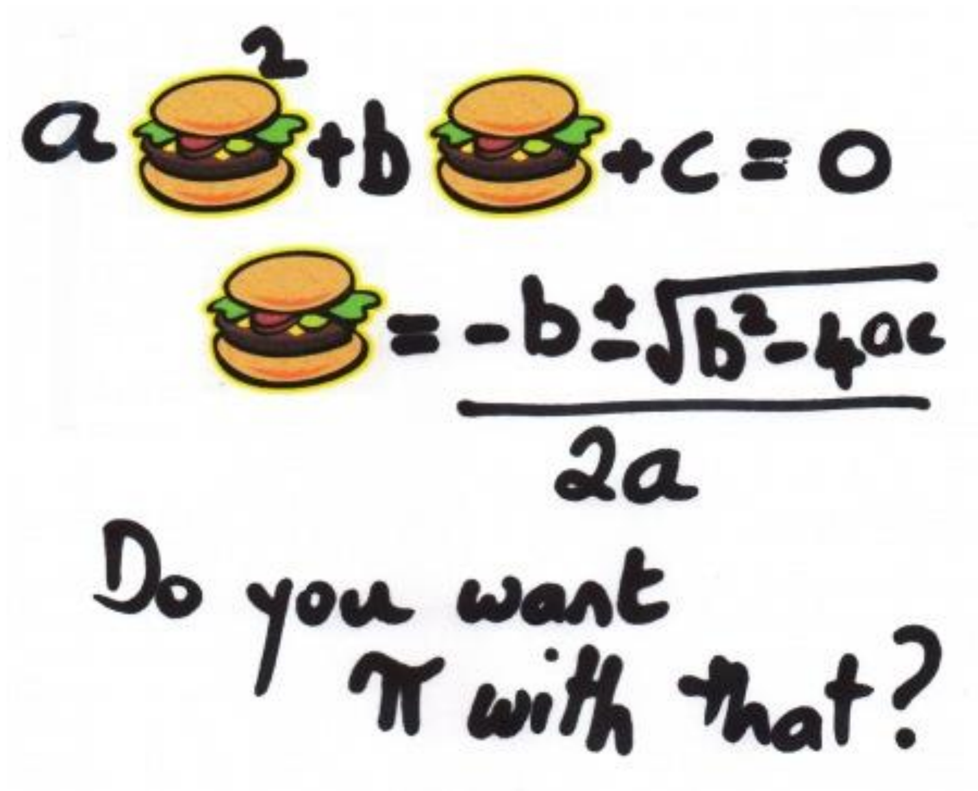


MCF3MI

Unit 3: Solving Quadratic Equations

UNIT 3: SOLVE QUADRATIC EQUATIONS – ESSENTIAL LEARNINGS

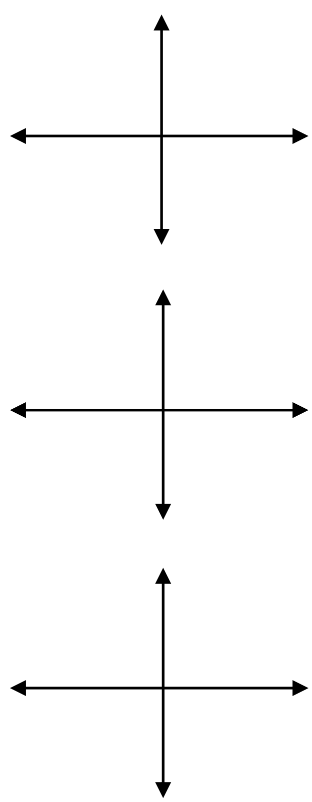
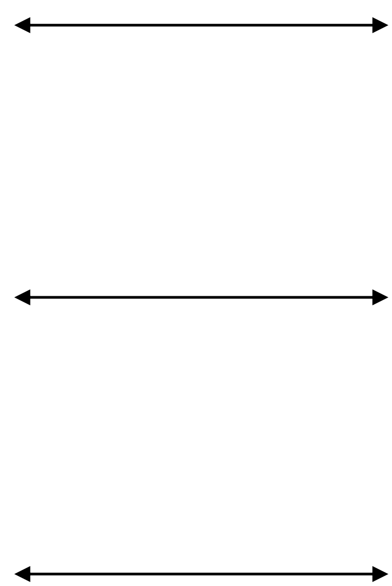
You will have at least three opportunities to demonstrate the essential learnings in this course. The first opportunity will occur during the unit with formative quizzes and homework checks. Your second opportunity will be on the unit test. Your third opportunity will be on the exam. You need to demonstrate all essential learnings to earn this credit.

Please take the opportunity to keep track of the essential learnings you've demonstrated during this unit using the checklist below.

Essential Learnings: Solve Quadratic Equations	***Homework***	***Associated Lessons***
☐ Solve quadratic equations by rearranging and square-rooting	*worksheet*	Lesson 3.1
☐ Solve quadratic equations by selecting and applying a factoring strategy	*pg. 112* #1ace, 2acegh, 4, 5, 7	Lesson 3.2
☐ Solve quadratic equations using the quadratic formula	*pg. 142 – 143* #1ace, 2aceg, 4bdfg, 5	Lesson 3.3
☐ Connect the number of real roots in a quadratic equation to the value of the discriminant	*pg. 150 – 152* #1 – 3, 4aceg, 8ace, 9, 10, 16, 17	Lesson 3.4
☐ Represent situations using quadratic expressions in one variable, then expand, simplify, and solve	*pg. 143 – 144* #7 – 9, 13 + *worksheet*	Lesson 3.5
Unit Review:	*worksheet* *pg. 115* #18, 19 *pg. 174* #4, 5, 7 *pg. 174* #6 *pg. 151 – 152* #6, 7, 18 *pg. 144* #16 *pg. 176* #7 *pg. 112* #7	

Quadratic Functions vs. Quadratic Equations

A *Quadratic Function* of the form $f(x) = ax^2 + bx + c$, where $f(x) = 0$ (*or* $y = 0$) is called a **Quadratic Equation**.

