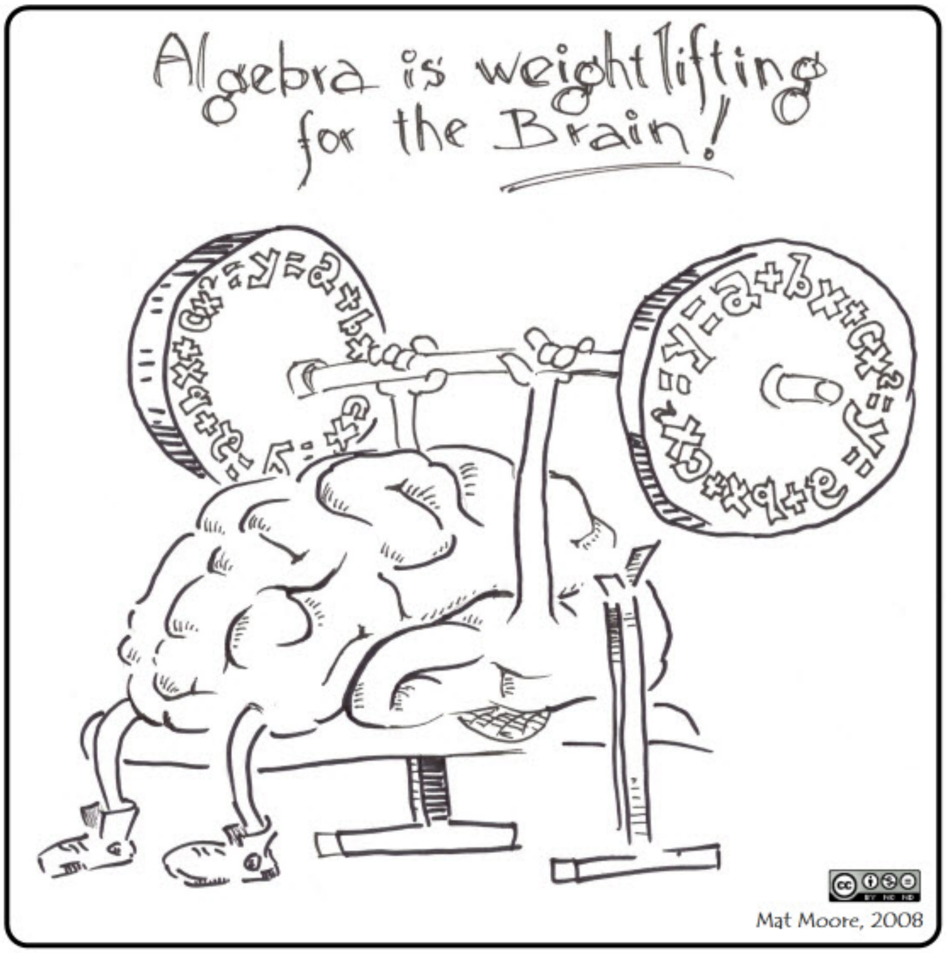

MCF3MI

Unit 0: Algebraic Tools

Polynomials Review – Adding, Subtracting, and Multiplying

A. **Polynomials** are expressions made up of a *finite* number of terms separated by ________________________ or ____________________________.

- A monomial is made up of _____ term, a binomial is made up of _____terms, and a trinomial is made up of _____ terms. (For expressions containing more than 3 terms, we simply use the prefix *poly* meaning "_____________".)

- The terms of a polynomial have ___________________ raised to whole-number exponents (0, 1, 2, . . .) and ______________________ (or numerical coefficients) which form products with the variables.

$$3x^2 - 5x + 4$$

- Terms that have the same variable factors (i.e. same *letters* with same *exponents*) are called ______________ _______________.

