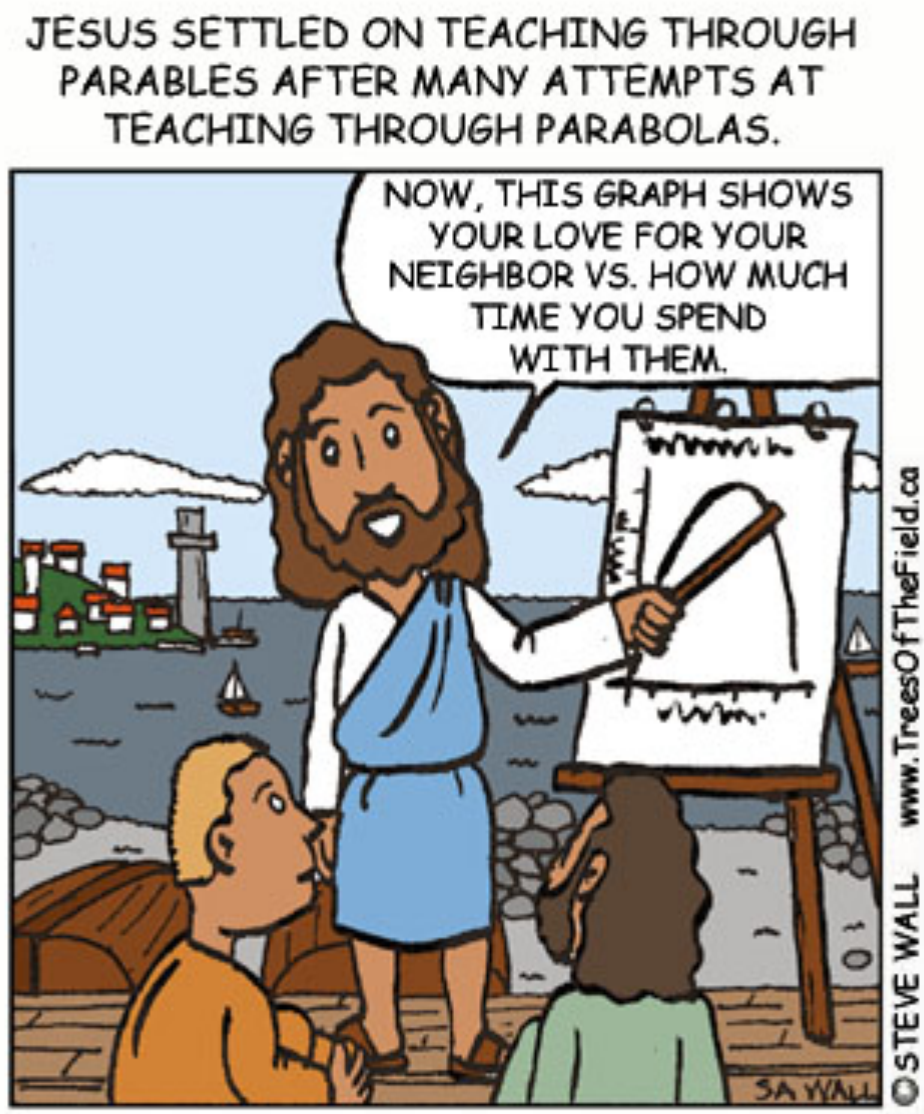


MCF3MI

Unit 2: Quadratic Functions – Forms and Transformations

UNIT 2B: QUADRATIC FUNCTIONS: FORMS – ESSENTIAL LEARNINGS

You will have at least three opportunities to demonstrate the essential learnings in this course. The first opportunity will occur during the unit with formative quizzes and homework checks. Your second opportunity will be on the unit test. Your third opportunity will be on the exam. You need to demonstrate all essential learnings to earn this credit.

Please take the opportunity to keep track of the essential learnings you've demonstrated during this unit using the checklist below.

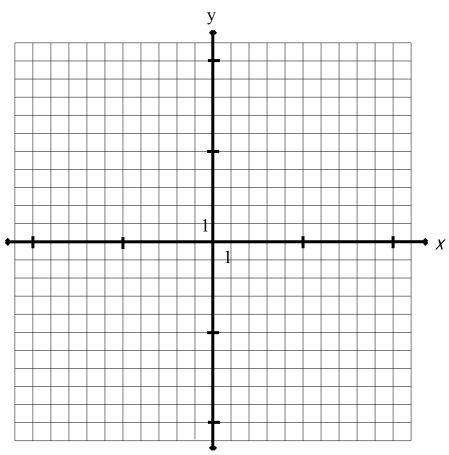
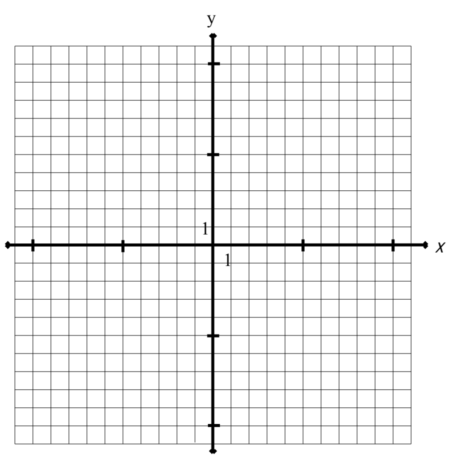
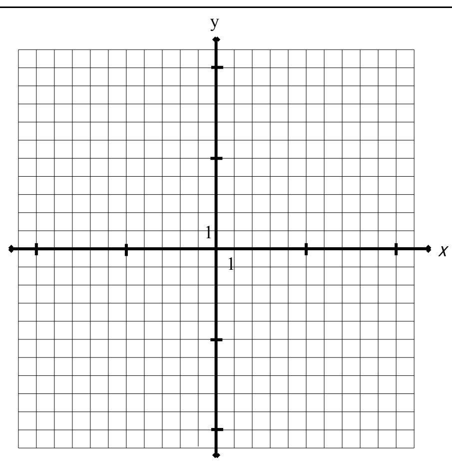
Essential Learnings: Connecting Graphs and Equations of Quadratic Functions & Solving Problems with Quadratic Functions	***Homework***	***Associated Lessons***
☐ Describe the information obtained by inspecting quadratic functions in all three forms (vertex, standard, and factored)	*pg. 71 – 72* #1, 2, 4, 6abc *pg. 83 – 84* #4bc, 6bd *pg. 96 – 97* #6aceg	Lesson 2.7
☐ Change accurately between quadratic equations in all three forms (vertex, standard, and factored)	*pg. 132 – 133* #3, 4, 9ace, 13, 14	Lesson 2.8
	worksheet	Lesson 2.9
☐ Sketch graphs in factored form or standard form, using various strategies	*worksheet*	Lesson 2.10
☐ Determine the equation of the quadratic function given a table of values or a graph	*pg. 169 – 170* #1 – 4, 8	Lesson 2.11
☐ Solve problems arising from real-word applications of quadratic functions	*pg. 72* #8, 9 *pg. 84 – 85* #12, 13, 16, 17	Lesson 2.12
Unit Review:	*pg. 114 – 115* #1, 2 (not g), 3 (not f) *pg. 116 – 117* #1 – 3, 7, 8, 10a, 11, 12 (not f), 13 (not g), 15 *pg. 174* #1	

Quadratic Functions: Comparing Three Forms

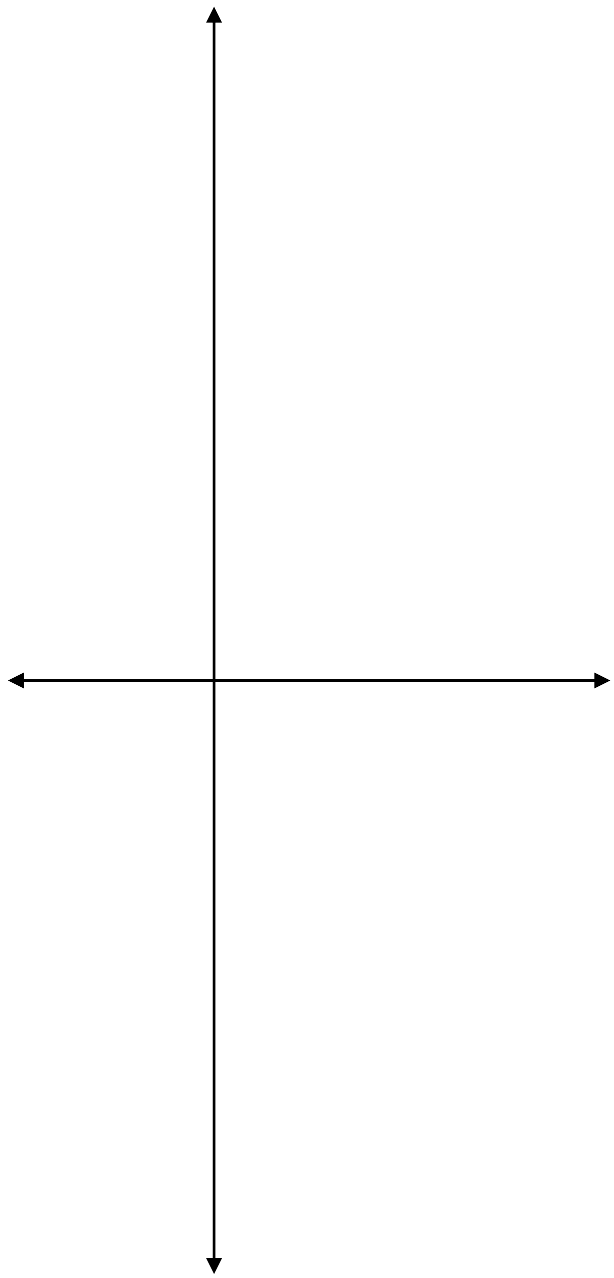
There are three algebraic forms commonly used to express a quadratic function, where $a \neq 0$:

- Factored Form: $f(x) = a(x - r)(x - s)$
- Vertex Form: $f(x) = a(x - h)^2 + k$
- Standard Form: $f(x) = ax^2 + bx + c$

Each <u>form</u> provides different kinds of information about the key features of the function.

