference is not a multiple of 360°.
e) coterminal; Their difference is 360°.
f) not coterminal; Their difference is not a multiple of 360°.

9. a) 60°; 120° **b)** 45°; 135°
c) 30°; 150° **d)** 15°; 165°
e) 80°; 100° **f)** 63°; 117°
g) 0°; 180°

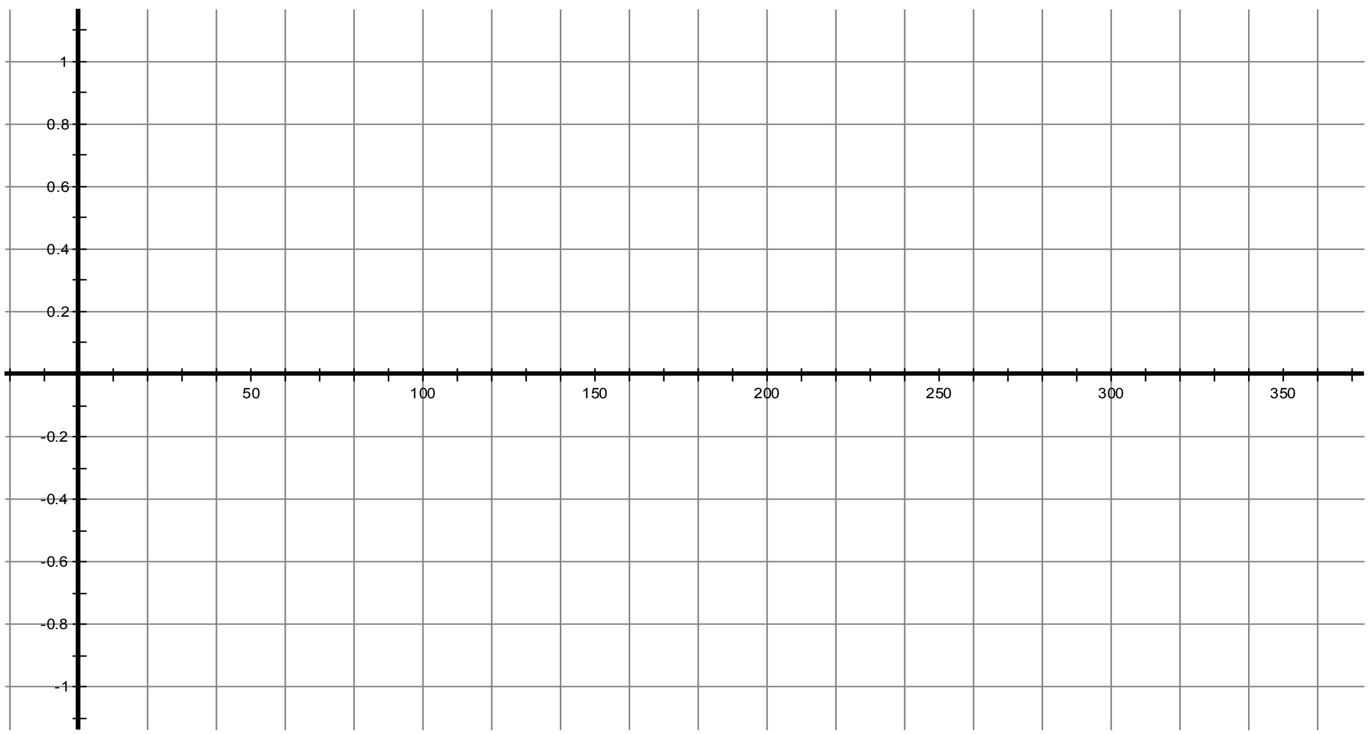
10. a) −60°; 300°; 240° **b)** −45°; 315°; 225°
c) −30°; 330°; 210° **d)** −15°; 345°; 195°
e) −80°; 280°; 260° **f)** −63°; 297°; 243°
g) −90°; 270°; none

Investigating the Sine Function

I. Recall the Graph of $y = \sin\theta$, where $0° \le \theta \le 360°$

Complete the table using a calculator and plot the points on the corresponding grid.

θ	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$	$210°$	$240°$	$270°$	$300°$	$330°$	$360°$
$y = \sin\theta$													



Is the above curve a ***function***? _____________ Why or why not? ____

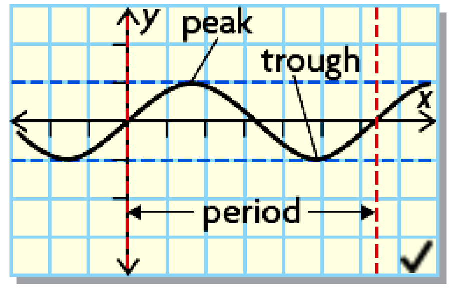
From the graph we can determine the <u>domain</u> and <u>range</u> of the *sine* function:

The **Domain** for the ***sine*** function is… D = _________________________________

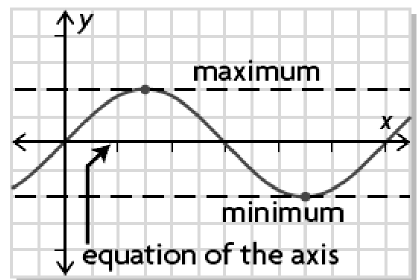
The **Range** for the ***sine*** function is… R = _________________________________

II. Terminology:

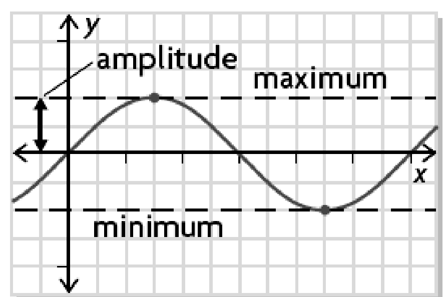
- **sine function:** a *sine* function is the graph of *f(x) = sin x*, where *x* is an angle measured in *degrees*; it is a **periodic function**



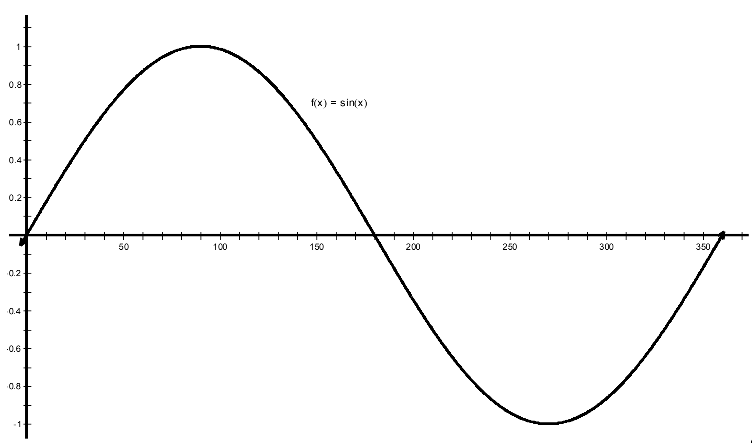
- **periodic**: happening or appearing at regular intervals; having repeated cycles
- **peak:** the highest point(s) on the graph (*maximum*)
- **trough**: the lowest point(s) on the graph (*minimum*)
- **period:** the interval of the independent variable (often time) needed for a repeating action to complete one cycle
- **cycle:** a series of events that are regularly repeated; a complete set of changes, starting from one point and returning to the same point in the same way
- **equilibrium axis**: the equation of the **horizontal line** halfway between the *maximum* and the *minimum*.
  It is determined by the **formula**…



- **Amplitude:** the distance from the function's equilibrium axis to either the *maximum* or the *minimum* value