	Quadratic Function	**Quadratic Equation**
Standard Form *(can also be given in vertex form or factored form)*	$f(x) = ax^2 + bx + c$ ***or*** $y = ax^2 + bx + c$ *(where $a \neq 0$)*	$0 = ax^2 + bx + c$ *(where $a \neq 0$)*
Unknown Variables	• 2 unknown variables ("x" and "y") • **any** value for "x" ($D = \{x \mid x \in R\}$) can be substituted into the function to find a value for "y"	• 1 unknown variable ("x") • only **specific** values for "x" can be substituted into the equation to get a true statement (*i.e.* LS = RS)
Characteristics	• the **function** is a *parabola* • it has a direction of opening • it increases/decreases on either side of the vertex • the *vertex* is a maximum/minimum value for the function • it has 0, 1, or 2 *x-intercepts* (*or "zeros")*	• the **equation** has 0, 1, or 2 real *solutions (or "roots")* • can be solved in 4 possible ways: 1. Rearranging & taking square roots 2. Factoring 3. Using the quadratic formula 4. Graphing
Graphical Representation		

1. Solving Quadratic Equations By Rearranging & Taking Square Roots

When Should I Solve Using This Method? a) When the equation is in vertex form

b) When the equation is in standard form with no "bx" term

How Do I Use This Method? 1) isolate the squared term

2) square root both sides (remember that your answer will be both **+** and **–**)

3) solve for ***x***

$x^2 = 25$	$2x^2 = 98$
$x^2 + 64 = 0$	$9x^2 - 16 = 0$
$x^2 + 9 = 25$	$(x - 2)^2 = 25$
$(x - 2)^2 + 9 = 25$	$4(x - 2)^2 + 9 = 25$

HW: Solve and check the equations on the accompanying worksheet and study the comparison table from this note.

 Name: ____________________

WORKSHEET: Solving Quadratic Equations by Rearranging & Taking Square Roots

Solve the following equations for $x \in R$ and check your solutions using *substitution*.

$200 = 8x^2$

$-44x^2 + 5384 = 60$

$11x^2 - 396 = 0$

$-4x^2 + 256 = 0$

$x^2 - 114 = -14$

$-3x^2 + 432 = 0$

$7x^2 - 63 = 0$

$4x^2 - 256 = 0$

$-10x^2 + 810 = 0$

$-800 = -8x^2$

$(x - 5)^2 - 100 = 0$

$3(x + 3)^2 - 12 = 0$

$4x^2 - 100 = -36$

$x^2 + 100 = 0$

$-2(x - 1)^2 + 27 = 9$

$2(x + 3)^2 - 98 = 0$

$9(x - 2)^2 = 16$

$3(x + 4)^2 - 9 = 99$

Solving Quadratic Equations by Factoring

1. *Recall*: Solving by Rearranging & Taking Square Roots

Quadratic equations in **vertex form** (no “bx” term) can be solved by rearranging and isolating “x”:

- isolate the squared term on one side of the equal sign
- take the square root of both sides (remember, the answer will be **+** and **–**)
- simplify to get 0, 1 or 2 real solutions for “x”

Ex. 1: Solve for ***x***.

a) $4x^2 = 0$

b) $x^2 = -16$

b) $16x^2 - 25 = 0$

2. Solving by Factoring

If **factorable**, quadratic equations can be solved by **setting each factor equal to zero** and isolating “x” (If at least one of the factors is equal to zero, the entire product is equal to zero). The solutions for “x” are also called the “roots” of the equation. The roots of the equation are the same values as the **x-intercepts** of the **function**.

Ex. 2: Solve by factoring.

a) $3x^2 + 15x = 0$

b) $x^2 - 6x = 0$

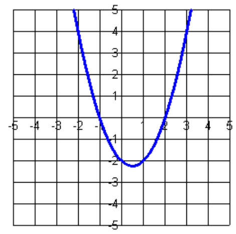
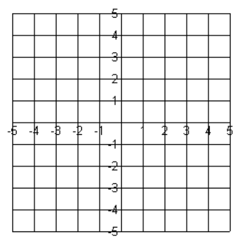
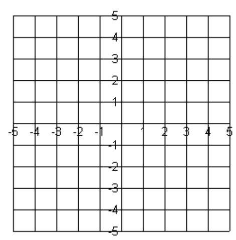
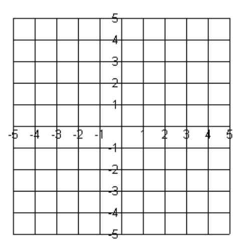
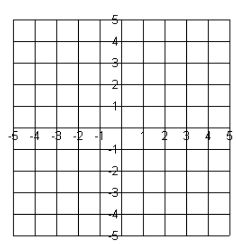
c) $x^2 + 8x + 16 = 0$

d) $5x^2 - 8x - 4 = 0$

e) $12x^2 + 5x - 2 = 0$

f) $16x^2 - 25 = 0$

Ex. 3: Fill in the values relating the roots (solutions) of the equation to the **x-intercepts** (zeros) of the **function**.

Algebraic Model of Quadratic Function	Graphical Model of Quadratic Function	x-intercepts/Zeros of Quadratic Function	Equation and Roots/Solutions of Quadratic Equation
$y = a(\quad)(\quad)$			***Equation:*** $0 = (x \quad)(x \quad)$ ***Roots:*** $x =$ or $x =$
$y = a(\quad)(\quad)$			***Equation:*** $0 = (x+2)(x-1)$ ***Roots:*** $x =$ or $x =$
$y = a(x+4)(x-3)$			***Equation:*** $0 = (x \quad)(x \quad)$ ***Roots:*** $x =$ or $x =$
$y = a(\quad)(\quad)$		*(-3, 0)* *and* *(5, 0)*	***Equation:*** $0 = (x \quad)(x \quad)$ ***Roots:*** $x =$ or $x =$
$y = a(\quad)(\quad)$			***Equation:*** $0 = (\quad)(\quad)$ ***Roots:*** $x = 4$ or $x = \frac{1}{2}$

HW: p. 112 # 1ace, 2acegh, 4, 5, 7 {if you want more practice factoring do all of #1 and #2}

1. Find the roots of each equation. Verify your answers.

a) $x(x + 4) = 0$

b) $(y - 3)(y + 2) = 0$

c) $(k - 9)(2k + 7) = 0$

d) $(3m + 1)(2m - 5) = 0$

e) $(3g + 12)(11g - 23) = 0$

f) $(3t + 3)(t + 5) = 0$

For help with question 2, refer to Example 2.