 	**Factored Form** $f(x) = (x + 2)(x - 4)$
 	**Vertex Form** $f(x) = (x - 1)^2 - 9$
 	**Standard Form** $f(x) = x^2 - 2x - 8$

In general, the three different forms can be summarized as follows:

<table>
<tr><td rowspan="6"></td><td>Factored Form
$f(x) = a(x - r)(x - s)$</td></tr>
<tr><td></td></tr>
<tr><td>Vertex Form
$f(x) = a(x - h)^2 + k$</td></tr>
<tr><td></td></tr>
<tr><td>Standard Form
$f(x) = ax^2 + bx + c$</td></tr>
<tr><td></td></tr>
</table>

What do all three forms have in common?

HW: p. 71-72 #1, 2, 4, 6abc; p. 83-84 #4bc, 6bd; p. 96-97 #6aceg

1. Identify the values of a, b, and c for each quadratic function.

a) $f(x) = 2x^2 + 4x + 8$

b) $y = x^2 + 9x - 6$

c) $h(x) = -x^2 - 2x$

d) $y = \frac{1}{2}x^2 - 16$

2. For each quadratic function in question 1, identify

a) the direction of opening

b) whether the vertex is a maximum or minimum

c) the y-intercept

4. For each quadratic function, identify

- the direction of opening
- the x-intercepts
- the coordinates of the vertex, and whether it is a maximum or a minimum value
- the axis of symmetry
- the y-intercept

a) $f(x) = (x - 3)(x - 7)$

b) $y = -(x + 1)(x + 9)$

c) $f(x) = -2(x - 4)(x + 2)$

d) $y = \frac{1}{2}x(x + 6)$

6. For each quadratic function, identify

- the direction of opening
- the coordinates of the vertex, and whether it is a maximum or a minimum value
- the axis of symmetry
- the y-intercept

a) $f(x) = (x - 3)^2 + 2$

b) $y = (x + 4)^2 - 5$

c) $f(x) = -3(x + 1)^2 - 6$

4. Write each quadratic function in standard form and identify the y-intercept. Then, sketch a graph of the function.

a) $h(x) = 3(x + 2)(x - 5)$

b) $y = -2(x + 7)(x - 2)$

c) $f(x) = 4(x + 2)^2$

6. Express each quadratic function in standard form and identify the y-intercept. Then, sketch a graph of the function.

a) $m(x) = (x + 2)^2 + 3$

b) $y = (x - 1)^2 - 4$

c) $d(x) = 3(x - 5)^2 + 8$

d) $y = -4(x + 3)^2 - 9$

6. Write each quadratic function in factored form. Then, identify the x-intercepts.

a) $y = x^2 + 10x + 21$

b) $y = x^2 - 4x - 12$

c) $y = 3x^2 - 18x + 15$

d) $y = 2x^2 - 8x + 6$

e) $f(x) = 5x^2 + 10x - 15$

f) $g(x) = x^2 - 3x - 18$

g) $r(x) = 3x^2 - 9x + 6$

1. a) $a = 2, b = 4, c = 8$
b) $a = 1, b = 9, c = -6$
c) $a = -1, b = -2, c = 0$
d) $a = \frac{1}{2}, b = 0, c = -16$

2. a) upward; vertex is a minimum; y-intercept is 8
b) upward; vertex is a minimum; y-intercept is -6
c) downward; vertex is a maximum; y-intercept is 0
d) upward; vertex is a minimum; y-intercept is -16

4. Sketches may vary.
a) $h(x) = 3x^2 - 9x - 30$; y-intercept $= -30$
b) $y = -2x^2 - 10x + 28$; y-intercept $= 28$
c) $f(x) = 4x^2 + 16x + 16$; y-intercept $= 16$
d) $y = -\frac{1}{2}x^2 + 6x - 18$; y-intercept $= -18$

6. Sketches may vary.
a) $m(x) = x^2 + 4x + 7$; y-intercept is 7
b) $y = x^2 - 2x - 3$; y-intercept is -3
c) $d(x) = 3x^2 - 30x + 83$; y-intercept is 83
d) $y = -4x^2 - 24x - 45$; y-intercept is -45

4.

	a) $f(x) = (x - 3)(x - 7)$	b) $y = -(x + 1)(x + 9)$	c) $f(x) = -2(x - 4)(x + 2)$	d) $y = \frac{1}{2}x(x + 6)$
Direction of opening	upward	downward	downward	upward
x-intercepts	$x = 3$, $x = 7$	$x = -1$, $x = -9$	$x = 4$, $x = -2$	$x = 0$, $x = -6$
Vertex, maximum/minimum	(5, −4) minimum	(−5, 16) maximum	(1, 18) maximum	(−3, −4.5) minimum
Axis of symmetry	$x = 5$	$x = -5$	$x = 1$	$x = -3$
y-intercept	21	−9	16	0

6. a) $y = (x + 3)(x + 7)$; $x = -7$, $x = -3$
b) $y = (x - 6)(x + 2)$; $x = -2$, $x = 6$
c) $y = 3(x - 5)(x - 1)$; $x = 1$, $x = 5$
d) $y = 2(x - 3)(x - 1)$; $x = 1$, $x = 3$
e) $f(x) = 5(x + 3)(x - 1)$; $x = -3$, $x = 1$
f) $g(x) = (x - 6)(x + 3)$; $x = -3$, $x = 6$
g) $r(x) = 3(x - 2)(x - 1)$; $x = 1$, $x = 2$
h) $m(x) = 2(x + 3)(x - 4)$; $x = -3$, $x = 4$

6.

	a) $f(x) = (x - 3)^2 + 2$	b) $y = (x + 4)^2 - 5$	c) $f(x) = -3(x + 1)^2 - 6$	d) $\frac{1}{2}y = (x - 2)^2 - 8$
Direction of opening	upward	upward	downward	upward
Vertex, maximum/minimum	(3, 2) minimum	(−4, −5) minimum	(−1, −6) maximum	(2, −16) minimum
Axis of symmetry	$x = 3$	$x = -4$	$x = -1$	$x = 2$
y-intercept	(0, 11)	(0, 11)	(0, −9)	(0, −8)

Converting from Standard Form to Vertex Form

A. Completing the Square

Completing the square is a process for <u>converting</u> a quadratic function from ***standard*** form, $y = ax^2 + bx + c$, into *vertex* form, $y = a(x - h)^2 + k$ by creating a **perfect square trinomial**.

Recall: A **perfect square trinomial** will have one of the following forms:

Expanded Form		***Factored Form***
$a^2 + 2ab + b^2$	=	$(a + b)^2$
	or	
$a^2 - 2ab + b^2$	=	$(a - b)^2$

Ex. 1: Factor the following.

a) $x^2 + 12x + 36$

b) $x^2 - 6x + 9$

Ex. 2: Find the value of "c" that makes the expression a perfect square trinomial. _________________

a) $x^2 + 16x +$ ______

b) $x^2 - 10x +$ ______

c) $x^2 + x +$ ______

Ex. 3: Complete the square on the following.

a) $y = x^2 + 6x + 7$

→To be a *perfect square trinomial*, what <u>should</u> "c" be?

→Let's add and subtract this value from the RS of the equation (to make a *net change* of zero).

→Group the perfect square trinomial in brackets and factor.

→Simplify.

b) $y = x^2 - 8x + 2$

c) $y = 2x^2 - 4x + 5$

B. Finding the Values of *h* and *k* Using Formulas

Given _standard_ form, $f(x) = ax^2 + bx + c$, we can find the vertex *(h, k)* by using the following formulas:

$$h = \frac{-b}{2a}$$ and $$k = f(h)$$

Ex. 4: Convert the following into vertex form.

a) $y = \frac{1}{2}x^2 + 4x$

b) $y = -2x^2 - 3x + 7$

For part a) above, is the vertex a maximum or a minimum?____________________________ What is the maximum/ minimum value and when does it occur? ____

For part b) above, is the vertex a maximum or a minimum?____________________________ What is the maximum/ minimum value and when does it occur? ____

HW: p. 132-133 #3, 4, 9ace, 13, 14

3. Find the value of k that makes each expression a perfect square trinomial.

a) $x^2 + 2x + k$

b) $x^2 - 8x + k$

c) $x^2 - 16x + k$

d) $x^2 - 9x + k$

e) $x^2 + 26x + k$

f) $x^2 + 13x + k$

4. Rewrite each quadratic function in the vertex form, $y = a(x - h)^2 + k$, by completing the square.

a) $y = x^2 + 10x + 4$

b) $y = x^2 + 16x + 1$

c) $y = x^2 - 8x + 13$

d) $y = 3x^2 + 6x + 1$

e) $y = -2x^2 + 4x + 3$

f) $y = 4x^2 - 16x - 2$

9. Find the maximum or minimum value of each quadratic function by completing the square.

a) $y = 2x^2 - x + 1$

b) $y = -3x^2 + x$

c) $y = \frac{1}{4}x^2 - 2x + 1$

d) $y = 4x^2 - x$

e) $y = -2x^2 + 5x - 1$

f) $y = \frac{3}{4}x^2 + 6x + 1$

g) $y = 5x^2 + 2x - 3$

h) $y = -\frac{2}{3}x^2 - x + 2$

13. In a simulation, the path of an aircraft after it has achieved weightlessness can be modelled by $h(t) = -10t^2 + 300t + 9750$, where $h(t)$ is the altitude of the aircraft, in metres, and t is the time, in seconds, after weightlessness is achieved.

a) Sketch the graph.

b) Find the maximum height of the aircraft by completing the square.

c) How long does it take to reach its maximum height?