Ex. 1: For the Sine function *y = sin x* …



a. The ***period*** is: ____________

b. The ***maximum*** value: is ____________

c. The ***minimum*** value is: ____________

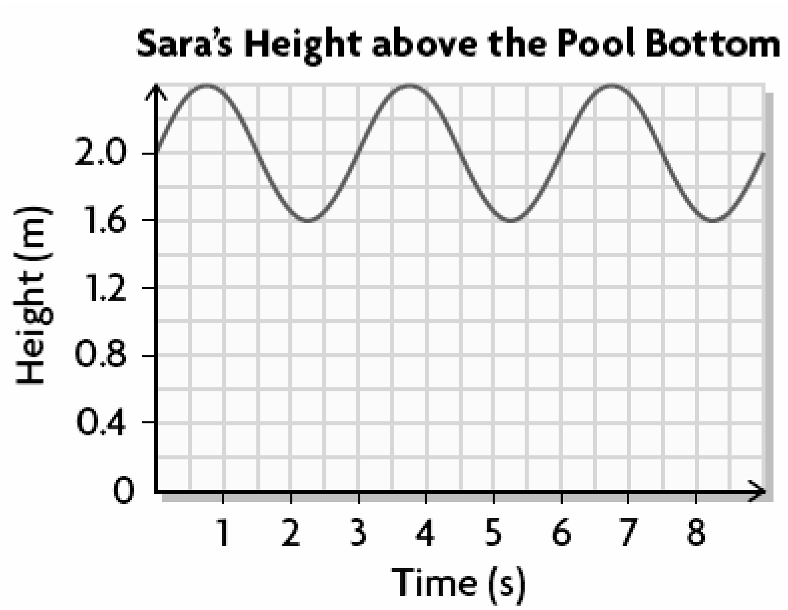
d. The ***equilibrium axis*** is: ____________

e. The ***amplitude*** is: ____________

III. Sinusoidal Functions

A **sinusoidal function** is a type of *periodic function* created by **transformations of *f(x) = sin x.***

Ex. 2: Sara is sitting in an inner tube in a wave pool. The depth of the water below her in terms of time can be represented by the graph shown below.



Determine each of the following:

a) The ***period***: ___________

b) The ***maximum*** height: ___________

c) The ***minimum*** height: ___________

d) The ***equilibrium axis*** is: ___________

e) The ***amplitude*** is: ___________

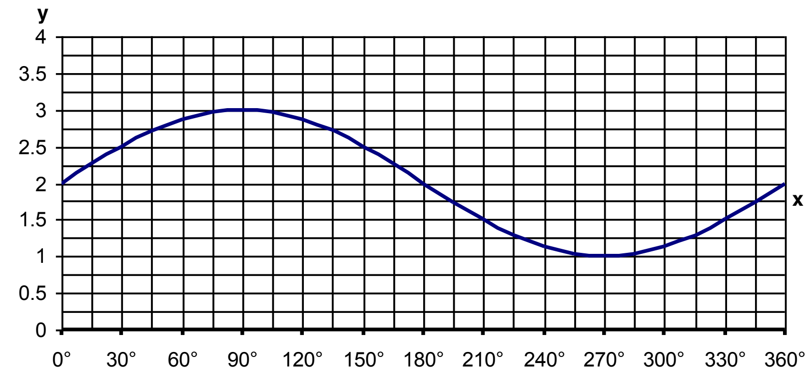
f) The ***domain*** is: ___________________________________

g) The ***range*** is: ___________________________________

 Name: ________________________

WORKSHEET: Characteristics of Sinusoidal Functions

Answer questions a–g for each of the following sinusoidal functions.



a) ***Period***: ______________________________

b) ***Maximum*** height: ____________________

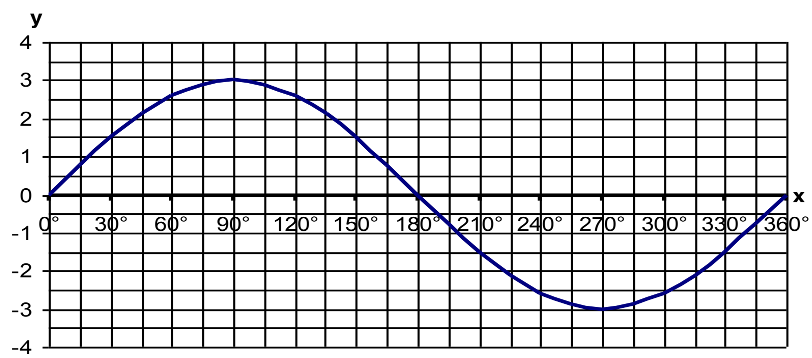
c) ***Minimum*** height: ____________________

d) ***Equilibrium axis***: ___________________

e) ***Amplitude***: ________________________

f) ***Domain***: ___________________________

g) ***Range***: ____________________________



a) ***Period***: ______________________________

b) ***Maximum*** height: ____________________

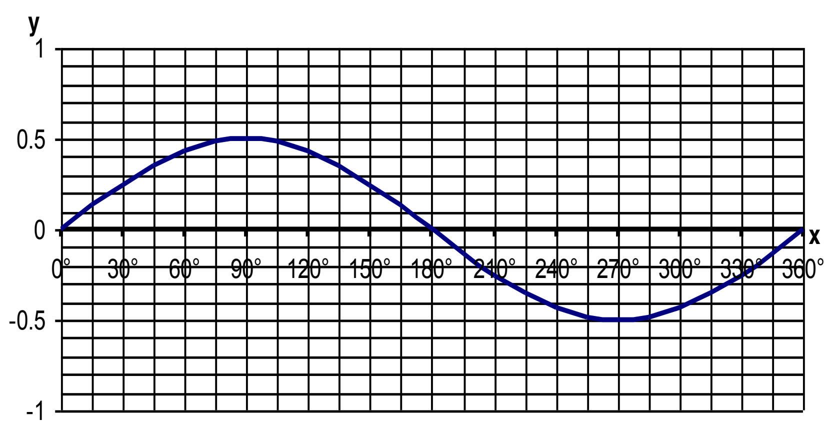
c) ***Minimum*** height: ____________________

d) ***Equilibrium axis***: ___________________

e) ***Amplitude***: ________________________

f) ***Domain***: ___________________________

g) ***Range***: ____________________________



a) ***Period***: ______________________________

b) ***Maximum*** height: ____________________

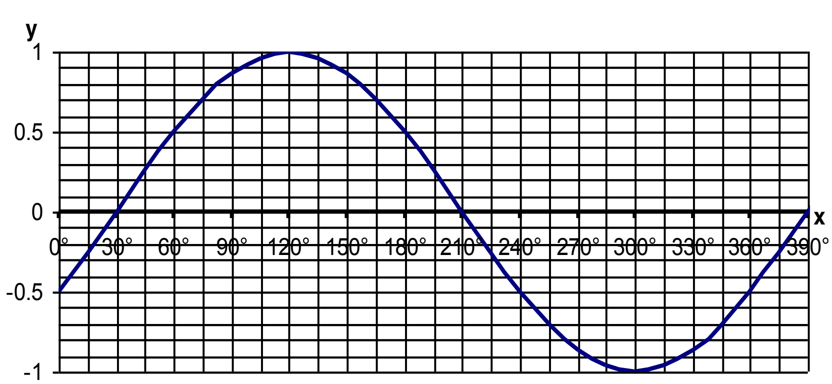
c) ***Minimum*** height: ____________________

d) ***Equilibrium axis***: ___________________

e) ***Amplitude***: ________________________

f) ***Domain***: ___________________________

g) ***Range***: ____________________________



a) ***Period***: ______________________________

b) ***Maximum*** height: ____________________

c) ***Minimum*** height: ____________________

d) ***Equilibrium axis***: ___________________

e) ***Amplitude***: ________________________

f) ***Domain***: ___________________________

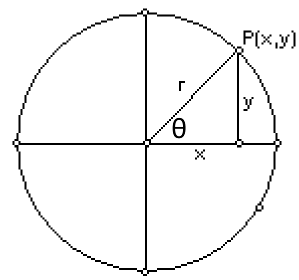
g) ***Range***: ____________________________