2. Find the roots of each equation. Verify your answers.

a) $x^2 + 13x + 22 = 0$

b) $p^2 - 4p - 45 = 0$

c) $2r^2 + 5r + 2 = 0$

d) $3d^2 - 7d - 6 = 0$

e) $9k^2 - 25 = 0$

f) $4q^2 - 28q + 49 = 0$

g) $7s^2 - 21s + 14 = 0$

h) $5w^2 - 45 = 0$

4. a) Create a quadratic function in standard form that has zeros at 5 and −7.

b) Explain how you produced this function.

c) Is your function the only possible correct answer to part a)? Explain.

5. A family of quadratic functions has zeros that are opposites, that is, the zeros have the same numerical value but are opposite in sign. What type of polynomials do these correspond to? Use several examples to explain how you know.

7. Consider the quadratic equation $x^2 - 49 = 0$.

a) Solve this equation by factoring.

b) Solve this equation by graphing the corresponding quadratic function.

c) Solve this equation by rearranging and taking square roots.

d) Do these methods all produce the same solutions? Explain.

e) Which method or methods do you prefer? Explain why.

1. a) $x = -4, x = 0$ **b)** $y = -2, y = 3$

c) $k = -\frac{7}{2}, k = 9$ **d)** $m = -\frac{1}{3}, m = 2.5$

e) $g = -4, g = \frac{23}{11}$ **f)** $t = -5, t = -1$

2. a) $x = -11, x = -2$ **b)** $p = -5, p = 9$

c) $r = -\frac{1}{2}, r = -2$ **d)** $d = -\frac{2}{3}, d = 3$

e) $k = -\frac{5}{3}, k = \frac{5}{3}$ **f)** $q = \frac{7}{2}$

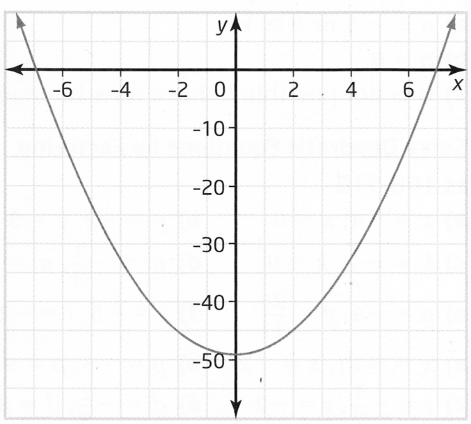
g) $s = 1, s = 2$ **h)** $w = -3, w = 3$

4. Answers may vary.

5. If the zeros of a quadratic function are opposite numbers, the quadratic must be in the form of a difference of squares if stated in standard or vertex form. If given in its factored form, the function is the product of a sum and difference of terms.

7. a) $x = -7, x = 7$

b)



c) $x^2 - 49 = 0$

$x^2 = 49$

$x = \pm 7$

The roots of the equation are -7 and 7.

d) All methods provide the same solutions.

e) Answers may vary.

Solving Quadratic Equations Using the Quadratic Formula

Part A: *Recall* – Solving by Rearranging & Taking Square Roots

a) isolate the squared term on one side of the equal sign

b) take the square root of both sides and solve for x

Part B: *Recall* – Solving by Factoring When the Expression is Factorable

a) set the expression equal to zero and factor

b) set each factor equal to zero and solve for x

Q: What do we do if the equation is neither in vertex form nor factorable?
A: We **graph** it and look for the x-intercepts **or** we use the **Quadratic Formula**!

Part C: Solving by Graphing (Using Technology)

For many examples in class, we have been **graphing** the function and looking at the x-intercepts to verify the solutions for the **equation**. Graphing is also a useful method for solving an equation when rearranging and factoring are not easy options. The difficulty with finding the solutions graphically is that some x-intercepts are not perfect integers! There are fractions and decimals to consider, and estimating them from a graph is not always precise . . . many graphing calculators have [ZERO] options that will help you find the x-intercepts, but what if you don't have access to a graphing calculator? Enter the Quadratic Formula!

4. Solving Using the Quadratic Formula

We know from last unit that we can convert standard form to vertex form by completing the square. If we complete the square on $ax^2 + bx + c = 0$ to get it into vertex form, we can rearrange it algebraically and solve for "x"! However, this is a lot of work to do each time we are given an equation in standard form that is not factorable! That's why we have the Quadratic Formula! Follow the examples in your text to see how the formula was *developed* (p. 136-137).

The Quadratic Formula:

Given: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex. 1: Solve the following equations using the quadratic formula.

a) $x^2 + 3x - 2 = 0$

b) $-3w^2 - 7w = -2$

c) $\frac{1}{10}x^2 + \frac{x}{5} - 1 = 0$

Check:

HW: p. 142-143 # 1ace, 2aceg, 4bdfg, 5

A

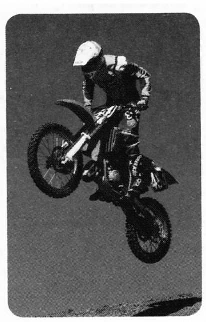
For help with questions 1 to 3, refer to Example 1.

1. Use the quadratic formula to find the roots of each equation. Express your answers as exact roots.
   - **a)** $x^2 - 7x + 4 = 0$
   - **b)** $x^2 + 3x - 2 = 0$
   - **c)** $x^2 + 9x + 6 = 0$
   - **d)** $x^2 - 5x - 3 = 0$
   - **e)** $x^2 - 2x - 14 = 0$
   - **f)** $x^2 + 9x + 1 = 0$

2. Use the quadratic formula to find the roots of each equation. Express answers as exact roots and as approximate roots to the nearest hundredth, where appropriate.
   - **a)** $4x^2 + 2x - 1 = 0$
   - **b)** $-3x^2 - 7x + 2 = 0$
   - **c)** $2x^2 - 8x - 3 = 0$
   - **d)** $5x^2 + 6x + 1 = 0$
   - **e)** $-6x^2 + x + 5 = 0$
   - **f)** $-6x^2 + 5x + 1 = 0$
   - **g)** $-6x^2 - 5x - 1 = 0$
   - **h)** $7x^2 + 3x - 1 = 0$