B

14. A ball is thrown vertically upward off the roof of a 34-m tall building. The height of the ball, $h(t)$, in metres, can be approximated by the function $h(t) = -5t^2 + 10t + 34$, where t is the time, in seconds, after the ball is thrown.

a) Sketch the graph.

b) How high is the ball after 2 s?

c) Find the maximum height of the ball.

3. a) $k = 1$ **b)** $k = 16$ **c)** $k = 64$
d) $k = 20.25$ **e)** $k = 169$ **f)** $k = 42.25$

4. a) $y = (x + 5)^2 - 21$ **b)** $y = (x + 8)^2 - 63$
c) $y = (x - 4)^2 - 3$ **d)** $y = 3(x + 1)^2 - 2$
e) $y = -2(x - 1)^2 + 5$ **f)** $y = 4(x - 2)^2 - 18$

9. a) minimum: $\frac{7}{8}$ **b)** maximum: $\frac{1}{12}$
c) minimum: -3 **d)** minimum: $-\frac{1}{16}$
e) maximum: 2.125 **f)** minimum: 11
g) minimum: -3.2 **h)** maximum: $2\frac{3}{8}$

13. a) Sketches may vary.
b) $h(t) = -10(t - 15)2 + 12\ 000$
The maximum height of the aircraft is 12 000 m.
c) 15 s

14. a) Sketches may vary.
b) 34 m **c)** 39 m

Converting from Factored or Vertex to Standard Form

A. Converting to Standard Form from Vertex Form

To complete our understanding of the relationship between the three common forms of quadratic functions, we need to be able to convert smoothly between them. *Recall*: Yesterday we converted from standard form to vertex form by completing the square.

Ex. 1: Complete the square on the following quadratic in standard form:

$y = x^2 - 12x + 9$

Today, we will focus on converting back to standard form from both vertex form and factored form. Let's begin with vertex form. To convert back to standard form from vertex form, you need to expand and simplify vertex form.

Ex. 2: Convert the following quadratic functions from vertex form to standard form.

a) $y = (x - 6)^2 - 27$

b) $y = -3(x + 2)^2 - 5$

B. Converting to Standard Form from Vertex Form

To convert to standard form from factored form, you need to expand and simplify factored form. Yes, it is the same process by which we convert from vertex form to standard form.

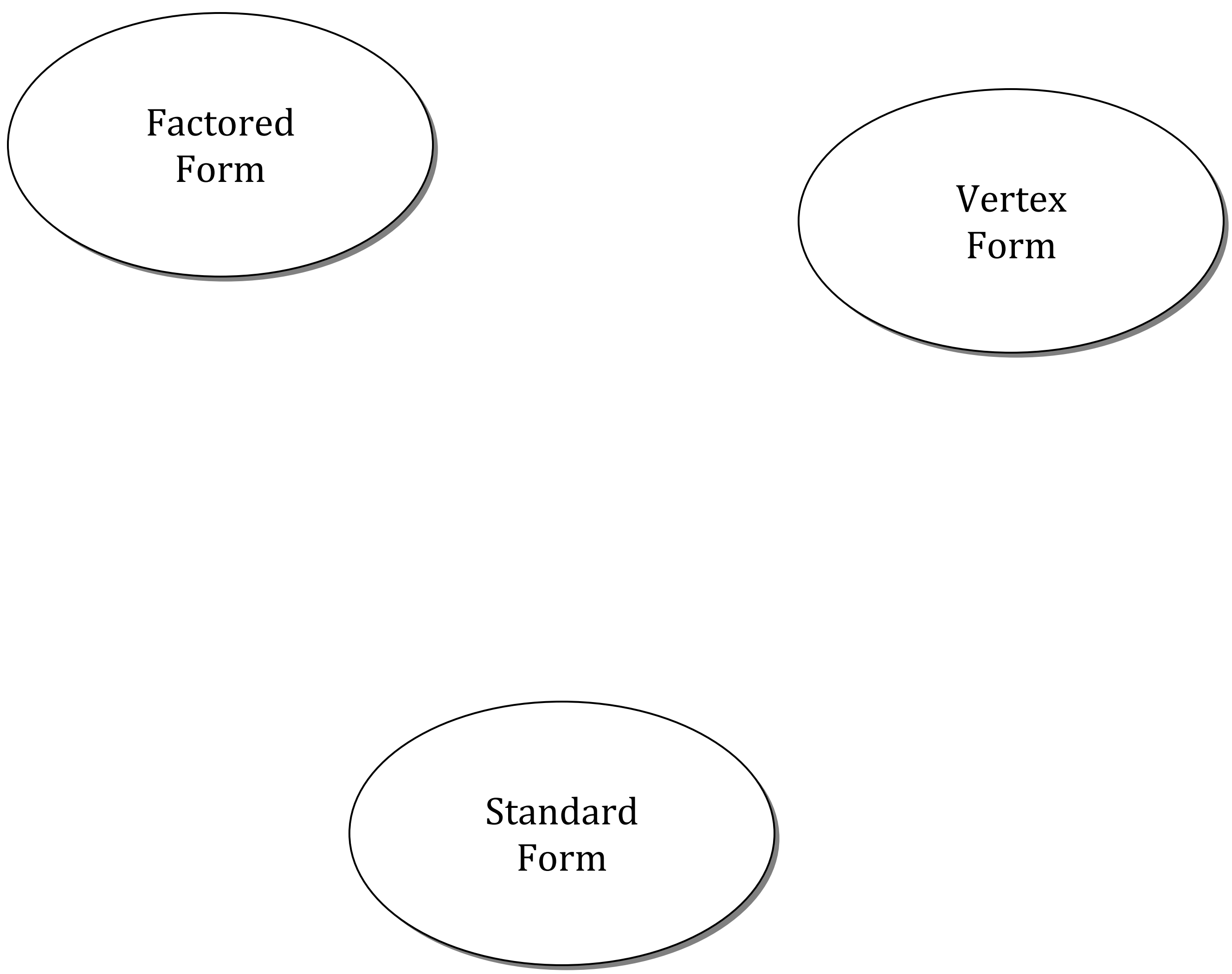
Ex. 3: Convert the following quadratic functions from factored form to standard form.

a) $y = (x - 5)(x + 4)$

b) $y = 2(x + 1)(x - 3)$

C. Converting Between All Three Forms

Now that we have worked through completing the square and expanding and simplifying, we have rehearsed all the conversion tools we need to move back and forth between all three forms of quadratic functions. Complete the following graphic organizer as a way of summarizing the conversion strategies.



HW: Worksheet "Unit 2 Lesson 9 Worksheet"

Converting Between All Three Form of Quadratic Functions

Given one of the forms of a quadratic function, convert to the other two forms.

1) $y = x^2 - 6x - 7$

Vertex Form: ___________________________________

Factored Form: ___________________________________

2) $y = 3(x - 1)^2 - 3$

Standard Form: ___________________________________

Factored Form: ___________________________________

3) $y = 2(x - 3)(x + 5)$

Vertex Form: ___________________________________

Standard Form: ___________________________________

4) $y = 2x^2 + 8x - 10$

Vertex Form: ________________________________

Factored Form: ________________________________

5) $y = 2(x + 1)^2 - 8$

Standard Form: ________________________________

Factored Form: ________________________________

6) $y = -3(x + 3)(x - 5)$

Vertex Form: ________________________________

Standard Form: ________________________________

Answers:

1. $y = (x - 3)^2 - 16$ $y = (x - 7)(x + 1)$
2. $y = 3x^2 - 6x$ $y = 3x(x - 2)$
3. $y = 2(x + 1)^2 - 32$ $y = 2x^2 + 4x - 30$
4. $y = 2(x + 2)^2 - 18$ $y = 2(x + 5)(x - 1)$
5. $y = 2x^2 + 4x - 6$ $y = 2(x + 3)(x - 1)$
6. $y = -3(x - 1)^2 + 48$ $y = -3x^2 + 6x + 45$

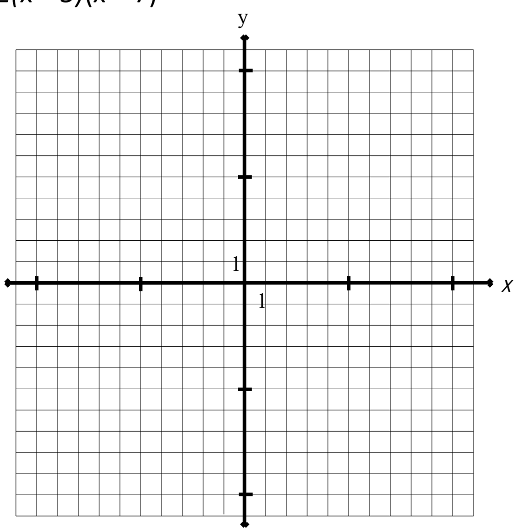
How to Graph a Quadratic Function (Given *Any* Form)!