Transformations of the Sine Function – Part I

I. Vertical Reflections and Stretches: *y* = *a* sin *x*

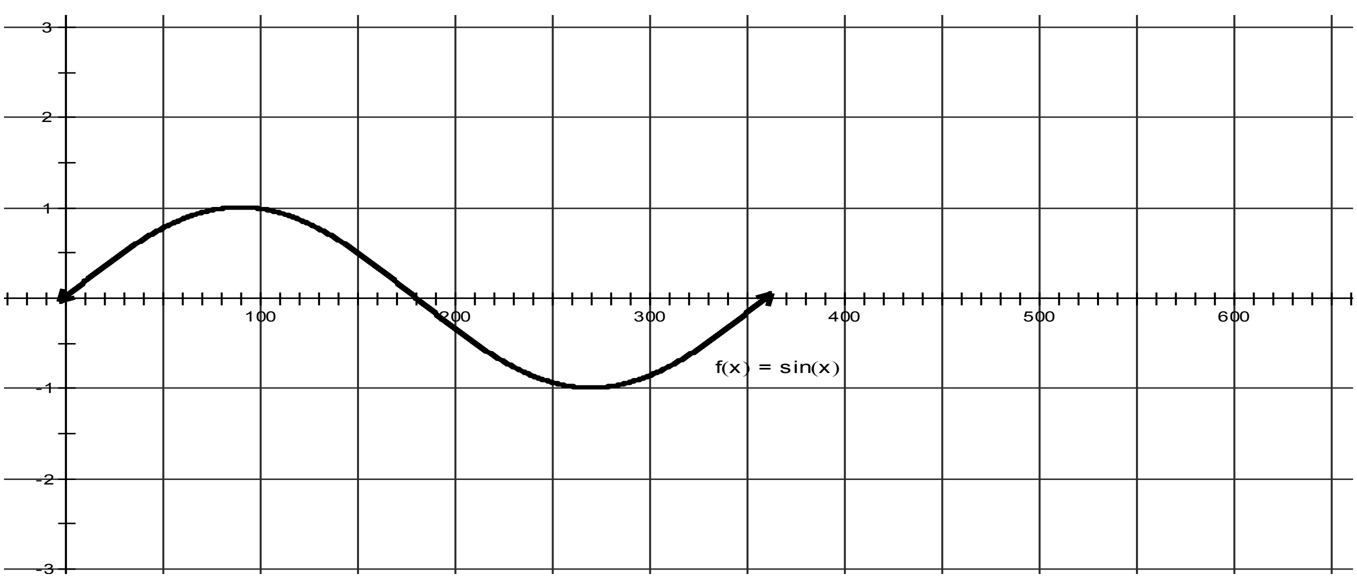
Let's reconsider our circle with centre (0, 0) on a coordinate grid …

- the point P(*x*, *y*) is any point on the **terminal arm of the circle**
- the letter *r* defines the radius of the circle
- not all circles will have a radius of 1 unit! So, how can we rewrite the sine ratio to isolate y?



Given the graph of *f(x)* = sin *x,* complete the table using a calculator and plot the points on the corresponding grid.

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
a) $f(x)=2\sin x$													
b) $f(x)=-\sin x$													
c) $f(x)=0.5\sin x$													



From the graph we can determine the range of each *sinusoidal* function:

a) For $f(x)=2\sin x$, **R =** ___________________________________

b) For $f(x)=-\sin x$, **R =** ___________________________________

c) For $f(x)=0.5\sin x$, **R =** ___________________________________

II. Summary

Reflections on the Function *y = sin x*:

Reflection	Mathematical Form	Effect
Vertical	$y = -\sin x$	Compared to $y = \sin x$, the graph of $y = -\sin x$ is a vertical reflection across the x- axis. The point (x, y) on $y = \sin x$ becomes the point (x, – y) on $y = -\sin x$.

Stretches on the Function *y* = sin *x*:

Stretch	Mathematical Form	Effect
Expansion	$y = a \sin x$ ***where a* > 1**	• If **a > 1**, the graph is **vertically expanded** by a factor of **a**. • The point (x, y) on $y = \sin x$ becomes the point (x, ay) on $y = a \sin x$. • Amplitude = a
Compression	$y = a \sin x$ **where 0 < *a* < 1**	• If **0 < a < 1**, the graph is **vertically compressed** by a factor of **a**. • The point (x, y) on $y = \sin x$ becomes the point (x, ay) on $y = a \sin x$. • Amplitude = a

One cycle of a sine function includes a *maximum*, a *minimum*, and *three zeros*. From this point forward, we will use a ***five-point method*** to sketch the graph of a sine function, using its amplitude and period.

Ex. 1: Sketch one cycle of the sine function ***y = 6 sin x*** and answer the following questions:

a. The ***period*** is: ____________

b. The ***maximum*** value: is _________

c. The ***minimum*** value is: __________

d. The ***equilibrium axis*** is: __________

e. The ***amplitude*** is: ____________

f. The ***domain*** is: ________________________

g. The ***range*** is: __________________________

Ex. 2: Sketch one cycle of the sine function ***y = – 0.25 sin x*** and answer the following questions:

a. The ***period*** is: ____________

b. The ***maximum*** value: is _________

c. The ***minimum*** value is: __________

d. The ***equilibrium axis*** is: __________

e. The ***amplitude*** is: ____________

f. The ***domain*** is: _________________________

g. The ***range*** is: ___________________________

HW: p. 261-263 #1, 5b, 6b, 7, 9ab, 11ac, 13a, 14

1. Sketch a graph of each function for $0° \leq x \leq 360°$. Determine the period, amplitude, domain, and range.

a) $y = 6 \sin x$

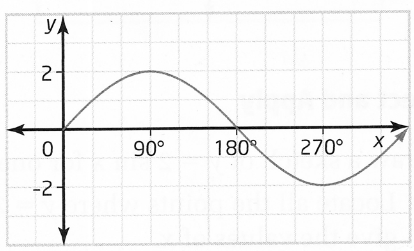
b) $y = \frac{1}{3} \sin x$

c) $y = -2 \sin x$

d) $y = -\frac{1}{2} \sin x$

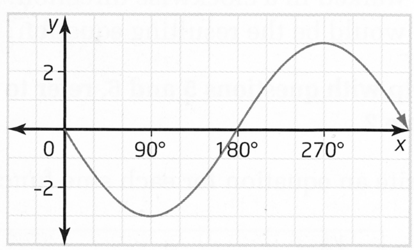
5. Write an equation for each sine function.

b)



6. Write an equation for each sine function.

b)



7. Draw a sketch of $y = 2 \sin x$ for one period.

a) Locate all the points where $y = 2$ and give the values of x.

b) Locate all the points where $y = -2$ and give the values of x.

9. Graph one cycle of each function. Label the x-intercepts, the maximum points, the minimum points , and the equation of the horizontal axis. Write the domain and range of the cycle.

a) $f(x) = 0.4 \sin x$

b) $f(x) = -4 \sin x$

11. For each of the functions, find the coordinates of the maximum and minimum points.

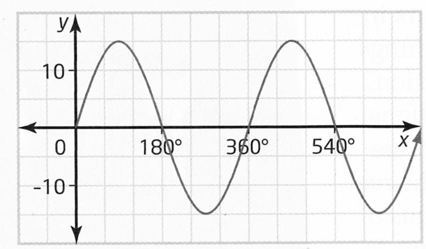
a) $y = \sin x$ **b)** $y = \sin x + 12$

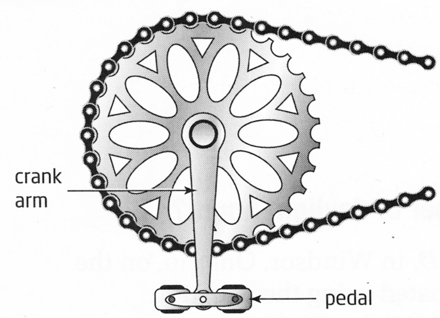
c) $y = -10 \sin x$ **d)** $y = \sin(x - 60°)$

13. Write an equation for each sine function. Indicate the intervals in which the function is increasing and decreasing over one period.

a) amplitude = 10, horizontal axis along the x-axis

14. **Chapter Problem** A physiotherapist establishes a treatment plan for a patient that includes the use of an exercise bicycle. The graph shows the height of a bicycle pedal above its crank arm's horizontal position, relative to the rotational angle, as the patient is pedalling it.





a) Identify the period, amplitude, and phase shift. What does each represent in this situation?

b) Write an equation that represents the function.