4. Solve each equation. Round your answers to the nearest hundredth, if necessary.
   - **a)** $-0.03x^2 - 0.6x + 0.5 = 0$
   - **b)** $-3x^2 + 0.06x + 4 = 0$
   - **c)** $x^2 - 0.8x + 0.08 = 0$
   - **d)** $\frac{2}{5}x^2 + 3x + 1 = 0$
   - **e)** $\frac{1}{10}x^2 + \frac{x}{5} - 1 = 0$
   - **f)** $\frac{1}{2}x^2 - x - \frac{5}{2} = 0$
   - **g)** $\frac{1}{3}x^2 - x - \frac{5}{3} = 0$

5. When Michael drives his dirt bike off a ramp, his flight path can be modelled by $h(d) = -0.57d^2 + 3.7d + 2.5$, where d is the horizontal distance from the ramp, in metres, and $h(d)$ is his height, in metres. How far away from the end of the ramp did he land, to the nearest tenth of a metre?



1. a) $x = \frac{7 + \sqrt{33}}{2}$ or $x = \frac{7 - \sqrt{33}}{2}$

b) $x = \frac{-3 + \sqrt{17}}{2}$ or $x = \frac{-3 - \sqrt{17}}{2}$

c) $x = \frac{-9 + \sqrt{57}}{2}$ or $x = \frac{-9 - \sqrt{57}}{2}$

d) $x = \frac{5 + \sqrt{37}}{2}$ or $x = \frac{5 - \sqrt{37}}{2}$

e) $x = \frac{2 + \sqrt{60}}{2}$ or $x = \frac{2 - \sqrt{60}}{2}$;
$x = 1 + \sqrt{15}$ or $x = 1 - \sqrt{15}$

f) $x = \frac{-9 + \sqrt{77}}{2}$ or $x = \frac{-9 - \sqrt{77}}{2}$

2. a) $x = \frac{-2 + \sqrt{20}}{8}$ or $x = \frac{-2 - \sqrt{20}}{8}$;
$x = \frac{-1 + \sqrt{5}}{4}$ or $x = \frac{-1 - \sqrt{5}}{4}$;
$x \doteq 0.31$ or $x \doteq -0.81$

b) $x = -\frac{7 + \sqrt{73}}{6}$ or $x = -\frac{7 - \sqrt{73}}{6}$;
$x \doteq 0.26$ or $x \doteq -2.59$

c) $x = \frac{8 + \sqrt{88}}{4}$ or $x = \frac{8 - \sqrt{88}}{4}$;
$x = \frac{4 + \sqrt{22}}{2}$ or $x = \frac{4 - \sqrt{22}}{2}$;
$x \doteq 4.35$ or $x \doteq -0.35$

d) $x = -1$ or $x = -\frac{1}{5}$

e) $x = -\frac{5}{6}$ or $x = 1$

f) $x = -\frac{1}{6}$ or $x = 1$

g) $x = -\frac{1}{3}$ or $x = -\frac{1}{2}$

h) $x = \frac{-3 + \sqrt{37}}{14}$ or $x = \frac{-3 - \sqrt{37}}{14}$;
$x \doteq 0.22$ or $x \doteq -0.65$

4. a) $x \doteq 0.80$ or $x \doteq -20.80$

b) $x \doteq 1.16$ or $x \doteq -1.14$

c) $x \doteq 0.68$ or $x \doteq 0.12$

d) $x \doteq -0.35$ or $x \doteq -7.15$

e) $x \doteq 2.32$ or $x \doteq -4.32$

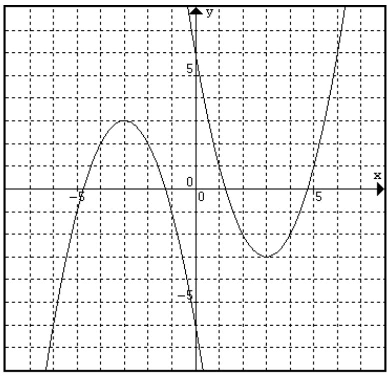
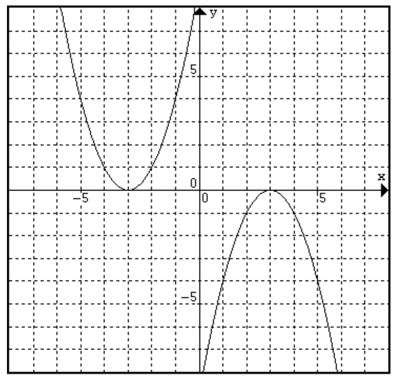
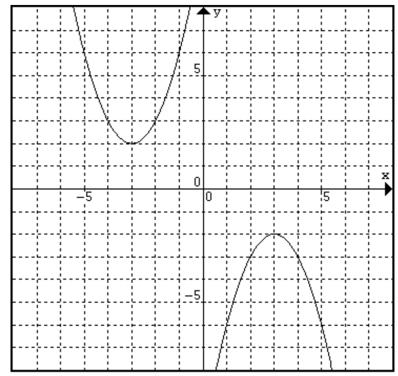
f) $x \doteq 3.45$ or $x \doteq -1.45$

g) $x \doteq 4.19$ or $x \doteq -1.19$

5. 7.1 m

Real Roots and the Discriminant

Recall that the "zeros" of a **quadratic function** (***y = ax² + bx + c***) refer to the **x-intercepts** of the graph.

Two Zeros (Two x-intercepts)	**One Zero (One x-intercept)**	**No Zeros (No x-intercepts)**
		

Zeros are the *x-values* that solve a **quadratic equation** (***ax² + bx + c = 0***). The method used to determine ***how many zeros*** a quadratic function has depends on the *form* of the equation.

1. Factored Form - $y = a(x-r)(x-s)$

$y = 3(x-4)(x+2)$ …solving for ***x*** produces *x* = ____ , *x* = ____ ∴ _______ zero(s)

$f(x) = -2x(x-5)$ …solving for ***x*** produces *x* = ____ , *x* = ____ ∴ _______ zero(s)

$y = 2(x+4)(x+4)$ …solving for ***x*** produces *x* = ____ , *x* = ____ ∴ _______ zero(s)

2. Vertex Form - $y = a(x-h)^2 + k$

To determine the number of zeros… find the **vertex**, determine the **direction of opening** and **sketch** (if necessary).