You will not be permitted to leave this course without the following skills!! Master them! ☺

<table>
<tr><th></th><th>Characteristics</th><th>What To Do . . .</th></tr>
<tr><td>Vertex Form</td><td>$f(x) = a(x - h)^2 + k$ or $y = a(x - h)^2 + k$
• In this form, the only <u>point</u> evident is the <u>vertex</u>, (h, k)</td><td>• Plot and label the vertex, (h, k)
• From the vertex, move horizontally (⇔) 1 unit, and vertically (⇕) $1^2(a)$ units (on either side); then, move horizontally 2 units, and vertically $2^2(a)$ units; [repeat with 3 units and $3^2(a)$ units, if necessary, to get accurate end behaviour]
• Sketch and label the axis of symmetry, $x = h$</td></tr>
<tr><td>Factored Form</td><td>$f(x) = a(x - r)(x - s)$ or $y = a(x - r)(x - s)$
• In this form, the only <u>points</u> evident are the <u>x-intercepts</u>, $(r, 0)$ and $(s, 0)$
• the x-value of the vertex ("h") can be calculated by finding the value of $\frac{r+s}{2}$
• the y-value of the vertex ("k") can be calculated by finding the value of $f(h)$</td><td>• Plot and label the x-intercepts, $(r, 0)$ and $(s, 0)$
• Find and plot the vertex, (h, k), using:
○ $h = \frac{r+s}{2}$
○ $k = f(h)$
• From the vertex, move horizontally (⇔) 1 unit, and vertically (⇕) $1^2(a)$ units (on either side); then, move horizontally 2 units, and vertically $2^2(a)$ units; [repeat with 3 units and $3^2(a)$ units, if necessary, to get accurate end behaviour]
• Sketch and label the axis of symmetry, $x = h$</td></tr>
<tr><td>Standard Form</td><td>$f(x) = ax^2 + bx + c$ or $y = ax^2 + bx + c$
• In this form, the only <u>point</u> evident is the <u>y-intercept</u>, $(0, c)$
• the x-value of the vertex ("h") can be calculated by finding the value of $\frac{-b}{2a}$
• the y-value of the vertex ("k") can be calculated by finding the value of $f(h)$</td><td>• Plot and label the y-intercept, $(0, c)$
• Find, plot, and label the vertex, (h, k), using:
○ $h = \frac{-b}{2a}$
○ $k = f(h)$
• From the vertex, move horizontally (⇔) 1 unit, and vertically (⇕) $1^2(a)$ units (on either side); then, move horizontally 2 units, and vertically $2^2(a)$ units; [repeat with 3 units and $3^2(a)$ units, if necessary, to get accurate end behaviour]
• Sketch and label the axis of symmetry, $x = h$</td></tr>
<tr><td>Any Form</td><td colspan="2">• any function can be graphed using a table of values
• any uncertain points on a graph can be checked using a table of values
• WARNING! Tables of values can be very time consuming, and should be used as a last resort.</td></tr>
</table>

1) Graph the following quadratic functions. 2) Check your answers using the graphing calculator.
3) Give yourself a mark out of 12 based on the graphing checklist and rubric. 4) Show your work to your teacher.

$f(x) = 2(x - 3)(x - 7)$



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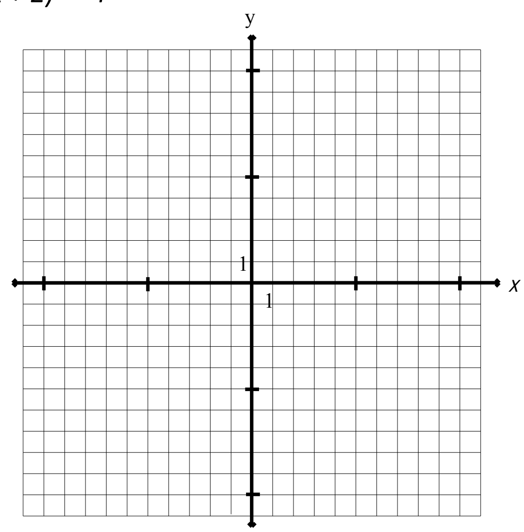
$f(x) = 3(x + 2)^2 - 4$



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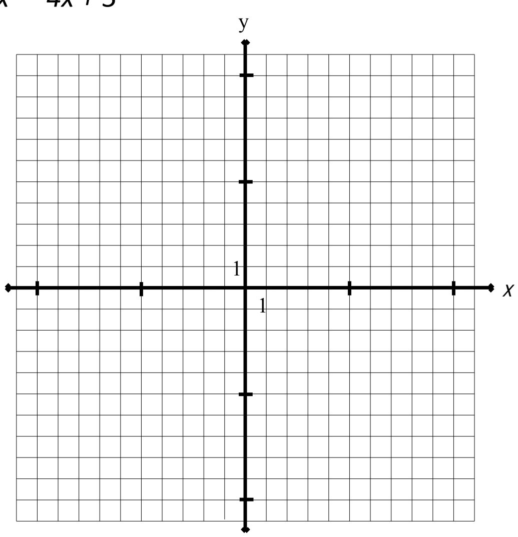
$f(x) = x^2 - 4x + 3$



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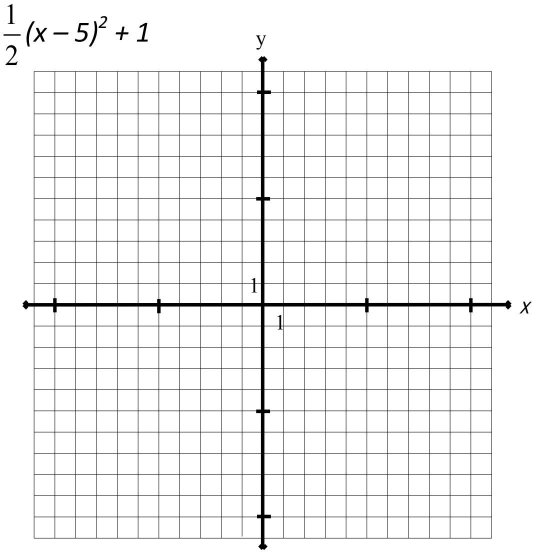
$f(x) = -\frac{1}{2}(x - 5)^2 + 1$



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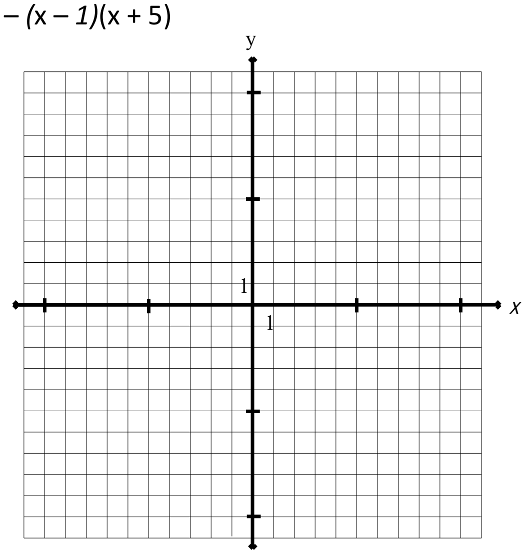
$f(x) = -(x - 1)(x + 5)$



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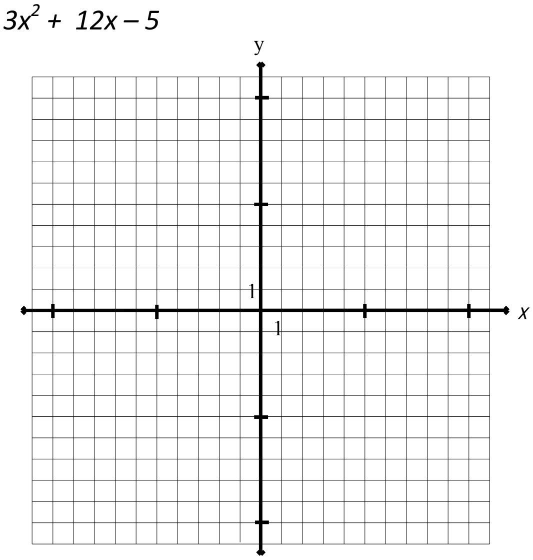
$f(x) = -3x^2 + 12x - 5$



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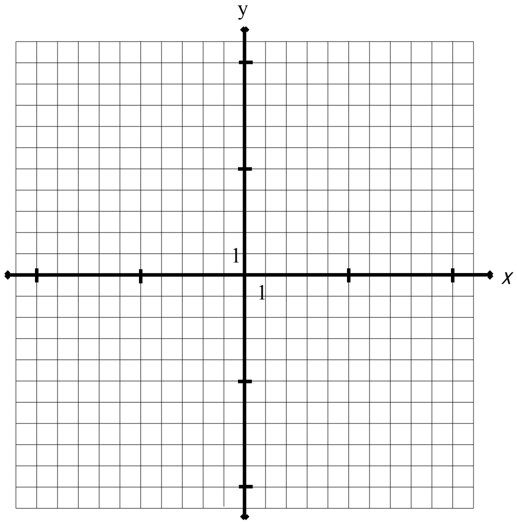
1) Graph the following functions.   2) Check your work against the rubric/checklist.   3) Practice!

f(x) = – *x*(x – 6)