B. To **simplify** a polynomial expression containing *like terms*, find the sum of their coefficients.

a) $2x^2 + 3x - 4 - x^2 + x + 9$

b) $10x^3 - 14x^2 + 3x - 4x^3 + 4x - 6$

C. To **add** polynomial expressions, remove the brackets and collect like terms.

a) $(4x^2 - 7x - 5) + (2x^2 - x + 3)$

D. To **subtract** polynomial expressions, remove the brackets in the second expression by multiplying each term by (– 1). Then, collect like terms.

a) $(4s^2 + 5st - 7t^2) - (6s^2 + 3st - 2t^2)$

E. To **multiply** polynomial expressions, remove the brackets using the **distributive property**. Then, collect like terms. (When more than one set of brackets is used, simplify to remove the innermost brackets first.)

a) $(2x + 5)(3x - 4)$

b) $2x(4x - 2) - 3x(x - 5) + 3(x + 6)$

c) $-3[2(x - 5) - 4(2x - 3)]$

d) $3(2x + 3)^2 - (x - 5)^2$

HW: p. 423 "Expanding Algebraic Expressions" #1abcd; p.430 "Multiplying Binomials" #1abcd; p. 83 #2; p. 84 #11

 Name____________________

Homework

Simplify each expression.

1) $(8 - x^2 - x^3) + (4x^2 - 6x^3 + 5)$

2) $(2x^2 - 4x^3 + 3x) + (4x^2 - x + 6x^3)$

3) $(k^2 - 7k^3 + 6) + (7k^3 + 8 - 2k^2)$

4) $(5 + 3b^2 + 3b) + (5 + b^2 + b)$

5) $(6n^2 + 6n - 4n^3) - (7n^2 + 7n^3 - 3n)$

6) $(a^2 + a^3 - 3a) - (5a^3 - 7a^2 + a)$

7) $(5v - 6v^2 + 3) - (7v - 4 + 7v^2)$

8) $(8b^2 - 5b + 2b^3) - (6b + 4b^3 + 7b^2)$

Find each product.

9) $(4n - 1)(n - 2)$

10) $(2r + 5)(4r + 7)$

11) $(6x + 2)(6x - 2)$

12) $(7x - 5)(x - 7)$

13) $3y(y + 1) - 2(y^2 + 3) + 6(2y - 1)$

14) $2x^2(x + 2) - 1(x^2 - 3x) + 5x^3$

15) $p(2p + 3) - p^2 + 5(p^2 - p)$

16) $8m(m^2 + 3m) - 5(m^3 + 2m^2) - 1(m^2 + 7)$

Answers to Homework

1) $-7x^3 + 3x^2 + 13$
2) $2x^3 + 6x^2 + 2x$
3) $-k^2 + 14$
4) $4b^2 + 4b + 10$
5) $-11n^3 - n^2 + 9n$
6) $-4a^3 + 8a^2 - 4a$
7) $-13v^2 - 2v + 7$
8) $-2b^3 + b^2 - 11b$
9) $4n^2 - 9n + 2$
10) $8r^2 + 34r + 35$
11) $36x^2 - 4$
12) $7x^2 - 54x + 35$
13) $y^2 + 15y - 12$
14) $7x^3 + 3x^2 + 3x$
15) $6p^2 - 2p$
16) $3m^3 + 13m^2 - 7$

Factoring I – Common Factoring and "Simple" Trinomials

A. Common Factoring

A **common factor** is a number and/or variable that divides evenly into <u>all terms</u> of the polynomial. When common factoring, always look for the **greatest common factor (GCF)**.

Factor the following:

a) $x^2 + x$ b) $2x – 8$ c) $ab – bc$ d) $9a^3 – 12a$

e) $2xy^2z + 6x^2yz – 8xy^2z^2$ f) $4a^3b^4 – 6a^2b^2 + 2ab$ g) $x(2x + 1) – 2(2x + 1)$

B. Factoring "Simple" Trinomials

A **simple trinomial** is one in the form $\boldsymbol{ax^2 + bx + c}$**, where** $\boldsymbol{a = 1}$. If it is factorable, a simple trinomial can be expressed as a product of two binomials. The constant terms of the binomials are integers that multiply to give "*c*" and add to give "*b*".