$y = 2(x-4)^2 - 3$ … Vertex (,) ; a = ______

⇒ The vertex is ___________the x-axis; parabola opens ________ ∴ _____ zero(s)

$y = -2(x-4)^2 - 3$ … Vertex (,) ; a = ______

⇒ The vertex is ___________the x-axis; parabola opens ________ ∴ _____ zero(s)

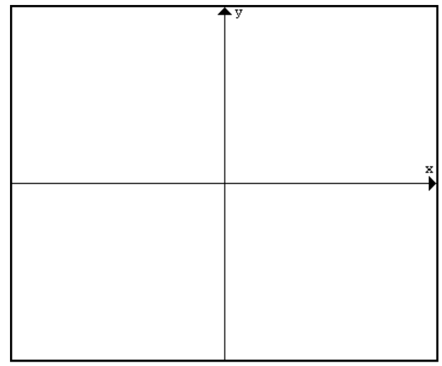
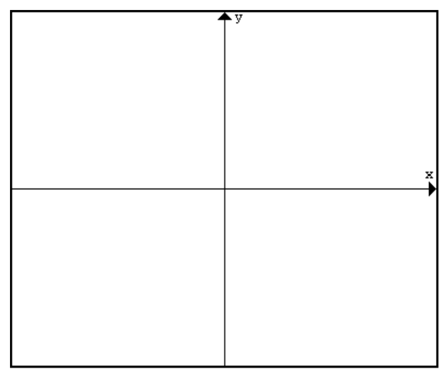
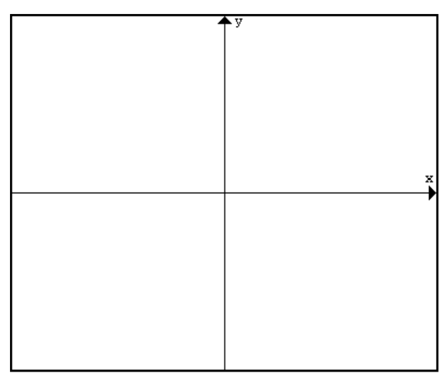
$y = 2(x+7)^2$ … Vertex (,) ; a = ______

⇒ The vertex is ___________the x-axis; parabola opens ________ ∴ _____ zero(s)

3. Standard Form - $y = ax^2 + bx + c$

To determine the ***number of zeros*** we can use: a) the **quadratic formula** ... $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

OR b) the graphing calculator to **sketch** the parabola.

Equation & Vertex	Quadratic Formula	Sketch
$4x^2 - 2x - 3 = 0$		
$5x^2 - 12x + 9 = 0$		
$x^2 - 2x + 1 = 0$		

4. Discriminant

From step 3, the portion of the quadratic formula **<u>under the square root sign</u>** provides a means to determine the number of roots. This portion, called the **discriminant, *D,*** *is equal to* ***$b^2 - 4ac$.***

Equation	Discriminant	Number of Roots
$4x^2 - 2x - 3 = 0$		
$5x^2 - 12x + 9 = 0$		
$x^2 - 2x + 1 = 0$		

Conclusion,

The Discriminant, *D* :
Given: $ax^2 + bx + c = 0$

$$D = b^2 - 4ac$$

If …	*…then the quadratic function has…*
$b^2 - 4ac > 0$	
$b^2 - 4ac < 0$	
$b^2 - 4ac = 0$	

<u>Examples</u>: Determine the **<u>number of roots</u>** *without solving or graphing*.

a) $x^2 - 4x + 3 = 0$

b) $-x^2 + 2x - 3 = 0$

HW: p. 150-152 # 1, 2, 3, 4aceg, 8ace, 9, 10, 16, 17

1. Identify the number of roots of the quadratic equation given the following values for the discriminant, D.

a) -10 **b)** 0

c) 7 **d)** $\frac{2}{9}$

e) -0.04 **f)** 9.57

2. Determine how many real roots each equation has.

a) $x^2 - 9x + 4 = 0$

b) $-4x^2 - 2x + 1 = 0$

c) $x^2 - 6x + 9 = 0$

d) $4x^2 - 12x + 9 = 0$

e) $5x^2 + x + 1 = 0$

f) $-x^2 + 2x - 4 = 0$

3. Verify your answers to question 2 graphically, with or without technology.

4. Determine the number of real roots each equation has. Then, find the roots of each equation by factoring.

a) $2x^2 = 9x - 4$

b) $x^2 + 8x = -7$

c) $3x^2 - 2x - 5 = 0$

d) $x^2 - 2x + 1 = 0$

e) $-8x^2 + 32 = 0$

f) $-x^2 - 25 = -10x$

g) $4x^2 + 2 = -6x$

h) $7x^2 - 14x = 0$

8. Find the x-intercepts of each quadratic function. Round your answers to the nearest hundredth, when necessary.

a) $y = 2x^2 - 5x + 6$

b) $y = x^2 + 6x + 1$

c) $y = x^2 + 3x - 12$

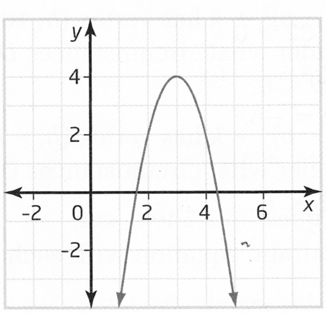
d) $y = 9x^2 - 24x + 16$

e) $y = -3x^2 + 2x - 1$

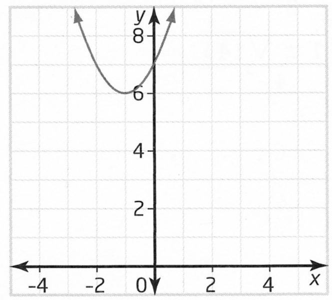
f) $y = 20x^2 - 3x - 9$

9. For each graph, determine whether $D = 0$, $D > 0$, or $D < 0$.

a)



b)