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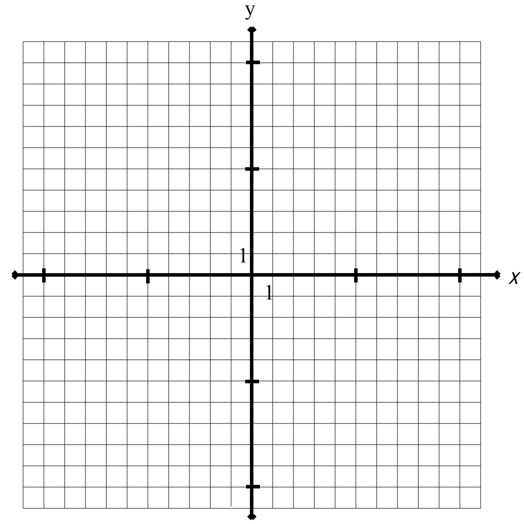
f(x) = $(x - 3)^2 + 2$



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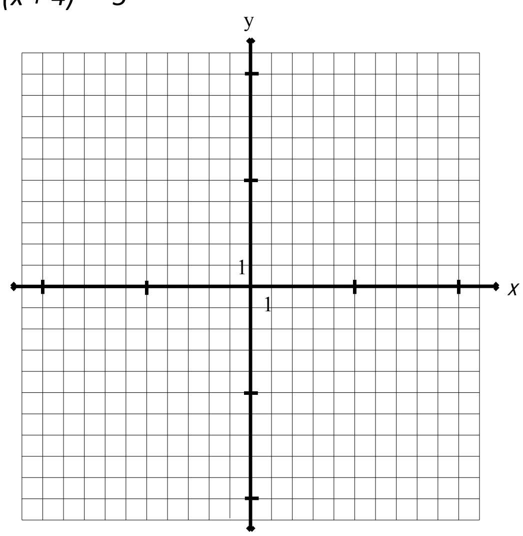
f(x) = $(x + 4)^2 - 5$



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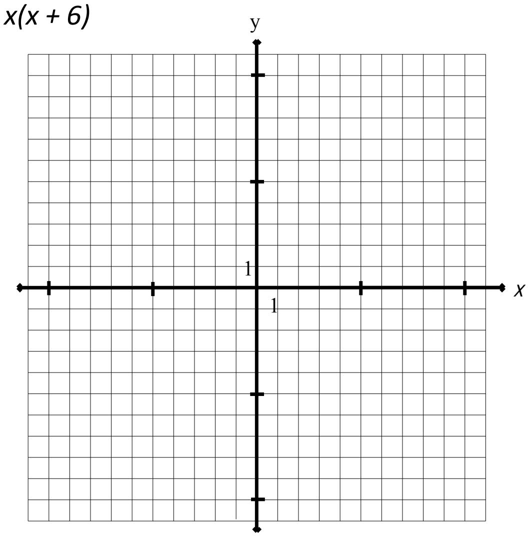
f(x) = $\frac{1}{2}x(x + 6)$



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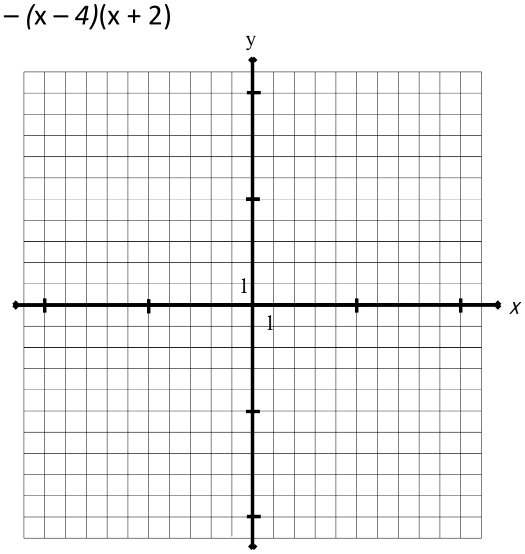
f(x) = – (x – 4)(x + 2)



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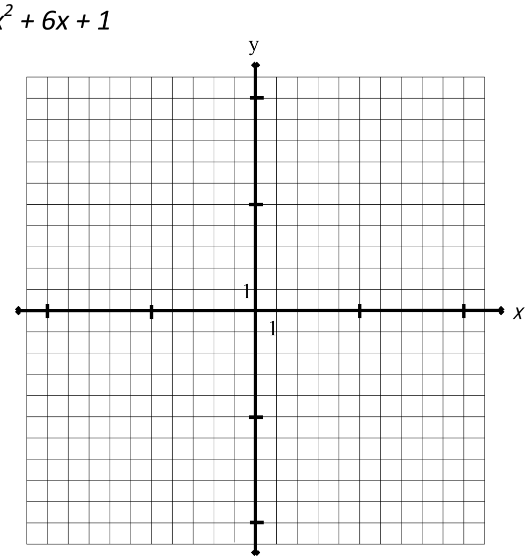
f(x) = $3x^2 + 6x + 1$



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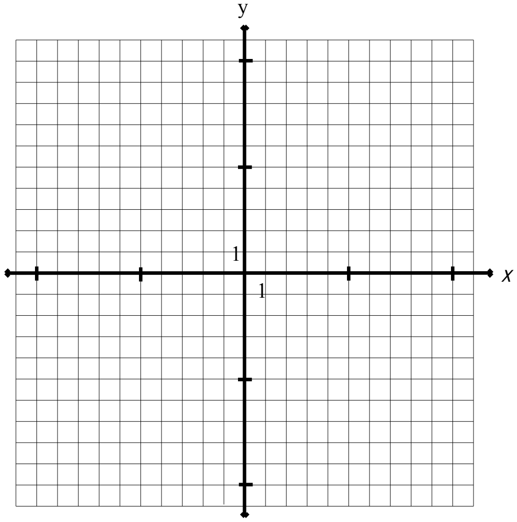
$f(x) = 2(x - 2)(x - 6)$



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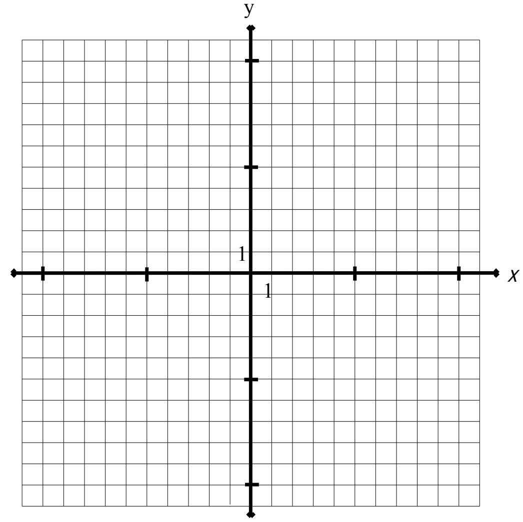
$f(x) = -3(x + 1)^2 - 6$



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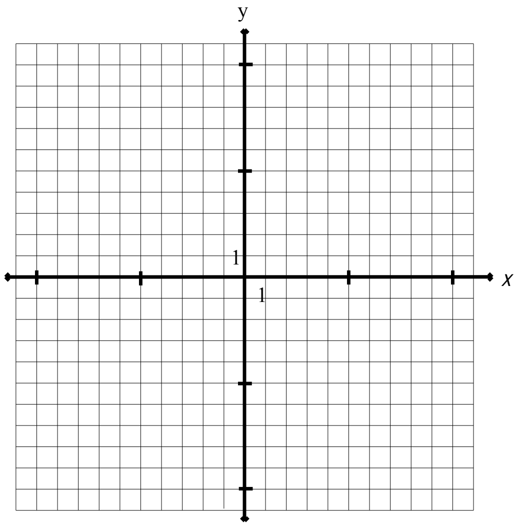
$f(x) = x^2 - 2x + 5$



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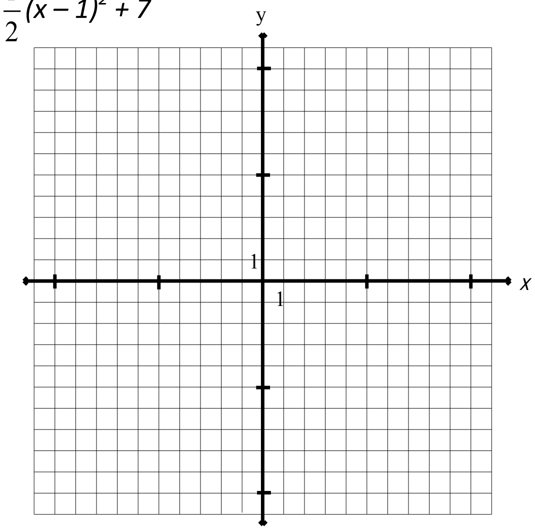
$f(x) = -\frac{1}{2}(x - 1)^2 + 7$



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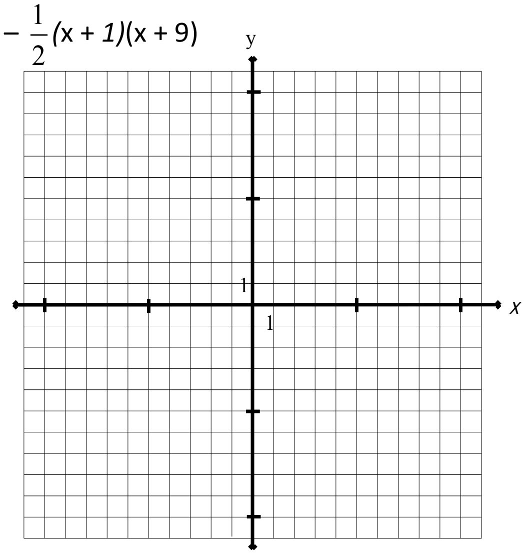
$f(x) = -\frac{1}{2}(x + 1)(x + 9)$



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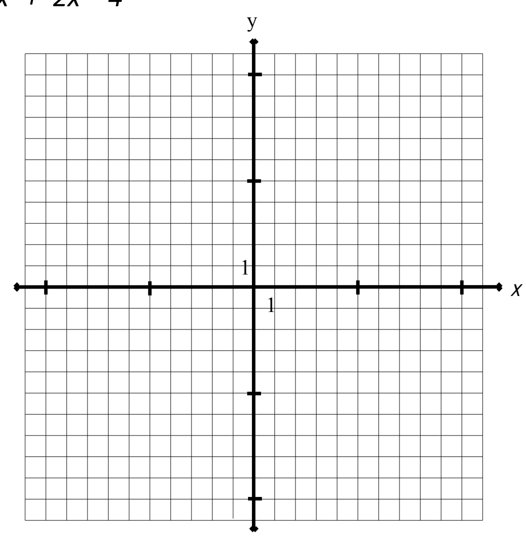
$f(x) = -x^2 + 2x - 4$



___ /12

How to Find the Equation of a Quadratic Function (Given a Graph or a Data Set)!