1. Sketches may vary.

a) period = 360°; amplitude = 6;
domain = $\{x \in \mathbf{R}\}$;
range = $\{y \in \mathbf{R} \mid -6 \leq y \leq 6\}$

b) period = 360°; amplitude = $\frac{1}{3}$;
domain = $\{x \in \mathbf{R}\}$;
range = $\{y \in \mathbf{R} \mid -\frac{1}{3} \leq y \leq \frac{1}{3}\}$

c) period = 360°; amplitude = 2;
domain = $\{x \in \mathbf{R}\}$;
range = $\{y \in \mathbf{R} \mid -2 \leq y \leq 2\}$

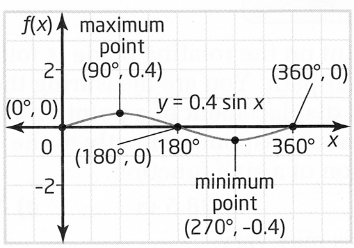
d) period = 360°; amplitude = $\frac{1}{2}$;
domain = $\{x \in \mathbf{R}\}$;
range = $\{y \in \mathbf{R} \mid -\frac{1}{2} \leq y \leq \frac{1}{2}\}$

5. a) $y = \sin x + 3$ **b)** $y = 2 \sin x$
c) $y = \sin(x - 90°)$

6. a) $y = \sin x - 3.5$ **b)** $y = -3 \sin x$
c) $y = \sin(x + 150°)$

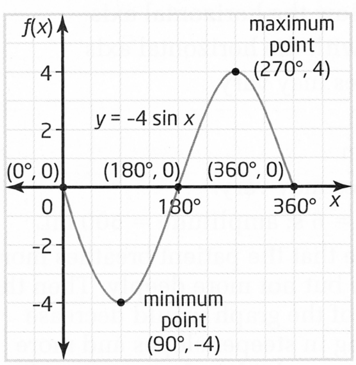
7. Sketches may vary.
a) $x = 90°$ **b)** $x = 270°$

9. a)



horizontal axis: $y = 0$;
domain = $\{x \in \mathbf{R} \mid 0° \leq x \leq 360°\}$;
range = $\{f(x) \in \mathbf{R} \mid -0.4 \leq f(x) \leq 0.4\}$

b)



horizontal axis: $y = 0$;
domain = $\{x \in \mathbf{R} \mid 0° \leq x \leq 360°\}$;
range = $\{f(x) \in \mathbf{R} \mid -4 \leq f(x) \leq 4\}$

11. a) maximum (90°, 1), minimum (270°, −1)
b) maximum (90°, 13), minimum (270°, 11)
c) maximum (270°, 10), minimum (90°, −10)
d) maximum (150°, 1), minimum (330°, −1)

13.

Function	Domain for One Period	Increasing Intervals	Decreasing Intervals
a) $y = 10 \sin x$	$0° \leq x \leq 360°$	$0° \leq x \leq 90°$, $270° \leq x \leq 360°$	$90° \leq x \leq 270°$
b) $y = \sin x - 8$	$0° \leq x \leq 360°$	$0° \leq x \leq 90°$, $270° \leq x \leq 360°$	$90° \leq x \leq 270°$
c) $y = \sin(x + 40°)$	$-40° \leq x \leq 320°$	$-40° \leq x \leq 50°$, $230° \leq x \leq 320°$	$50° \leq x \leq 230°$
d) $y = \sin(x - 100°)$	$100° \leq x \leq 460°$	$100° \leq x \leq 190°$, $370° \leq x \leq 460°$	$190° \leq x \leq 370°$

14. a) period = 360°; This represents the number of degrees through which the crank arm rotates for each rotation of the pedal; amplitude = 15 cm; This represents the length of the crank arm and corresponds to the maximum displacement of the pedal from the horizontal position of the arm; There is no phase shift because the horizontal displacement is 0 cm when the angle of the arm to horizontal is 0°.
b) $y = 15 \sin x$

Transformations of the Sine Function – Part II

I. Vertical Translations and Phase Shifts on *y = a* sin (*x – d) + c*

Like quadratic functions, sine functions can undergo translations up/down and left/right according to the function *y = a* sin (*x – d) + c*, where *a* denotes the amplitude, *d* denotes a phase shift (horizontal translation left or right) and *c* denotes a vertical translation up or down.

Transformation	**Transformed Function**	**Effect on *y = sin x***
Horizontal Translation or PHASE SHIFT	*y = sin (x – d)*	If ***d* > 0**, the **PHASE SHIFT** of the graph is *d* degrees to the **right** If ***d* < 0**, the **PHASE SHIFT** of the graph is *d* degrees to the **left**
Vertical Translation	*y = sin x + c*	If ***c* > 0**, the graph is **vertically translated up** *c* units If ***c* < 0**, the graph is **vertically translated down** *c* units Axis of equilibrium is: **y = *c***

Building on our use of a five-point method to sketch the graph of a sine function, we will be adding *phase shift and vertical translation* to our considerations (we used *period* and *amplitude* in the last lesson!).

II. Graph following functions using transformations on *y = sin x*. Label the equilibrium axis.

a) y = sin x + 1 A = ______; P = _______; P.S. = ___________________ ; V.T = ___________________
Range = ____

b) y = sin (x – 20°) A = ______; P = _______; P.S. = ___________________ ; V.T = ___________________
Range = ____

c) y = 3 sin x – 2 A = ______; P = _______; P.S. = ___________________ ; V.T = ___________________
Range = ____

d) y = – sin (x + 40°) + 3 A = ______; P = _______; P.S. = ___________________ ; V.T = ______________
Range = ____

III. Write the equation of a sinusoidal function undergoing the following transformations on *y = sin x*.

a) amplitude of 7 units, vertical translation down 15 units, and phase shift 120° left.

b) reflection across the x-axis, amplitude of 0.15 units, vertical translation up 4 units, and phase shift 10° right.

c) reflection across the x-axis, amplitude of 55 units, vertical translation down 25 units.

HW: p. 261-262 #2a, 3b, 5, 6, 9cdef, 10

2. Compare the graphs of each pair of functions for $0° \le x \le 360°$. Determine the period, amplitude, domain, and range, as well as the equation of the horizontal axis.

a) $y = \sin x + 6$ and $y = \sin x - 6$

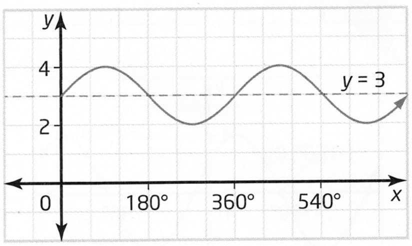
3. Compare the graphs of each pair of functions for $0° \le x \le 360°$. Determine the period, amplitude, phase shift, domain, and range.

a) $y = \sin(x - 60°)$ and $y = \sin(x + 60°)$

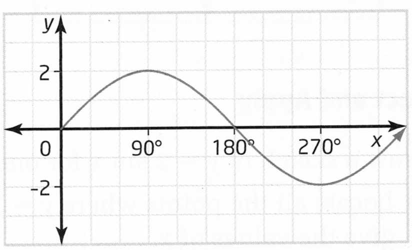
b) $y = \sin(x - 270°)$ and $y = \sin(x - 90°)$

5. Write an equation for each sine function.

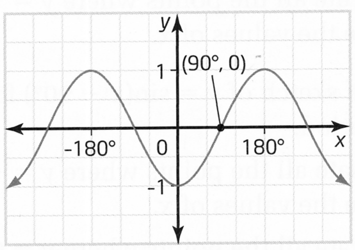
a)



b)

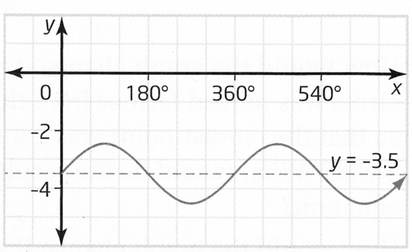


c)

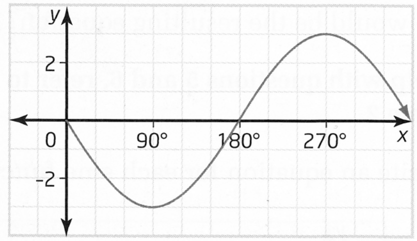


6. Write an equation for each sine function.

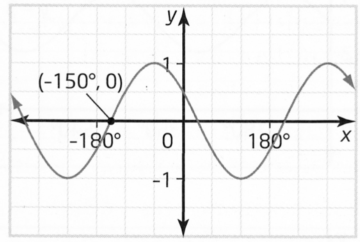
a)



b)



c)



9. Graph one cycle of each function. Label the x-intercepts, the maximum points, the minimum points , and the equation of the horizontal axis. Write the domain and range of the cycle.

a) $f(x) = 0.4 \sin x$

b) $f(x) = -4 \sin x$

c) $f(x) = \sin x - 5$

d) $f(x) = \sin x + 8$

e) $f(x) = \sin(x - 45°)$

f) $f(x) = \sin(x + 90°)$

B

10. Without graphing, consider each sine function. Identify the period, amplitude, phase shift, domain, and range, as well as the equation of the horizontal axis.

a) $y = \sin x - 30$ **b)** $y = \sin(x - 30)$

2. a)