Expand and simplify the following:

a) $(x + 3)(x + 5)$ b) $(w – 4)(w+ 3)$ c) $(z – 1)(z – 6)$

Factor the following and check your work:

a) $y^2 + 7y + 10$ b) $a^2 - a - 6$ c) $x^2 - 6x + 9$

HW: p. 420 "Common Factoring" #1; p. 424 "Factoring Trinomials in the Form $ax^2 + bx + c$" #1; p. 96 #2, 8ab

Homework

Factor each completely.

1) $n^2 + 12n + 20$

2) $x^2 + 10x + 24$

3) $r^2 + 2r - 35$

4) $x^2 + 9x - 10$

5) $p^2 + 12p + 32$

6) $b^2 - 15b + 54$

7) $x^2 + x - 30$

8) $n^2 + 3n + 2$

9) $n^2 - 10n + 9$

10) $n^2 + 2n - 63$

11) $12x - 16y$

12) $x^2 - 5x$

13) $4x^2 - 6xy$

14) $2a - 6b - 4c$

15) $6x + 11x^2 - 12xy$

16) $6x^2 + 15x + 9$

17) $10ab - 20ac - 15a^2$

18) $2x^2y^2 - 5xy^2 - 7xy$

19) $10a^2b^2 + 4ab - 6ab^2$

20) $21x^3y^2 - 14x^2y + 28x^2y^2$

Answers to Homework

1) $(n+10)(n+2)$
2) $(x+6)(x+4)$
3) $(r-5)(r+7)$
4) $(x-1)(x+10)$
5) $(p+8)(p+4)$
6) $(b-9)(b-6)$
7) $(x+6)(x-5)$
8) $(n+1)(n+2)$
9) $(n-1)(n-9)$
10) $(n+9)(n-7)$
11) $4(3x - 4y)$
12) $x(x - 5)$
13) $2x(2x - 3y)$
14) $2(a - 3b - 2c)$
15) $x(6 + 11x - 12y)$
16) $3(2x^2 + 5x + 3)$
17) $5a(2b - 4c - 3a)$
18) $xy(2xy - 5y - 7)$
19) $2ab(5ab + 2 - 3b)$
20) $7x^2y(3xy - 2 + 4y)$

Factoring II – Factoring by Grouping and "Tricky" Trinomials

A. Factoring by Grouping

Factoring by grouping is a form of factoring where selected terms in an expression are grouped together and factored separately from the other terms. Usually, those new factors are also factorable! *Hint*: We often look to factor by grouping when we see four or more terms in an expression.

Factor the following:

a) $x^3 - 4x^2 + 3x - 12$

b) $3ax - 3ay - 6bx + 6by$

c) $3x^5 - 6x^3 - x^2 + 2$

d) $(x^2 - 9) + (x^2 - 6x + 9)$

B. Recall "Simple" Trinomials

A **simple trinomial** is one in the form $\boldsymbol{ax^2 + bx + c}$**, where $\boldsymbol{a = 1}$**. We factor it by:

1. finding two integers that multiply to give "*c*" and add to give "*b*"
2. creating two binomials, each with a variable term and one of the integers

Factor the following:

a) $x^2 + 7x + 12$

b) $b^2 - b - 30$

C. Factoring "Tricky" Trinomials

A **tricky trinomial** is one in the form $\boldsymbol{ax^2 + bx + c}$**, where** $\boldsymbol{a \neq 1}$. We factor it by using a method of **systematic trial and error**. This means that, given a trinomial is the product of two binomials, we work through the possible answers to find the correct answer.

Factor the following:

a) $2x^2 + 11x + 12$

b) $6h^2 - 17h + 5$

c) $6w^2 - 15w - 9$

d) $12x^2 + 5x - 3$

HW: **Part A:** ***Factor the following by grouping:***

a) $x^3 - 5x^2 + 6x - 30$

b) $2x^3 - 6x^2 - 3x + 9$

c) $2x^3 - 4x^2 + 10x - 20$

Part B: p. 96-97 #4, 5, 8cd, 14abc

Homework

Factor each completely.