You will not be permitted to leave this course without the following skills!! Master them! ☺

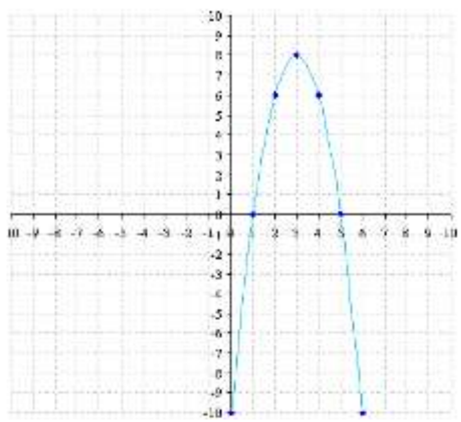
Find an Equation From a Graph

Steps:

1. Locate the <u>vertex</u>, (h, k)
2. Locate <u>another point</u> on the curve, (x, y)
3. Substitute x, y, h, and k into the vertex form $[y = a(x - h)^2 + k]$ and rearrange to <u>solve for "a"</u>
4. <u>Substitute</u> the values for a, h, and k back into $f(x) = a(x - h)^2 + k$ ***or*** $y = a(x - h)^2 + k$

Example:

Ex. 1: Find the equation of the parabola given its graph.



Find an Equation That Models a Data Set

Steps:

1. Create a <u>scatter plot</u> of the data by hand or using technology
2. Locate the <u>vertex</u>, (h, k)
3. Locate <u>another point</u> on the curve, (x, y)
4. Substitute $x, y, h,$ and k into the vertex form $[y = a(x - h)^2 + k]$ and rearrange to <u>solve for "a"</u>
5. <u>Substitute</u> the values for a, h, and k back into $f(x) = a(x - h)^2 + k$ ***or*** $y = a(x - h)^2 + k$

Note: *using Excel, you can find the equation by adding a trendline to your scatter plot using a <u>polynomial regression</u> with a degree of "2".*

Example:

Ex. 2: Find an equation that models the following data set, with and without technology. Compare the models.

x	*y*
0	0
7	23
18	47
24	56
33	46
44	19
50	0

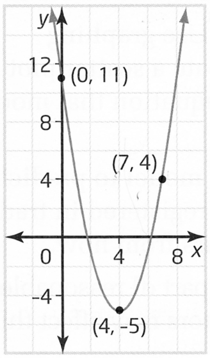
HW: p.169-171 # 1-4, 8

A

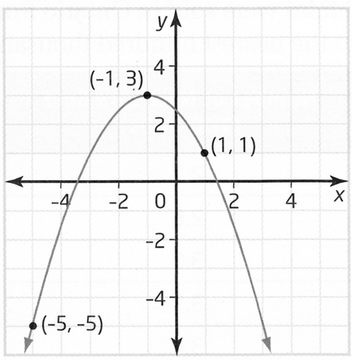
For help with questions 1 and 2, refer to Example 1.

1. Write an equation for each parabola given its graph.

a)

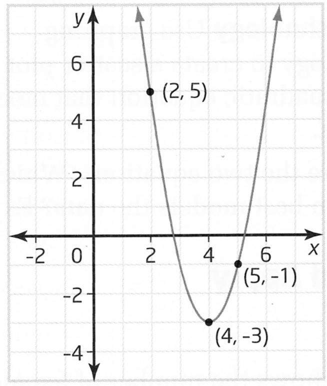


b)

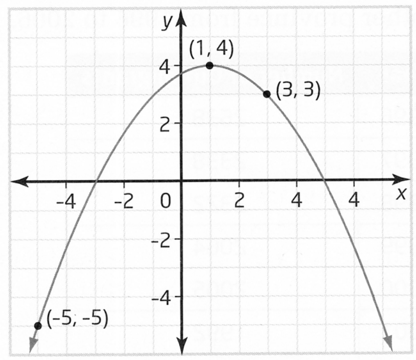


2. Write an equation for each parabola given its graph.

a)



b)



3. The table shows the curve of a parabolic arch at the entrance to a park, where x is the horizontal distance from one side of the arch and h is the height of the arch above the ground, both in metres.

x	h
0	0
2	7
6	16
9	19
15	17
18	12
22	0

a) Make a scatter plot of the data.

b) Estimate the coordinates of the vertex and find an equation that models the data.

c) **Use Technology** Use graphing technology to create a scatter plot and find a quadratic equation that models the data.

d) Compare the two equations. Which equation best models the data? Explain.

4. The table shows the number of females aged 15–19 who migrated from Ontario to another province from 1996 to 2006.

Year	Number of Out-migrants
1996	2638
1997	2338
1998	2322
1999	2064
2000	2005
2001	1992
2002	2232
2003	2178
2004	2150
2005	2282
2006	2799

Source: Statistics Canada, CANSIM Table 051-0012

a) Make a scatter plot of the data and draw a curve of best fit.

b) Estimate the coordinates of the vertex and find an equation that models the data.

c) **Use Technology** Use graphing technology to create a scatter plot and find a quadratic equation that models the data.

d) Does a quadratic model represent the data well? Explain.

8. The table shows data collected at a test track that is used for measuring the length of a skid mark versus the speed of a car.

Speed (km/h)	Length of Skid (m)
0	0.0
10	0.8
20	3.2
30	7.1
40	12.3
50	18.6
60	25.4
70	32.6
80	47.1
90	59.2
100	70.8

a) Make a scatter plot of the data and draw a curve of best fit.

b) Use the coordinates of the vertex to find an equation that models the data.

c) Use the algebraic equation to estimate the length of the skid mark for a car travelling at 120 km/h.

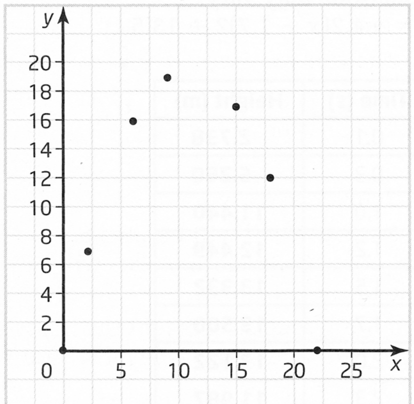
d) **Use Technology** Use graphing technology to find a quadratic equation that models the data.

e) Use the quadratic regression equation to determine the length of the skid for a car travelling at 120 km/h. Compare the result with your answer for part c). How far off was your estimate?

1. a) $y = (x - 4)^2 - 5$ **b)** $y = -\frac{1}{2}(x + 1)^2 + 3$

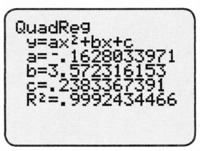
2. a) $y = 2(x - 4)^2 - 3$ **b)** $y = -\frac{1}{4}(x - 1)^2 + 4$

3. a)



b) Estimates and equations may vary; vertex: (11, 20); $y = -\frac{20}{121}(x - 11)^2 + 20$

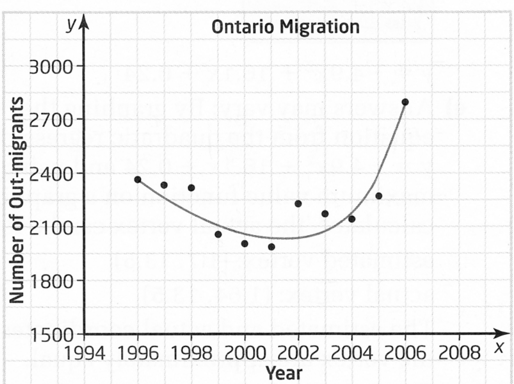
c)



$y = -0.162\,803\,397\,1x^2 + 3.572\,316\,153x + 0.238\,336\,739\,1$

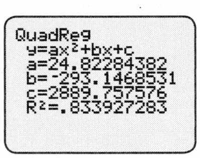
d) The equation derived from the quadratic regression on the data is the best model. When graphed, the parabola passes through all of the points.

4. a)



b) Estimates and equations may vary; vertex: (2001, 1992); $y = \frac{16}{5}(x - 2001)^2 + 1992$

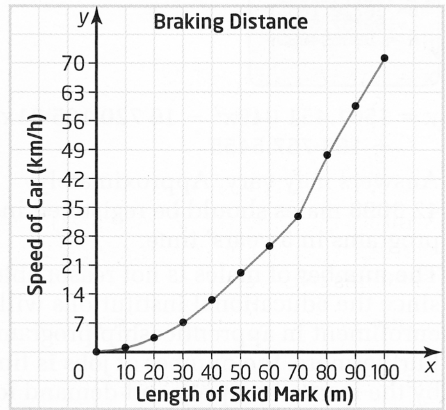
c)



$y = 24.822\,843\,82x^2 - 293.146\,853\,1x + 2889.757\,576$

d) A quadratic model fits the data well since the parabola passes through several points and the number of points above and below the parabola are equal. The parabola represents the average distances between the points.