	Period (°)	Amplitude	Horizontal Axis	Domain	Range
$y = \sin x + 6$	360	1	$y = 6$	$x \in \mathbf{R}$	$5 \le y \le 7$
$y = \sin x - 6$	360	1	$y = -6$	$x \in \mathbf{R}$	$-7 \le y \le -5$

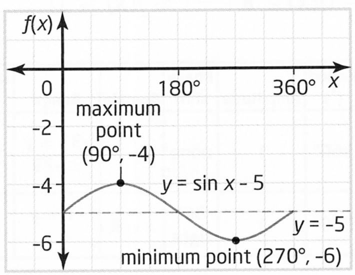
3. b)

	Period (°)	Amplitude	Phase Shift	Domain	Range
$y = \sin(x - 270°)$	360	1	270° right	$x \in \mathbf{R}$	$-1 \le y \le 1$
$y = \sin(x - 90°)$	360	1	90° right	$x \in \mathbf{R}$	$-1 \le y \le 1$

5. a) $y = \sin x + 3$ **b)** $y = 2 \sin x$
c) $y = \sin(x - 90°)$

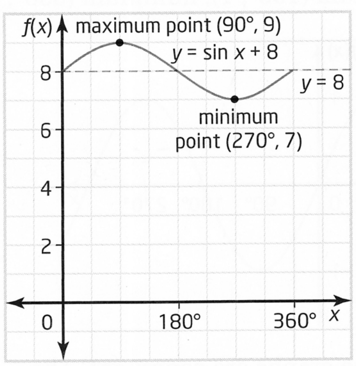
6. a) $y = \sin x - 3.5$ **b)** $y = -3 \sin x$
c) $y = \sin(x + 150°)$

9. c)



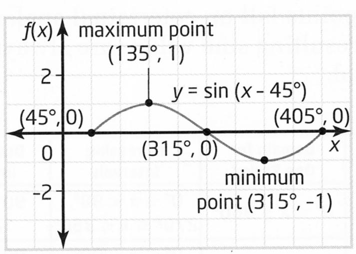
horizontal axis: $y = -5$;
domain = $\{x \in \mathbf{R} \mid 0° \le x \le 360°\}$;
range = $\{f(x) \in \mathbf{R} \mid -6 \le f(x) \le -4\}$

d)



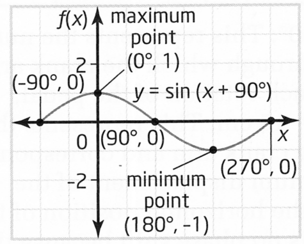
horizontal axis: $y = 8$;
domain = $\{x \in \mathbf{R} \mid 0° \le x \le 360°\}$;
range = $\{f(x) \in \mathbf{R} \mid 7 \le f(x) \le 9\}$

e)



horizontal axis: $y = 0$;
domain = $\{x \in \mathbf{R} \mid 45° \le x \le 405°\}$;
range = $\{f(x) \in \mathbf{R} \mid -1 \le f(x) \le 1\}$

f)



horizontal axis: $y = 0$;
domain = $\{x \in \mathbf{R} \mid -90° \le x \le 270°\}$;
range = $\{f(x) \in \mathbf{R} \mid -1 \le f(x) \le 1\}$

10.

Function	Period (°)	Amplitude	Phase shift	Domain	Range	Horizontal Axis
a) $y = \sin x - 30$	360°	1	none	$x \in \mathbf{R}$	$-31 \le y \le -29$	$y = -30$
b) $y = \sin(x - 30)$	360°	1	30° right	$x \in \mathbf{R}$	$-1 \le y \le 1$	$y = 0$

5.5 Make Connections With Sine Functions

How can you model periodic phenomena such as the number of daylight hours in a year or the volume of air in your lungs as you breathe? In the previous section, you investigated single transformations of the sine function. However, accurate models of periodic behaviours in the real world typically involve combinations of transformations of the sine function.

Tools

- graphing calculator

Technology Tip

You can find the number of days between dates in the format DDMM.YY using the dbd(function. For example, find the number of days between January 1, 2007, and May 13, 2007, as follows:

- Press 2nd [CATALOG] and select **dbd(**.
- Enter 0101.07. Press , . Enter 1305.07 and press) ENTER.

May 13, is day 132 in 2007.

Investigate

How can you model the number of daylight hours?

The number of daylight hours, D, in Windsor, Ontario, on the nth day of the year can be estimated using the equation $D(n) = 2.9607\sin(0.9863n - 77.8374) + 12.0318$.

1. Use a graphing calculator to graph this function for two years. Think about the appropriate window settings before you start. Make sure your calculator is in Degree mode.
2. Predict when the maximum number of hours will occur. Use the Maximum operation of the graphing calculator to check. When does this occur?
3. Predict when the minimum number of hours will occur. Check your prediction using the Minimum operation of the graphing calculator. When does this occur?
4. When is the number of daylight hours increasing? Decreasing?
5. Identify the horizontal axis of the graph. What does it represent?
6. What is the period of this graph? Explain.

Getting to Know Your Free Online Graphing Calculator

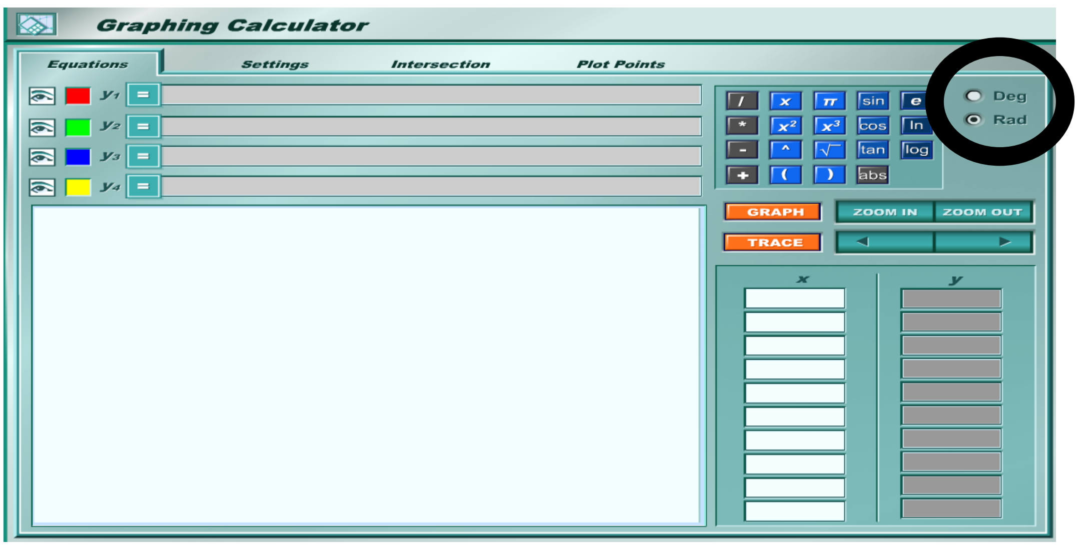
Today, we will be using the free online graphing calculator extensively and, as there is an entire component on your unit test devoted to work with this online graphing calculator, it's an excellent opportunity to learn how to use it more effectively and efficiently!

As we move through an exploration of the online graphing calculator, we will be working through the "Investigate" question from your textbook, on page 264. Let's go there now!

<u>Before You Get Started:</u>

1.) How to change the units from radians ("Rad") to degrees ("Deg"):

- Go to the top right corner of the free online graphing calculator and change from radians to degrees



<u>Investigate:</u>

2.) Please read the question in its entirety (that means not only the set-up, but also questions 1 through 6! Now read it a second time, to pick up any details you may have missed.

3.) We've been asked to graph this function: D(*x*) = 2.9607*sin(0.9863x – 77.8374) + 12.0318. Let's type that equation into the first line of the graphing calculator.

***IMPORTANT**! You need to type an asterisk ("*") between the "a" value and "sin" for the free online graphing calculator to work properly! Also, replace the "n" with an "x"!*

4.) We've been asked to graph this function for two years. How do you represent that value on the x-axis? We also want to make sure that we can see where the action is taking place on the graph. These considerations inform how you "set the window" of your graphing calculator.