1) $9x^3 - 24x^2 - 24x + 64$

2) $15k^3 + 10k^2 + 6k + 4$

3) $12n^3 - 15n^2 + 4n - 5$

4) $56n^3 + 35n^2 - 64n - 40$

5) $2v^2 - 7v - 4$

6) $3x^2 - x - 30$

7) $7v^2 - 23v + 6$

8) $5a^2 - 31a - 28$

9) $2n^2 - 5n - 25$

10) $3k^2 - 28k + 9$

11) $6n^2 - 7n + 2$

12) $4a^2 + a - 14$

13) $6x^2 - 23x + 20$

14) $6r^2 - 19r + 3$

15) $4k^2 - 15k + 14$

16) $4x^2 - 12x + 5$

Answers to Homework

1) $(3x^2 - 8)(3x - 8)$
2) $(5k^2 + 2)(3k + 2)$
3) $(3n^2 + 1)(4n - 5)$
4) $(7n^2 - 8)(8n + 5)$
5) $(2v + 1)(v - 4)$
6) $(3x - 10)(x + 3)$
7) $(7v - 2)(v - 3)$
8) $(5a + 4)(a - 7)$
9) $(2n + 5)(n - 5)$
10) $(3k - 1)(k - 9)$
11) $(2n - 1)(3n - 2)$
12) $(a + 2)(4a - 7)$
13) $(2x - 5)(3x - 4)$
14) $(r - 3)(6r - 1)$
15) $(k - 2)(4k - 7)$
16) $(2x - 1)(2x - 5)$

Factoring III – Special Cases: Difference of Squares and Perfect Square Trinomials

A. Difference of Squares

A **difference of squares** polynomial is exactly that: a "difference" (subtraction) between two perfect squares! Look to factor a difference of squares when you see **two terms** separated by **subtraction**. (If it is *not* a difference of squares, the only type of factoring you can complete will be common factoring!)

Expand and simplify the following:

a) (x + 3)(x – 3)

b) (2x – 3)(2x + 3)

c) (3x – 5y)(3x + 5y)

To factor a difference of squares,

$$a^2 - b^2 = (a+b)(a-b)$$

Factor the following:

a) $p^2 - 9$

b) $36m^2 - 100$

c) $6x^2 - 150$

B. Perfect Square Trinomials

Expand and simplify the following:

a) $(x + 7)^2$ b) $(2x + 5)^2$ c) $(3x - 5y)^2$

A **perfect square trinomial** can be factored just like any other trinomial (simple or tricky) BUT you can save time if you notice that your trinomial has one of the following forms:

Expanded Form		***Factored Form***
$a^2 + 2ab + b^2$	=	$(a + b)^2$
	or	
$a^2 - 2ab + b^2$	=	$(a - b)^2$

Factor the following:

a) $x^2 - 6x + 9$ b) $25y^2 + 20y + 4$ c) $2v^2 - 16v + 32$

HW: p. 105-106 #2, 3, 9

 Name____________________________________

Homework

Factor each completely.

1) $4p^2 - 4$

2) $n^2 + 4n + 4$

3) $2x^2 + 40x + 200$

4) $a^2 - 1$

5) $4n^2 - 49$

6) $2x^2 - 18$

7) $20a^2 - 45$

8) $27n^2 - 75$

9) $16b^2 - 40b + 25$

10) $4x^2 + 4x + 1$

11) $4c^2 - 12c + 9$

12) $49^2e + 28e + 4$

Answers to Homework

1) $4(p+1)(p-1)$
2) $(n+2)^2$
3) $2(x+10)^2$
4) $(a-1)(a+1)$
5) $(2n-7)(2n+7)$
6) $2(x-3)(x+3)$
7) $5(2a-3)(2a+3)$
8) $3(3n-5)(3n+5)$
9) $(4b-5)^2$
10) $(2x+1)^2$
11) $(2c-3)^2$
12) $(7e+2)^2$