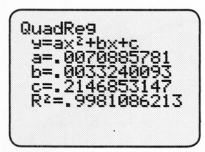
8. a)



b) Equations may vary; $y = 0.008x^2$

c) Answers may vary; 115.2 m

d)



$y = 0.007\,088\,578\,1x^2 + 0.003\,324\,009\,3x + 0.214\,685\,3147$

e) $y \doteq 102.7$

The difference between the estimate and calculated value is $115.2 - 102.7$ or 12.5 m. This is a difference of almost 11%.

Applications of Quadratic Functions

Quadratic functions can model many real-life situations:

- the path of a flare being fired into the sky
- the arc of a volleyball being bumped or set
- the path of Tarzan swinging on a rope from one tree to the next (okay, this one's not real life)
- and lots more!

Let's look at a compelling real-world example:

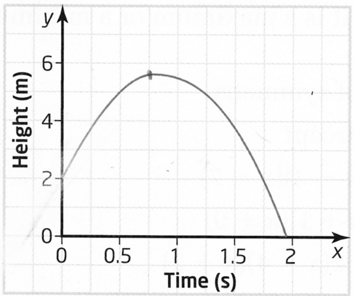
The path of my little toe as I jump off the platform in our classroom can be modeled by a quadratic function. We can use some key features of the function to find out valuable information about the behaviour of my toe as it careens through space!

1. What variable is being measured on the **x-axis**? __________________________
2. What variable is being measured on the **y-axis**? __________________________
3. What is the **direction of opening** for this parabola? __________________________
4. Which point on the graph will tell us what the height of the **platform** is? __________________________
5. Which point will tell us what my toe's **maximum** height was during its flight? _________________________
6. How can we find out **how long** the toe was in the air? ____
7. Sketch a graph showing the path of my little toe. Label the axes, give the graph a title, and mark the key features describing the toe's behaviour.
8. For this real-life situation, what is the domain? _______________________________________
9. What is the range? _____________________________________

Let's complete question #12abc together from p. 73 of your textbooks . . .

HW: p. 72 #8, 9 and p. 84-85 #12, 13, 16, 17

8. This graph shows the height–time relationship of a bottle rocket.

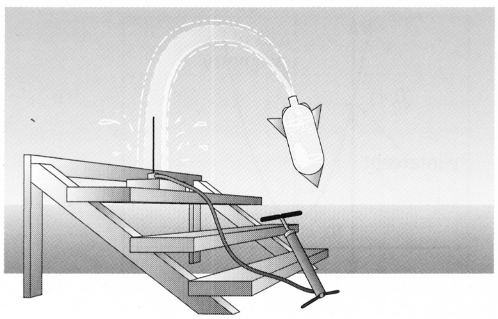


a) From what height above ground was this bottle rocket launched?

b) What is the approximate maximum height reached?

c) Estimate the hang time of the rocket.

Explain how you found each of your answers.



9. Refer to question 8.

a) Copy the graph and extend it into the second quadrant (to the left of the y-axis) to locate a second x-intercept. Does this point have any physical meaning in the given situation? Explain.

b) Use both x-intercepts to find an approximate value for the vertex. Use the vertex to find a quadratic function in vertex form that models this graph.

c) Check the accuracy of your function by substituting coordinates from the graph. How accurate is the function?

d) Suppose that the launching platform were lowered. Describe how this would affect

- the y-intercept of the corresponding graph
- the location of the vertex and the maximum height achieved
- the x-intercept of the corresponding graph and the hang time of the rocket

12. An object is thrown into the air and returns to the ground. The equation relating height and time for this object is $h = -4.9t(t - 2)$, where h is the height, in metres, and t is the time, in seconds.

a) In which form is this quadratic function expressed? Justify your answer.

b) Find the zeros, or t-intercepts, of this function. Explain their significance.

c) Write the function in standard form.

d) Find the coordinates of the vertex. Is this vertex a maximum or minimum? Explain the significance of this point.

e) Graph the function.

13. Could golfing on the moon improve your score? The function $h(t) = -0.8(t - 25)^2 + 500$ gives the height $h(t)$, in metres, as a function of the time t, in seconds, for a typical golf shot on the moon.

a) In which form is this quadratic function expressed? Justify your answer.

b) What are the coordinates of the vertex? Is the vertex a maximum or minimum? Explain what the coordinates of the vertex tell you about the golf shot.

c) Write the function in standard form, and identify the y-intercept.

d) Graph the function.

e) Identify the intervals for which the function is

i) positive or negative

ii) increasing or decreasing

16. Use Technology A company wants to sell T-shirts at a high school. Research suggests that if each T-shirt costs \$15, the company will sell 100 T-shirts. For every dollar increase in price they will sell 3 fewer T-shirts.

a) Use the formula Revenue = (price of one T-shirt)(number of T-shirts sold) to explain why revenue is given by $R(x) = (15 + x)(100 - 3x)$, where R is the revenue and x is the increase in price, both in dollars.

b) Use a graphing calculator or CAS to graph the function and find the y-intercept. What is the significance of the y-intercept?

c) Write the function in standard form. Explain how the standard form can be used to verify your answer to part a).

d) Find the T-shirt selling price that will maximize revenue. Find the total expected revenue if T-shirts are sold at that price.

17. Use Technology Refer to question 16. The company is considering decreasing their price. Research suggests that for every dollar decrease in price, the company will sell 18 more T-shirts.

a) Write the equation that expresses revenue as a function of x, where x is the price reduction in dollars.

b) If the company decreases the price, what is the price of a T-shirt that will maximize revenue? What is the maximum revenue the company can expect at that reduced selling price?

c) Compare these results to your results from question 16 part d). Should the company increase or decrease price? Explain why.

8. a) 2 m **b)** 5.5 m **c)** 1.9 s

9. a) $x = -0.15$ s; not meaningful; Explanations may vary.

b) (0.875, 5.5); $y = -5(x - 0.875)^2 + 5.5$

c) Estimates and functions may vary depending on the value obtained for the first x-intercept and the value assigned for the maximum height.

d) The y-intercept of the graph would be lowered to the height chosen for the new pad. The maximum height would be reduced by the same amount as was taken from the launching pad. However, the time required to get to the maximum height would not change. The x-intercept of the new graph would be smaller as would the hang time for the rocket associated with it.

12. a) factored form; It corresponds most closely to the form $f(x) = a(x - r)(x - s)$.

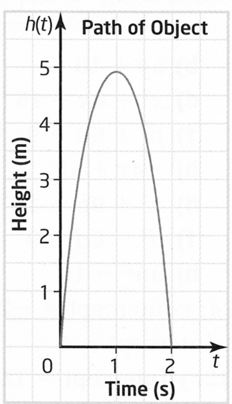
b) $t = 0$; $t = 2$; These values indicate the time at which the object leaves the ground and the time at which it returns.

c) $h(t) = -4.9t^2 + 9.8t$

d) (1, 4.9); It is a maximum. Since $a = -4.9$, the parabola opens downward.

The object reaches its maximum height of 4.9 m at a time of 1 s after it is thrown.

e)



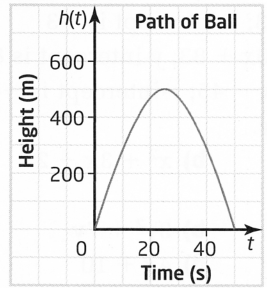
13. a) vertex form; It corresponds directly to the form $f(x) = a(x - h)^2 + k$.

b) (25, 500); It is a maximum; Since $a = -0.8$, the parabola opens downward.

The ball will reach a maximum height of 500 m at a time of 25 s after it is hit.

c) $h(t) = -0.8t^2 + 40t$; y-intercept = 0

d)



e) i) The function is positive for $0 \text{ s} < t < 50$ s.

ii) The function is increasing for $0 \text{ s} < t < 25$ s.

The function is decreasing for $25 \text{ s} < t < 50$ s.

16. a) Explanations may vary.

b) y-intercept = 1500; It represents the revenue generated when there is no increase in the price of a T-shirt.

c) $R(x) = -3x^2 + 55x + 1500$; Explanations may vary.

d) vertex: approximately (9, 1752); The maximum revenue will be \$1752 when the price of a T-shirt is \$24.

17. a) $R(x) = (15 - x)(100 + 18x)$

b) Since only whole dollar decreases are allowed, the maximum revenue will be \$1900 when the price of a T-shirt is \$10.

c) decrease; the company will generate a greater revenue

Review for Unit 2 Part B Test

Representative Questions for Review:

1) Convert the following quadratic into standard form and factored form:

   $y = 2(x - 1)^2 + 4$

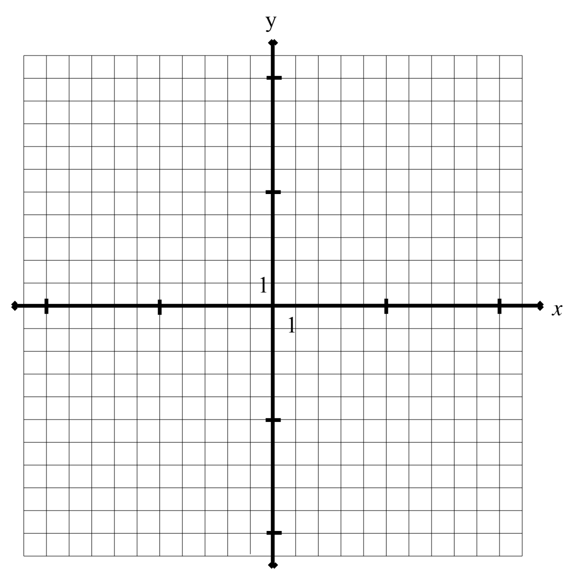
2) Convert the following quadratic into standard form and vertex form:

   $y = (x - 4)(x - 2)$

3) Given $y = -2x^2 + 8x + 1$, what is the initial value?