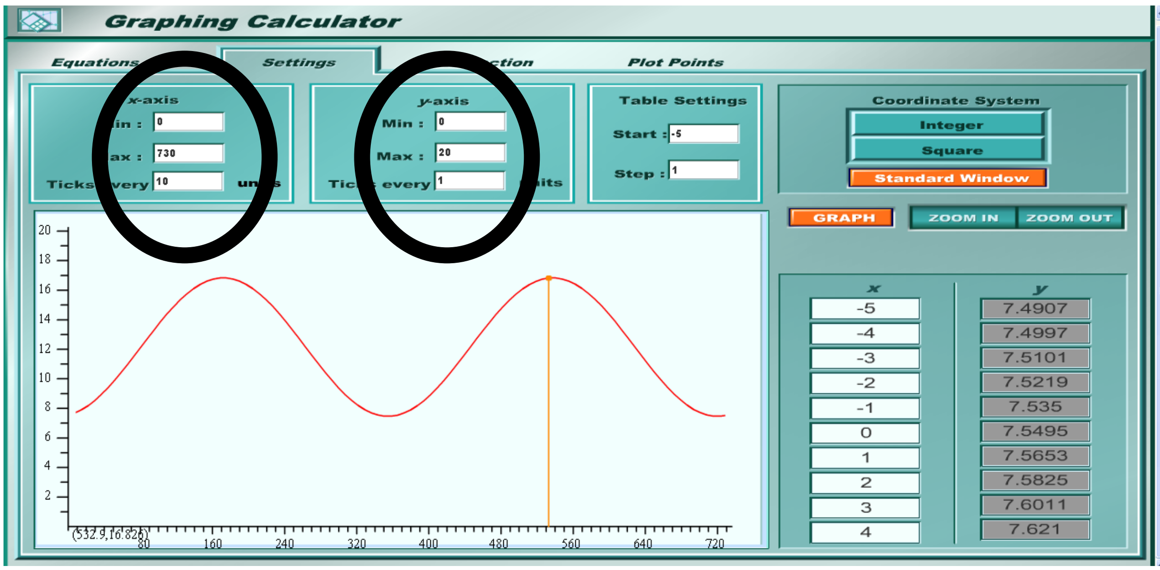
Given what we know about the key features of a sine function, what can we say about the range of the function in this question? How can we figure it out? What pieces of the equation can help us?

Take a moment to make a few rough calculations and see if you can figure out the range.

5.) How to set the window for the x-values you want to see:

- To make any changes to the viewing window in our free online graphing calculator, you need to click into "Settings", the second tab from the left at the top.

- For this investigation, we are asked to graph the function for two years. Since our x-axis represents number of days, you need to model the interval from 0 days to 730 days (730 days is equal to two years, since there are 365 days in a year).
- Set the "X-Axis Min" value = 0 and set the "X-Axis Max" value = 730.



6.) How to set the window for the y-values so you want to see:

- For this investigation, we are asked to model the number of hours of daylight in Windsor; this information will be on the y-axis.
- Recall that we can determine the range of our function by looking at the "a" value (which is our amplitude, or height above/below our equilibrium axis) and the "c" value (which gives us our equilibrium axis).
- If the "c" value is about 12, and the "a" value is about 3, then we can determine our range to be 15 hours on the high end (12 + 3) and 9 hours on the low end (12 – 3)
- Set the "Y-Axis Min" value = 9 and set the "Y-Axis Max" value = 15.

7.) Click the "Graph" button and you will complete the first question in our investigation.

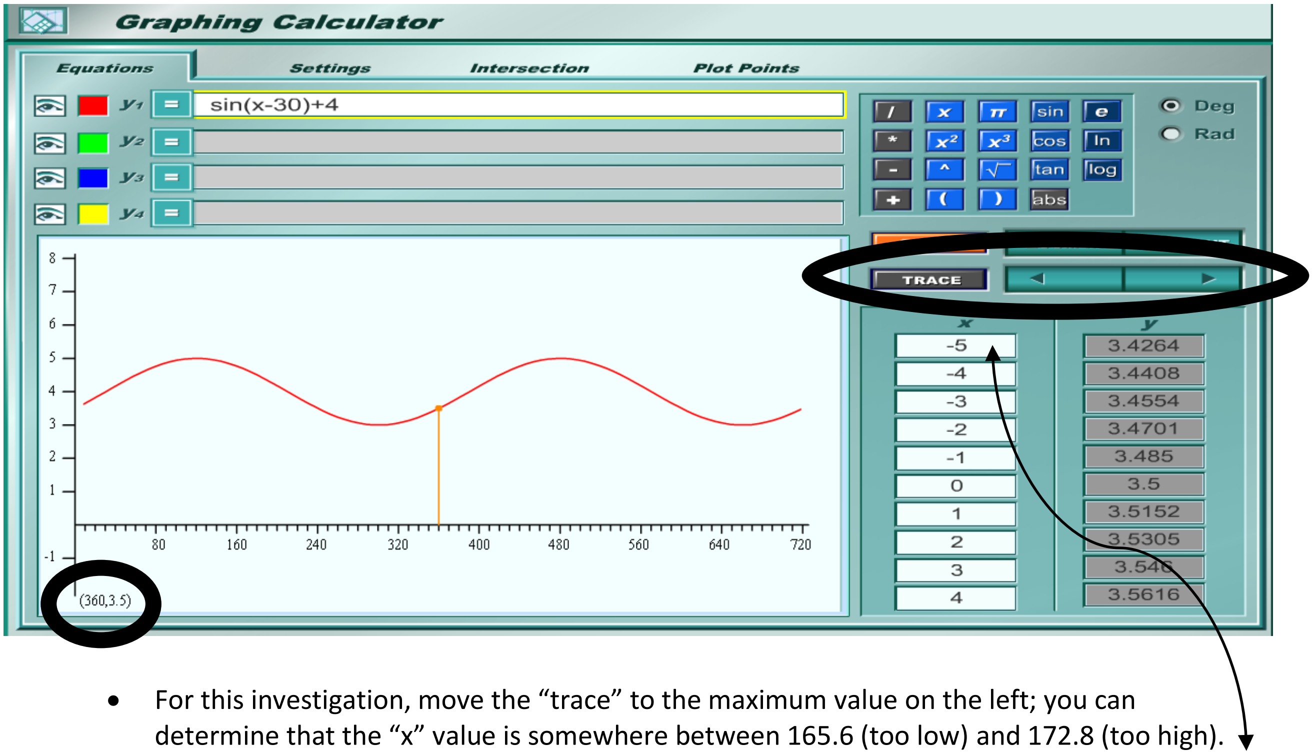
8.) We've been asked to predict when both the maximum and minimum number of hours of daylight will occur. Even before we use the graph or our knowledge of sine functions, let's *think* about this.
We're looking for the longest day of the year in Canada – when is that?
We're also looking for the shortest day of the year in Canada – any ideas?
Google it, people!

9.) Hopefully, the Internet has given you an answer, but let's use some mathematics and arrive at the same answer another way. We are looking for the maximum value and the minimum value on this sine function. Those points represent the "peak" and "trough" of our graph, which we've already calculated as a part of our range: the "peak" should be around 15 hours and the "trough" around 9 hours.

So, if we've got an approximate "y" value, how can find the corresponding "x" value?

10.) How to use the "Trace" function to find specific points on the graph:

- Click the "Equations" tab in the top left corner of the graphing calculator
- Click the "Trace" button and watch a magic orange vertical line appear in the graph
- Move this line left and right along the curves of the graph using the two arrows to the right of the "Trace" button
- Notice in the bottom right corner, the coordinates of the point that is being mapped are displayed



- For this investigation, move the "trace" to the maximum value on the left; you can determine that the "x" value is somewhere between 165.6 (too low) and 172.8 (too high).
- You can zero in on the exact value by typing specific numbers (for example, 168) into the table of values to the bottom right of the free online graphing calculator; it will then produce the corresponding "y" value
- Try the numbers between 166 and 172 and see which one produces the largest "y" value

11.) Use the same methods to make predictions about the minimum value.
Ultimately, you should find that the maximum value occurs around 170 (which is June 19th – summer solstice) and the minimum value occurs around 353 (which is December 19th – winter solstice)
Check it out: http://mistupid.com/calendar/dayofyear.htm, http://en.wikipedia.org/wiki/Summer_solstice, http://en.wikipedia.org/wiki/Winter_solstice

That takes care of questions 2 and 3.

12.) Question 4 involves observations of this function. The number of daylight hours is increasing when the function is increasing: between 0 and 170 (which is January 1 to June 19), between 353 and 535 (which is December 20 to June 19), and between 719 and 730 (which is December 20 to December 31). The number of daylight hours is decreasing when the function is decreasing: between 171 and 352 (which is June 20 to December 19) and between 536 and 718 (which is June 20 to December 19).