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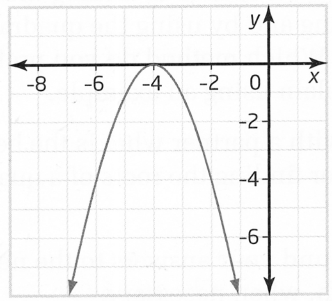

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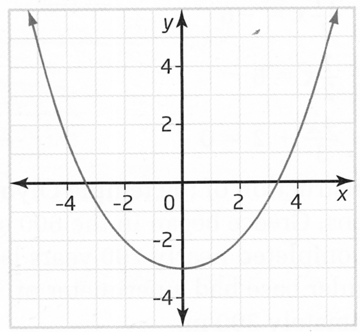

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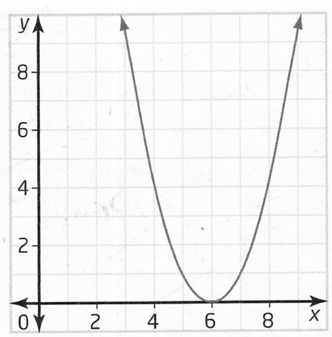


B

10. Explain under what conditions $ax^2 + bx + c = 0$ can be solved by factoring. Illustrate your explanation with an example.

16. If $y = a(x - h)^2 + k$ has vertex $(h, 10)$, explain under what circumstances $D = 0$, $D > 0$, or $D < 0$.

17. If $y = a(x - h)^2 + k$ has vertex $(h, -7)$, explain under what circumstances $D = 0$, $D > 0$, or $D < 0$.

1. a) $D < 0$, so there are no real roots.
b) $D = 0$, so there is one real root.
c) $D > 0$, so there are two real roots.
d) $D > 0$, so there are two real roots.
e) $D < 0$, so there are no real roots.
f) $D > 0$, so there are two real roots.

2. a) $D > 0$, so there are two real roots.
b) $D > 0$, so there are two real roots.
c) $D = 0$, so there is one real root.
d) $D = 0$, so there is one real root.
e) $D < 0$, so there are no real roots.
f) $D < 0$, so there are no real roots.

4. a) $D > 0$, so there are two real roots; $x = 4$, $x = \frac{1}{2}$
b) $D > 0$, so there are two real roots; $x = -1$, $x = -7$
c) $D > 0$, so there are two real roots; $x = -1$, $x = \frac{5}{3}$
d) $D = 0$, so there is one real root; $x = 1$
e) $D > 0$, so there are two real roots; $x = 2$, $x = -2$
f) $D = 0$, so there is one real root; $x = 5$
g) $D > 0$, so there are two real roots; $x = -1$, $x = -\frac{1}{2}$
h) $D > 0$, so there are two real roots; $x = 0$, $x = 2$

8. a) There are no x-intercepts.
b) -0.17 and -5.83
c) -5.27 and 2.27
d) The x-intercept is $\frac{4}{3}$.
e) There are no x-intercepts.
f) The x-intercepts are $\frac{3}{4}$ and $-\frac{3}{5}$.

9. a) $D > 0$ **b)** $D < 0$ **c)** $D = 0$
d) $D > 0$ **e)** $D = 0$ **f)** $D < 0$

10. Explanations and examples may vary.
$ax^2 + bx + c$ is factorable if the factors of the product of $a \times c$ have a sum resulting in b.
$20x^2 - 3x - 9 = (4x - 3)(5x + 3)$

16. $y = a(x - h)^2 + 10$
The discriminant cannot be zero for this equation.
Since the vertex is $(h, 10)$, the graph will not have any x-intercepts for $a > 0$, so $D < 0$.
h can take on any value.
The graph will have two x-intercepts for $a < 0$, so $D > 0$.

17. $y = a(x - h)^2 - 7$
The discriminant cannot be zero for this equation.
Since the vertex is $(h, -7)$, the graph will not have any x-intercepts for $a < 0$, so $D < 0$.
h can take on any value.
The graph will have two x-intercepts for $a > 0$, so $D > 0$.

Solving Word Problems Using Quadratic Equations!

"***Quadratum***" is Latin for "*square*". When the side lengths of a square are unknown ("x"), the area of that square becomes a quadratic expression ($A = x^2$). This makes quadratic equations perfect for solving area problems!

Strategies for solving word problems:

- write what you are given as you read it
- write what you are required to find
- provide a "Let . . ." statement to assign your variables
- draw a diagram and label it with your assigned variables
- write the equation(s) of the relationship(s) between your variables
- solve and check that you have answered the question fully
- provide a "Therefore . . ." statement

Ex. 1: A rectangular pool measures 10m by 5m. A deck, of uniform width, is to be built all the way around the pool such that the total area of the pool and deck will be 126 m^2. Set up and solve a quadratic equation to determine the width of the deck.

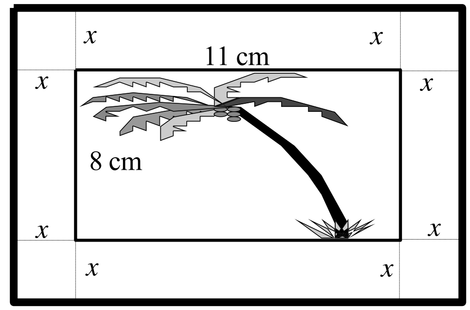
Ex. 2: A garden measuring 12 meters by 16 meters is to have a pedestrian pathway installed all around it, increasing the total area to 285 square meters. What will be the width of the pathway?

Ex. 3: A dining room measures 5 m by 4 m. A strip of uniform width is added to two adjacent sides to increase the area to 25 m^2. Find the width of the strip to 1 decimal place.