Solving Literal Equations

Formulas are *equations* that express a *relationship* between more than one letter or variable. A formula is also called a **literal equation** when it involves several letters or variables. Sometimes algebra is needed to change the formula to a more useful equivalent equation, which is solved for a particular letter or variable. Let's consider a few examples:

1. Solve $d = rt$ for *r*

2. Solve $P = 2l + 2w$ for *w*

3. Solve $Q = \frac{c+d}{2}$ for *d*

4. Solve $V = \frac{3k}{t}$ for *t*

5. Solve $A = \frac{\pi r^2 S}{360}$ for *S*

6. Solve $A = \frac{(a+b)h}{2}$ for *a*

HW {no calculator!}: WORKSHEET "Solving Literal Equations" #1-7, 9, 10, 12, 13, 16-18, 21, 22

WORKSHEET: Solving Literal Equations

Solve the following literal equations:

1. Solve $d = rt \;\; for \;\; r$
2. Solve $P = \frac{144p}{y} \;\; for \;\; p$
3. Solve $R = \frac{CS}{d} \;\; for \;\; C$
4. Solve $P = a + b + c \;\; for \; b$
5. Solve $T = m - n \;\; for \; n$
6. Solve $A = \frac{a+b}{2} \;\; for \; b$
7. Solve $V = lwh \;\; for \; w$
8. Solve $m = \frac{y_2 - y_1}{x_2 - x_1} \;\; for \;\; y_2$
9. Solve $ax + by = c \;\; for \;\; y$
10. Solve $A = \frac{a+b+c+d}{4} \;\; for \;\; c$
11. Solve $S = 2(lw + lh + wh) \; for \;\; w$
12. Solve $P = 2(l + w) \;\; for \;\; l$
13. Solve $d = \frac{C}{\pi} \;\; for \;\; \pi$
14. Solve $\frac{1}{f} = \frac{1}{a} + \frac{1}{b} \;\; for \;\; f$
15. Solve $A = p(1 + rt) \;\; for \;\; t$
16. Solve $I = prt \;\; for \;\; r$
17. Solve $ax + b = c \;\; for \;\; a$
18. Solve $S = 2\pi rh \;\; for \;\; h$
19. Solve $A = 2\pi r^2 + 2\pi rh \;\; for \;\; h$
20. Solve $y - y_1 = m(x - x_1) \;\; for \;\; x$
21. Solve $R = \frac{l+3w}{2} \;\; for \;\; w$
22. Solve $ax + by + c = 0 \;\; for \;\; y$
23. Solve $C = \frac{5}{9}(F - 32) \;\; for \;\; F$

Answers:

1. $r = \frac{d}{t}$
2. $p = \frac{Py}{144}$
3. $C = \frac{Rd}{S}$
4. $b = P - a - c$
5. $n = m - T$
6. $b = 2A - a$
7. $w = \frac{V}{lh}$
8. $y_2 = mx_2 - mx_1 + y_1$
9. $y = \frac{c - ax}{b}$
10. $c = 4A - a - b - d$
11. $w = \frac{S - 2lh}{2l + 2h}$
12. $l = \frac{P - 2w}{2}$
13. $\pi = \frac{C}{d}$
14. $f = \frac{ab}{b + a}$
15. $t = \frac{A - p}{pr}$
16. $r = \frac{I}{pt}$
17. $a = \frac{c - b}{x}$
18. $h = \frac{S}{2\pi r}$
19. $h = \frac{A - 2\pi r^2}{2\pi r}$
20. $x = \frac{y - y_1 + mx_1}{m}$
21. $w = \frac{2R - l}{3}$
22. $y = \frac{-ax - c}{b}$
23. $F = \frac{9}{5}C + 32$

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