13.) Question 5 involves the equilibrium axis, which the textbook calls the "horizontal axis". What does it represent in this equation? It represents the average number of daylight hours per year in Windsor.

14.) Question 6 involves the period of the graph. In this investigation, the period of the graph is a calendar year, which is 365 days.

Applications of Sine Functions

Work ***with a partner*** or ***on your own*** to complete the following questions:

Using a graphing calculator:

- p. 266-267
  # 1, 2, 6, 8

Sketching by hand:

- p. 267
  # 5

You will submit your work at the end of the class for grading. Happy graphing!

1. The number of daylight hours, D, in Edmonton, Alberta, on the nth day of the year can be approximated by the equation $D(n) = 4.6855\sin(0.9856n - 79.1491) + 12.1512$.
   a) Graph the function for two years.
   b) Determine the maximum number of daylight hours, and when it occurs.
   c) Determine the minimum number of daylight hours, and when it occurs.
   d) Compare the number of daylight hours in Edmonton to that in Windsor, season by season.
   e) Edmonton is situated at approximately 53° north latitude. What change would need to be made to the equation for daylight hours in Punta Arenas, Chile, at 53° south latitude?

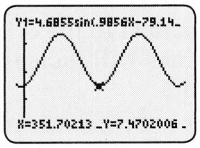
2. A wind turbine uses rotating blades to produce electricity. The equation $h(\theta) = 8.5\sin(\theta + 180°) + 30$ can be used to find the height, $h(\theta)$, in metres, of a point on a given blade, where θ is the angle the blade makes with the horizontal.
   a) Graph the function. Set Xmin = 0 and Xmax = 720, and use ZoomFit to view the graph. Sketch the graph.
   b) Explain why a sine function was used to model the height of a point on a blade.
   c) What was the initial height of the point?
   d) What is the maximum height of the point?
   e) How high above the ground is the axis of a blade?

6. The CN Tower in Toronto, Ontario is approximately 554 m tall. In strong winds, the top of the communication tower will sway up to 2 m. On a particular day, the displacement of the top of the tower in metres, relative to the normal position, is modelled by $f(t) = 0.9\sin(2t)$, where t is the time, in minutes.
   a) Sketch a graph that shows the swaying of the top of the communication tower over 8 h.
   b) How wide is the sway?
   c) What is the period of the sway?

8. The current, i, in amperes (A), passing through a wire is given by the equation $i = 6\sin\left(\frac{2}{3}t + 30\right)$, where t is the time, in seconds.
   a) Graph the function.
   b) Determine the period, amplitude, and phase shift of the current from the graph.

5. A crank rotates such that the height of the handle, h, in metres is given by $h(\theta) = 2\sin(\theta - 90°)$, where θ is the rotational angle relative to the horizontal.
   a) Sketch a graph of this function for two rotations.
   b) At what height was the handle when the rotation began?
   c) What was the position of the handle relative to the horizontal, when the rotation began?

1. a)



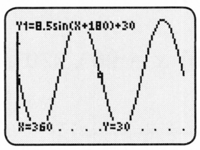
b) approximately 16.8 h on June 20

c) approximately 7.5 h on December 20

d) Through spring and summer there will be more hours of daylight in Edmonton than in Windsor. However, for fall and winter there will be fewer hours of daylight in Edmonton than in Windsor.

e) $y = -4.6855 \sin(0.9856n - 79.1491) + 12.1512$

2. a)



b) The point on the rotating blade is following a circular path. You are measuring the vertical displacement of a point on the path that corresponds directly to the definition of a sine function.

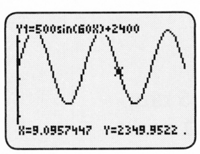
c) 30 m **d)** 38.5 m **e)** 30 m

6. a) Sketches may vary.

b) 1.8 m

c) 180 min

8. a)



b) period = 540 s; amplitude = 6 μA; phase shift = 30° to the left

5. a) Sketches may vary.

b) 2 m below the horizontal axis

c) at 90° with the horizontal axis

Review Day #1

1. Complete the ***Frayer Model* graphic organizer** to help you explain the functions of *a*, *d*, and *c* in the transformation of $y = a \sin(x - d) + c$. Include at least one worked example on the graphic organizer. **You may use this during your test!**

2. Complete the following review questions from the text:
   - p. 268 – 269
     #6, 7
     #8 (online graphing calculator)
     #9 (online graphing calculator)

   - p. 275
     # 9, 13

6. Sketch a graph of each function for one cycle. Determine the period, amplitude, phase shift, domain, range, and the equation of the horizontal axis.

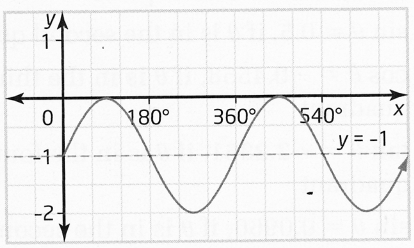
a) $y = 5 \sin x$ **b)** $y = -\frac{1}{4} \sin x$

c) $y = \sin x + 12$ **d)** $y = \sin x - 2.5$

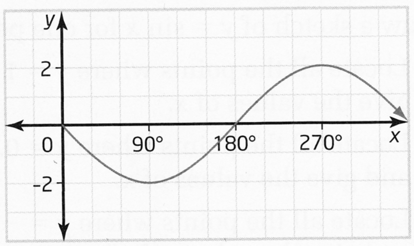
e) $y = \sin(x - 60°)$ **f)** $y = \sin(x + 90°)$

7. Write an equation for each sine function.

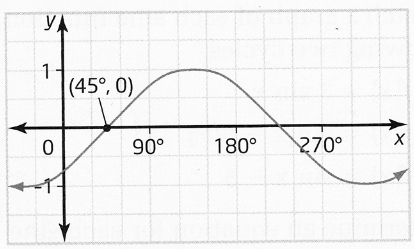
a)



b)



c)



8. Use Technology According to the biorhythm theory, three cycles affect people's lives, giving them favourable and non-favourable days. The physical cycle can be modelled as $y = \sin\left(\frac{360}{23}t\right)$, where t represents a person's age, in days. Similarly, the emotional cycle can be modelled as $y = \sin\left(\frac{90}{7}t\right)$ and the intellectual cycle as $y = \sin\left(\frac{120}{11}t\right)$. Use a graphing calculator to compare the three cycles for $0 \le t \le 60$.

9. Use Technology The height of a rider on a Ferris wheel, in metres, can be modelled using the function $h(\theta) = 10 \sin(\theta - 90°) + 12$, where θ is the angle of rotation.

a) Use a graphing calculator to graph the function.

b) What is the radius of the Ferris wheel?

c) At what height was the rider when the ride began?

d) How would the function and the graph change if the Ferris wheel turned in the opposite direction?

9. Which pairs of angles are coterminal? Justify your answer.

a) 40° and 220° **b)** 65° and 785°

c) 115° and −245° **d)** −35° and −235°

13. Sketch a graph of each function for one cycle. Determine the period, amplitude, phase shift, domain, range, and the equation of the horizontal axis.

a) $y = \sin x + 3$ **b)** $y = -2 \sin x$

c) $y = \frac{1}{2} \sin x$ **d)** $y = \sin(x + 30°)$

6. For **a)** to **f)**, sketches may vary.

a) period = 360°; amplitude = 5;
domain = $\{x \in \mathbf{R} \mid 0° \leq x \leq 360°\}$;
range = $\{y \in \mathbf{R} \mid -5 \leq y \leq 5\}$ $y = 0$

b) period = 360°; amplitude = 0.25;
domain = $\{x \in \mathbf{R} \mid 0° \leq x \leq 360°\}$;
range = $\{y \in \mathbf{R} \mid -0.25 \leq y \leq 0.25\}$ $y = 0$

c) period = 360°; amplitude = 1;
domain = $\{x \in \mathbf{R} \mid 0° \leq x \leq 360°\}$;
range = $\{y \in \mathbf{R} \mid 11 \leq y \leq 13\}$ $y = 12$

d) period = 360°; amplitude = 1;
domain = $\{x \in \mathbf{R} \mid 0° \leq x \leq 360°\}$;
range = $\{y \in \mathbf{R} \mid -3.5 \leq y \leq -1.5\}$
$y = -2.5$

e) period = 360°; amplitude = 1;
domain = $\{x \in \mathbf{R} \mid 60° \leq x \leq 420°\}$;
range = $\{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$ $y = 0$,
Phase shift = 60° right

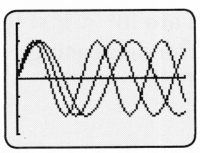
f) period = 360°; amplitude = 1;
domain = $\{x \in \mathbf{R} \mid -90° \leq x \leq 270°\}$;
range = $\{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$ $y = 0$;
phase shift = 90° left

7. a) $f(x) = \sin x - 1$

b) $f(x) = -2 \sin x$ or $f(x) = 2 \sin (x - 180°)$

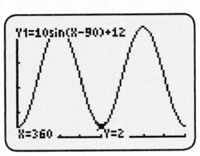
c) $f(x) = \sin(x - 45°)$

8.