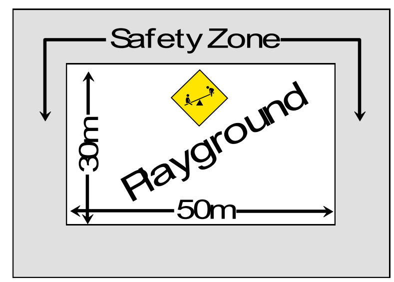
HW: WORKSHEET: Solving Area Problems Using Quadratic Equations and p. 143-144 # 7, 8, 9, 13* (*challenge)

WORKSHEET: Solving Area Problems Using Quadratic Equations

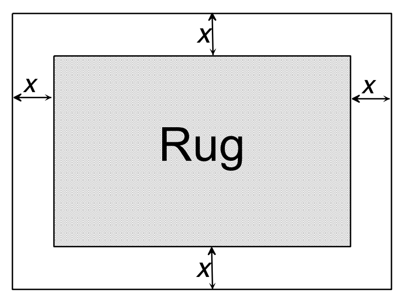
1) A photograph 8 cm by 11 cm will be framed as shown in the diagram. The combined area of the frame and photograph will be 180 cm^2. Algebraically determine the outside dimensions of the frame.



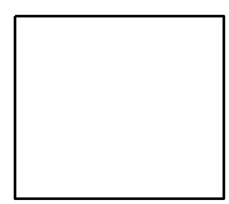
2) A school playground is rectangular and has a length of 50 m and a width of 30 m as shown. A safety zone of uniform width surrounds the playground. If the area of the safety zone equals the area of the playground, what is the width of the safety zone, to 2 decimal places?



3) A rectangular rug 4 m by 2 m is placed in a room with floor area 24 m^2 such that a strip of bare floor of uniform width surrounds the rug. Set up an equation that models this situation and use it to algebraically determine the width of the strip of bare floor.



4) A square is transformed into a rectangle by increasing the length by 8 m and the width by 5 m. If the area of the resulting rectangle is 108 m^2, algebraically determine the length of each side of the original square.



Answers: ***1) 2 cm*** ***2) 7.84 m*** ***3) 1 m*** ***4) 4 m***

Unit 3 Review Assignment

1. **Solving by Rearranging & Taking Square Roots**
   a) *WORKSHEET*: "Solving by Rearranging and Taking Square Roots"
   b) Solve the following:
   i) $3(x+3)^2 - 27 = 0$ iii) $(x-5)^2 - 100 = 0$
   ii) $2(x-3)^2 - 32 = 0$ iv) $(x-2)^2 - 25 = 0$

   Answers: ***i) x = 0 or x = -6*** ***ii) x = 7 or x = -1*** ***iii) x = 15 or x = -5*** ***iv) x = 7 or x = -3***

2. **Solving by Factoring**
   p. 115 # 18, 19

3. **Solving by Graphing or Using the Quadratic Formula**
   p. 174 # 4, 7

4. **Real Roots and the Discriminant**
   p. 174 # 6
   p. 151-152 # 6, 7, 18
   p. 144 # 16

5. **Solving Area Problems with Quadratic Equations**
   a) p. 174 # 5

   b) Complete the following:

   i) A rectangular swimming pool measuring 10 m by 4 m is to be surrounded by a deck of uniform width. The combined area of the deck and pool should be 135 m^2. What is the width of the deck? ***(2.5 m)***

   ii) A framed picture has a border of uniform width with the same area as the picture. If the picture is 12 cm by 17 cm, find the width of the border to two decimal places. ***(2.93 cm)***

   iii) Eric wants to build a rectangular playpen with an area of 20 m^2 and a perimeter of 16 m.

   a) Write a quadratic equation for the area of the playpen that uses the width, *w*, as the unknown variable and the length, *l,* as (8 – w). ***($w^2 - 8w + 20 = 0$)***
   b) What is the discriminant of the quadratic equation? ***(D = -16)***
   c) Can Eric build a playpen with these constraints? ***(Nope! Not if he wants to use real numbers!)***
   d) Suppose Eric changes the required perimeter to 18 m. What are the dimensions of the new playpen, assuming that *w* is the width and *(9 – w)* is the length? ***(w = 4 m and l = 5 m)***

6. **Putting It All Together:**
   p. 176 # 7
   p. 112 # 7

*A good study strategy would be to review the notes for the lesson before doing the associated review questions. You should be confident with ALL of the examples in the notes before completing the review.

 Name___________________________________

Solving by Rearranging and Taking Square Roots

Solve each equation by taking square roots.

1) $m^2 = 64$

2) $r^2 = -3$

3) $x^2 = 100$

4) $a^2 = 25$

5) $n^2 + 4 = 104$

6) $-x^2 = -36$

7) $r^2 - 7 = 42$

8) $4b^2 = 16$

9) $3x^2 = 48$

10) $10k^2 = 360$

11) $9n^2 + 2 = 11$

12) $6p^2 + 4 = 0$

13) $8v^2 - 6 = 794$

14) $7p^2 - 6 = 442$

Answers to Solving by Rearranging and Taking Square Roots

1) $\{8, -8\}$
2) No solution.
3) $\{10, -10\}$
4) $\{5, -5\}$
5) $\{10, -10\}$
6) $\{6, -6\}$
7) $\{7, -7\}$
8) $\{2, -2\}$
9) $\{4, -4\}$
10) $\{6, -6\}$
11) $\{1, -1\}$
12) No solution.
13) $\{10, -10\}$
14) $\{8, -8\}$

18. Find the roots of each equation.

a) $x^2 - 11x + 30 = 0$

b) $3p^2 + 13p - 10 = 0$

c) $3q^2 - 13q + 10 = 0$

d) $5m^2 + 14m - 3 = 0$

e) $3r^2 + 8r + 4 = 0$

f) $4n^2 - 4n - 3 = 0$

19. A javelin is thrown from a raised platform. Its height as a function of time is given by the equation $h(t) = -5t^2 + 20t + 25$, where $h(t)$ is the height of the javelin, in metres, and t is the time, in seconds.

a) Write this equation in factored form.

b) What is the hang time of the javelin? Explain how you found your answer.