   Find the vertex.

   Graph the quadratic with all appropriate labels.



Textbook review questions:

p. 114-115 #1, 2 (not g), 3 (not f)

p. 116-117 #1-3, 7, 8, 10a, 11, 12 (not f), 13 (not g), 15

p. 174 #1

2.1 Quadratic Functions: Exploring Forms, pages 64-75

1. Consider this quadratic function.
$f(x) = x^2 + 5x - 8$

a) In which form is this function?

b) Identify the values of a, b, and c. Does this parabola open upward or downward?

c) What is the y-intercept?

d) Create a table of values and graph the function. Use enough values to get a good idea of the shape of the graph.

e) Estimate the values of the

i) x-intercepts **ii)** vertex

f) Is the vertex a maximum or a minimum? Explain.

2. Consider this quadratic function.
$f(x) = -(x - 3)(x + 5)$

a) In which form is this function? Does this parabola open upward or downward?

b) What are the x-intercepts?

c) Use the x-intercepts to find the coordinates of the vertex. Is the vertex a maximum or a minimum? Explain.

d) What is the axis of symmetry?

e) What is the y-intercept?

f) Graph the function.

g) Identify the intervals for which the function is

i) positive or negative

ii) increasing or decreasing

3. Consider this quadratic function.
$y = 2(x - 1)^2 - 3$.

a) In which form is this function? Does this parabola open upward or downward?

b) What are the coordinates of the vertex? Is the vertex a maximum or a minimum? Explain.

c) What is the axis of symmetry?

d) What is the y-intercept?

e) Graph the function.

f) Identify the intervals for which the function is increasing or decreasing.

1. Which statement is true for
$f(x) = (x + 3)^2 - 4$?

A The x-intercepts are 3 and -4.

B The x-intercepts are -3 and -4.

C The vertex is $(3, -4)$.

D The vertex is $(-3, -4)$.

2. Which statement is true for
$y = 2(x + 5)(x - 8)$?

A The x-intercepts are 5 and -8.

B The x-intercepts are -5 and 8.

C The vertex is $(5, -8)$.

D The vertex is $(-5, 8)$.

3. Which statement is true for
$m(x) = -3x^2 + x - 4$?

A The y-intercept is -4 and the parabola opens upward.

B The y-intercept is -4 and the parabola opens downward.

C The y-intercept is -3 and the parabola opens upward.

D The y-intercept is -3 and the parabola opens downward.

7. Write each quadratic function in standard form. Then, identify the y-intercept.

a) $f(x) = -2(x - 3)(x + 4)$

b) $g(x) = 3(x - 2)^2 + 5$

8. Factor each expression, if possible. If it is not possible, write "not factorable" and explain why.

a) $4y^2 - 36$ **b)** $h^2 - 4h - 32$

c) $p^2 - 20p + 100$ **d)** $w^2 + 8w + 6$

e) $3k^2 + 16k - 12$ **f)** $2f^2 + 4f - 7$

10. Write each quadratic function in factored form. Then, identify the x-intercepts.

a) $h(x) = x^2 + x - 42$

b) $f(x) = 2x^2 - 9x - 18$

11. An arrow is fired from a raised platform. Its height as a function of time is given by $h(t) = -5t^2 + 10t + 40$, where t is the time, in seconds, and $h(t)$ is the height, in metres. Find the length of time the arrow is in the air.

12. Consider the quadratic function $y = -2(x - 1)^2 - 3$.

a) In which form is this function? Does this parabola open upward or downward?

b) What are the coordinates of the vertex? Is the vertex a maximum or a minimum? Explain.

c) What is the axis of symmetry?

d) What is the y-intercept?

e) Graph the function.

f) Identify the intervals for which the function is increasing or decreasing.

13. Consider the quadratic function $w(x) = (x - 1)(x + 5)$.

a) In which form is this equation? Does this parabola open upward or downward?

b) What are the x-intercepts?

c) Find the coordinates of the vertex. Is the vertex a maximum or a minimum? Explain.

d) What is the axis of symmetry?

e) What is the y-intercept?

f) Graph the function.

15. Digital music player cases sell for \$20 each. Research by the seller has shown that for each \$10 increase in price, 100 fewer cases will be sold. The revenue generated from case sales can be represented by the function $R(x) = (2 + x)(6 - x)$, where $R(x)$ is the revenue, in thousands of dollars, and x is the number of \$10 price increases.

a) What are the x-intercepts of this function?

b) Use the x-intercepts to find the equation of the axis of symmetry and the coordinates of the vertex.

c) Is the vertex a maximum or a minimum?

d) How many \$10 price increases should there be in order to maximize revenue? What is the selling price that gives the maximum revenue?

1. Rewrite each quadratic function in the vertex form, $y = a(x - h)^2 + k$, by completing the square.

a) $y = x^2 + 8x - 7$

b) $y = -2x^2 + 16x - 3$

c) $y = \frac{1}{2}x^2 - 4x + 1$

1. a) standard form

b) $a = 1$, $b = 5$, $c = -8$; upward

c) -8

d)

x	y
-5	-8
-4	-12
-3	-14
-2	-14
-1	-12
0	-8
1	-2

e) i) $x = -6.3$, $x = 1.3$

ii) $(-2.5, -14.25)$

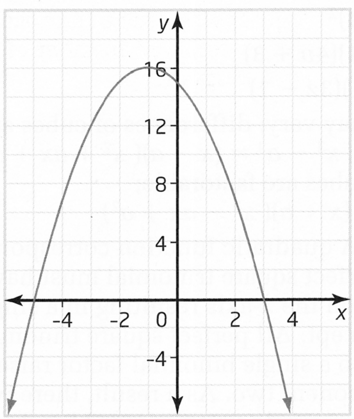
f) The vertex is a minimum. The graph opens upward.

2. a) factored form; downward

b) $x = 3$, $x = -5$ **c)** $(-1, 16)$; maximum

d) $x = -1$ **e)** y-intercept $= 15$

f)



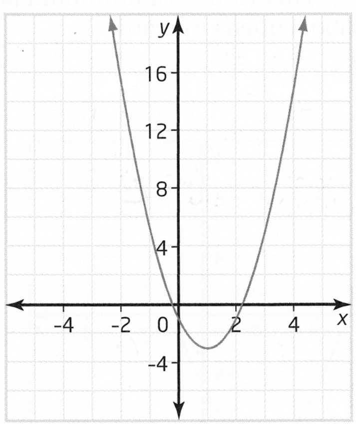
3. a) vertex form; upward

b) $(1, -3)$; minimum

c) $x = 1$

d) y-intercept $= -1$

e)



1. D

2. B

3. B

7. a) $f(x) = -2x^2 - 2x + 24$; y-intercept $= 24$

b) $g(x) = 3x^2 - 12x + 17$; y-intercept $= 17$

8. a) $4(y + 3)(y - 3)$ **b)** $(h - 8)(h + 4)$

c) $(p - 10)^2$

d) not factorable; There are no factor pairs of 6 that have a sum of 8.

e) $(k + 6)(3k - 2)$

f) not factorable; There are no factor pairs of -14 that have a sum of 4.

10. a) $h(x) = (x + 7)(x - 6)$; $x = -7$, $x = 6$

b) $f(x) = (x - 6)(2x + 3)$; $x = -\frac{3}{2}$, $x = 6$

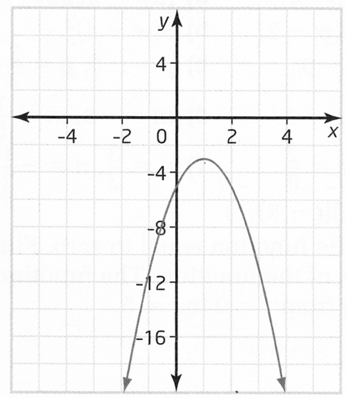
11. 4 s

12. a) vertex form; downward

b) $(1, -3)$; The vertex gives a maximum value for y because the parabola opens downward.

c) $x = 1$ **d)** -5

e)



13. a) factored form; upward

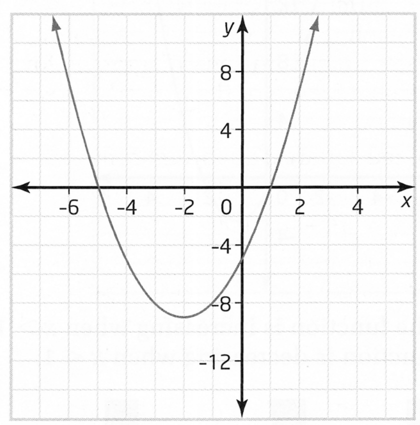
b) 1 and -5

c) $(-2, -9)$; This value is a minimum because the parabola opens upward.

d) $x = -2$

e) -5

f)



15. a) $x = -2$, $x = 6$

b) $x = 2$; $(2, 16)$

c) maximum

d) Two \$10 price increases (or a price of \$40) will result in a maximum possible revenue.

1. a) $y = (x + 4)^2 - 23$

b) $y = -2(x - 4)^2 + 29$

c) $y = \frac{1}{2}(x - 4)^2 - 7$