All three functions have the same amplitude, horizontal axis, and initial value. They vary with respect to their periods. The physical cycle has the shortest period of about 23 days, the emotional cycle has the intermediate period of about 28 days, and the intellectual cycle has the longest period of about 33 days. At first all three cycles have positive values. At about 20 days, all the cycles are near their minimum values. As time progresses, the difference in period tends to separate the three curves.

9. a)



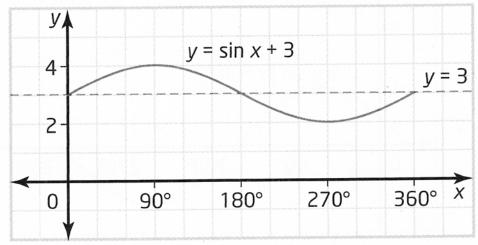
b) 10 m **c)** 2 m

d) Answers may vary.

9. a) not coterminal **b)** coterminal

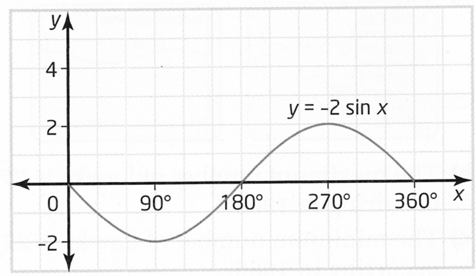
c) coterminal **d)** not coterminal

13. a)



period = 360°
amplitude = 1
phase shift = 0°
domain = $\{x \in \mathbf{R} \mid 0° \leq x \leq 360°\}$
range = $\{y \in \mathbf{R} \mid 2 \leq y \leq 4\}$
The horizontal axis has equation $y = 3$.

b)



period = 360°
amplitude = 2
phase shift = 0°
domain = $\{x \in \mathbf{R} \mid 0° \leq x \leq 360°\}$
range = $\{y \in \mathbf{R} \mid -2 \leq y \leq 2\}$
The horizontal axis has equation $y = 0$.

Review Day #2

Complete a **gallery walk** for your completed *Frayer Model*, where you leave your organizer on your desk and you rotate around the room looking at the organizers that other students created. Look for at least **one** improvement you can make to your own *Frayer Model* and then make that change. On the back of your *Frayer Model*, write what you did to make yours better based on the idea(s) you got from others.

Representative Questions for Review:

1) Sketch the following angles in standard position and label their quadrants. Then sketch 1 co-terminal angle for each:

a) 200° b) – 220°

2) Describe the transformations (in order!) to the following sine function: $y = -2 \sin(x - 30°) + 4$

3) Sketch and appropriately label the following sine function: $y = 3 \sin(x + 15°) - 2$

Textbook review questions:

p. 270 – 271 #4, 5, 8, 9, 10

#13 (online graphing calculator)

#14 (online graphing calculator)

4. Which angle is not coterminal with 150°?

A 510°

B 870°

C 1230°

D 330°

5. Sketch each angle in standard position. Then, find two coterminal angles for each given angle.

a) 30° **b)** 65° **c)** 160°

d) 180° **e)** 200° **f)** 305°

g) 193° **h)** 270°

8. Sketch a graph of each sine function, showing two cycles.

a) $f(x) = 3 \sin x$

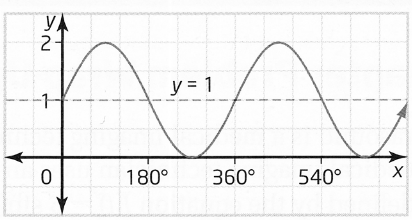
b) $f(x) = \sin(x - 60°)$

c) $f(x) = \sin x - 5$

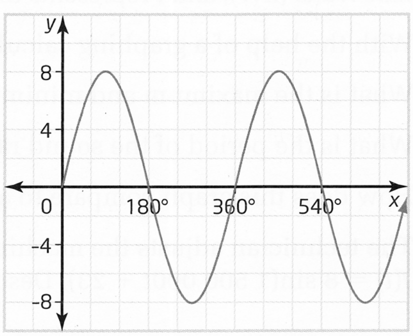
9. Determine an equation for each sine function.

a) amplitude $= \frac{1}{2}$, reflected in the x-axis

b)



c)



10. Write an equation for each sine function. Indicate the intervals in which the function is increasing and decreasing over one period.

a) amplitude = 5, horizontal axis along the x-axis

b) amplitude = 1, horizontal axis along $y = 3$

c) amplitude = 1, horizontal axis along the x-axis, phase shift of 90° to the right

d) amplitude = 1, horizontal axis along the x-axis, phase shift of 30° to the left

13. **Use Technology** The height, h, in metres, of a point on a boat's propeller, relative to the surface of the water is defined by $h(\theta) = -0.12 \sin(\theta + 90°) - 0.43$, where θ is the rotational angle, in degrees.

a) With the help of a graphing calculator, sketch a graph of the function for three rotations of the propeller.

b) What would be the maximum and minimum depths of the propeller?

c) Determine the depth of the given point on the propeller after rotations of 240° and 600°. Explain the results.

Achievement Check

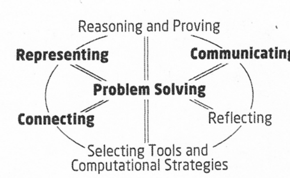
14. **Use Technology** The number of people, in millions, who used public transit in a large city during any given month can be modelled by the function $f(x) = 2.3 \sin(30x + 30) + 4.7$, where x represents the month, with January = 1, February = 2, and so on.

a) Use a graphing calculator to graph the function for one year.

b) Approximately how many people used public transit in August?

c) During which month was ridership at its highest level?

d) In which two months did about 4 000 000 people use public transit?



e) In what ways might global warming affect this model?

4. D

5. Sketches and answers may vary.

a) $-330°$; $390°$; $750°$ **b)** $-295°$; $425°$; $785°$

c) $-200°$; $520°$; $880°$ **d)** $-180°$; $540°$; $900°$

e) $-160°$; $560°$; $920°$ **f)** $-55°$; $665°$; $1025°$

g) $-167°$; $553°$; $913°$ **h)** $-90°$; $630°$; $990°$

8. For a) to c), sketches may vary.

9. a) $f(x) = -\frac{1}{2} \sin x$ **b)** $f(x) = \sin x + 1$

c) $f(x) = 8 \sin x$

10. a) $f(x) = 5 \sin x$

The increasing intervals are $\{0° < x < 90°\}$ and $\{270° < x < 360°\}$; The decreasing interval is $\{90° < x < 270°\}$.

b) $f(x) = \sin x + 3$

The increasing intervals are $\{0° < x < 90°\}$ and $\{270° < x < 360°\}$; The decreasing interval is $\{90° < x < 270°\}$.

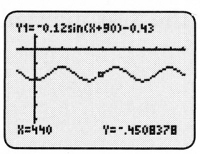
c) $f(x) = \sin (x - 90°)$

The increasing intervals are $\{90° < x < 180°\}$ and $\{360° < x < 450°\}$; The decreasing interval is $\{180° < x < 360°\}$.

d) $f(x) = \sin (x + 30°)$

The increasing intervals are $\{-30° < x < 60°\}$ and $\{240° < x < 330°\}$; The decreasing interval is $\{60° < x < 240°\}$.

13. a)



b) maximum depth = 55 cm; minimum depth = 31 cm

c) 37 cm; 240° and 600° are coterminal angles.

Name: _______________

Summarizing the roles of a, d & c in $y = a \sin (x - d) + c$

Complete the following graphic organizer to summarize the roles of *a*, *d* and *c* in the transformation and resulting sketch of a sine function.

Role of *a*:	**Role of *d*:**
Role of *c*:	**Example**

$y = a \sin (x - d) + c$