4. Solve the following equations using the quadratic formula. Express answers as exact roots and as approximate roots, rounded to the nearest hundredth.

a) $x^2 + 5x - 1 = 0$

b) $5x^2 - 2x - 4 = 0$

c) $-\frac{1}{6}x^2 + \frac{1}{3}x + 2 = 0$

d) $-0.2x^2 + 0.7x + 1.1 = 0$

7. Find the x-intercepts of each quadratic function. Round answers to the nearest hundredth, when necessary.

a) $y = 2x^2 - 5x + 7$

b) $y = x^2 + 4x + 1$

c) $y = 9x^2 - 30x + 25$

d) $y = 15x^2 - 26x + 8$

6. Determine how many real roots each equation has. Then, find the roots of each equation by factoring.

a) $2x^2 + 11x + 5 = 0$

b) $4x^2 - 4x + 1 = 0$

c) $2x^2 + 7x = 0$

d) $9x^2 - 64 = 0$

6. Find the x-intercepts of each quadratic function by graphing, with or without technology.

a) $y = x^2 - 8x - 20$

b) $y = x^2 + 11x - 42$

c) $y = -x^2 + x + 2$

d) $y = x^2 + x + 4$

7. Calculate the value of the discriminant, D, for each of the functions in question 6. Relate this to the number of roots for each related quadratic equation.

18. Find a quadratic equation with each pair of roots.

a) $x = \dfrac{14 \pm \sqrt{168}}{2}$

b) $x = \dfrac{-11 \pm \sqrt{145}}{4}$

16. Under what circumstances will the equation $ax^2 + bx + c = 0$ have only one solution? Explain.

5. Qin's rectangular rose garden which measures 5 m by 11 m is to be doubled in area by extending each side length by an equal amount.

a) Sketch and label a diagram to represent this situation.

b) Find how much each side is extended, to the nearest metre.

c) Find the dimensions of the new rose garden, to the nearest metre.

7. Solve each equation using the most appropriate method.

a) $3x^2 - 13x + 4 = 0$

b) $2x^2 - 14x = 0$

c) $6x^2 + 2x - 1 = 0$

d) $4x^2 - 12x + 9 = 0$

e) $3(4x^2 + x) = (x - 1)^2$

7. Consider the quadratic equation $x^2 - 49 = 0$.

a) Solve this equation by factoring.

b) Solve this equation by graphing the corresponding quadratic function.

c) Solve this equation by rearranging and taking square roots.

d) Do these methods all produce the same solutions? Explain.

e) Which method or methods do you prefer? Explain why.

18. a) $x = 5, x = 6$ **b)** $p = -5, p = \frac{2}{3}$

c) $q = \frac{10}{3}, q = 1$ **d)** $m = -3, m = \frac{1}{5}$

e) $r = -2, r = \frac{2}{3}$ **f)** $n = \frac{3}{2}, n = -\frac{1}{2}$

19. a) $-5(t + 1)(t - 5)$

b) 5 s; Set the function equal to zero. Find the zeros of the function. The function has meaning from $t = 0$ to $t = 5$.

4. a) exact roots: $x = \frac{-5 + \sqrt{29}}{2}$ or $x = \frac{-5 - \sqrt{29}}{2}$
approximate roots: $x \doteq 0.19$ or $x \doteq -5.19$

b) exact roots: $x = \frac{2 + \sqrt{84}}{10}$ or $x = \frac{2 - \sqrt{84}}{10}$; $x = \frac{1 + \sqrt{21}}{5}$ or $x = \frac{1 - \sqrt{21}}{5}$
approximate roots: $x \doteq 1.12$ or $x \doteq -0.72$

c) exact roots: $x = \frac{2 + \sqrt{52}}{2}$ or $x = \frac{2 - \sqrt{52}}{2}$; $x = 1 + \sqrt{13}$ or $x = 1 - \sqrt{13}$
approximate roots: $x \doteq 4.61$ or $x \doteq -2.61$

d) exact roots: $x = \frac{7 + \sqrt{137}}{4}$ or $x = \frac{7 - \sqrt{137}}{4}$
approximate roots: $x \doteq 4.68$ or $x \doteq -1.18$

7. a) no roots; $D < 0$

b) $x \doteq -0.27$ or $x \doteq -3.73$

c) $x = \frac{5}{3}$

d) $x = \frac{4}{3}$ or $x = \frac{2}{5}$

6. a) $D > 0$, so there are two real roots; $x = -5$, $x = -\frac{1}{2}$

b) $D = 0$, so there is one real root; $x = \frac{1}{2}$

c) $D > 0$, so there are two real roots; $x = 0$, $x = -\frac{7}{2}$

d) $D > 0$, so there are two real roots; $x = -\frac{8}{3}$, $x = \frac{8}{3}$

6. a) The x-intercepts are 10 and -2.

b) The x-intercepts are 3 and -14.

c) The x-intercepts are 2 and -1.

d) There are no x-intercepts.

7. a) $D > 0$, so there are two real roots.

b) $D > 0$, so there are two real roots.

c) $D > 0$, so there are two real roots.

d) $D < 0$, so there are no real roots.

18. a) $0 = x^2 - 14x + 7$ **b)** $0 = 2x^2 + 11x - 3$

16. The equation $y = ax^2 + bx + c$ will have only one solution when $b^2 - 4ac = 0$.

5. a) Sketches may vary.

b) Each side is to be extended by 3 m.

c) 14 m by 8 m

7. a) $x = 4, x = \frac{1}{3}$

b) $x = 0, x = 7$

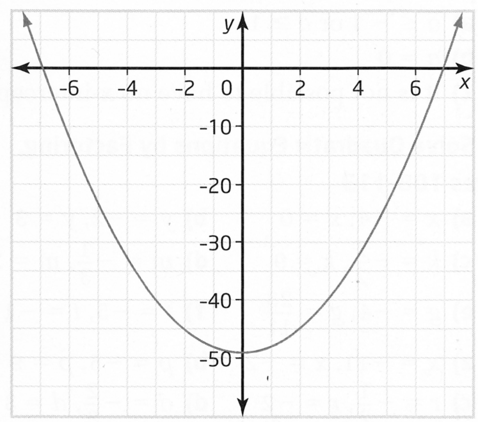
c) $x = \frac{-1 + \sqrt{7}}{6}$, or $x = \frac{-1 - \sqrt{7}}{6}$

d) $x = \frac{3}{2}$

e) $x = \frac{-5 + \sqrt{69}}{22}$ or $x = \frac{-5 - \sqrt{69}}{22}$

7. a) $x = -7, x = 7$

b)



c) $x^2 - 49 = 0$
$x^2 = 49$
$x = \pm 7$
The roots of the equation are -7 and 7.

d) All methods provide the same solutions.

e) Answers may vary.