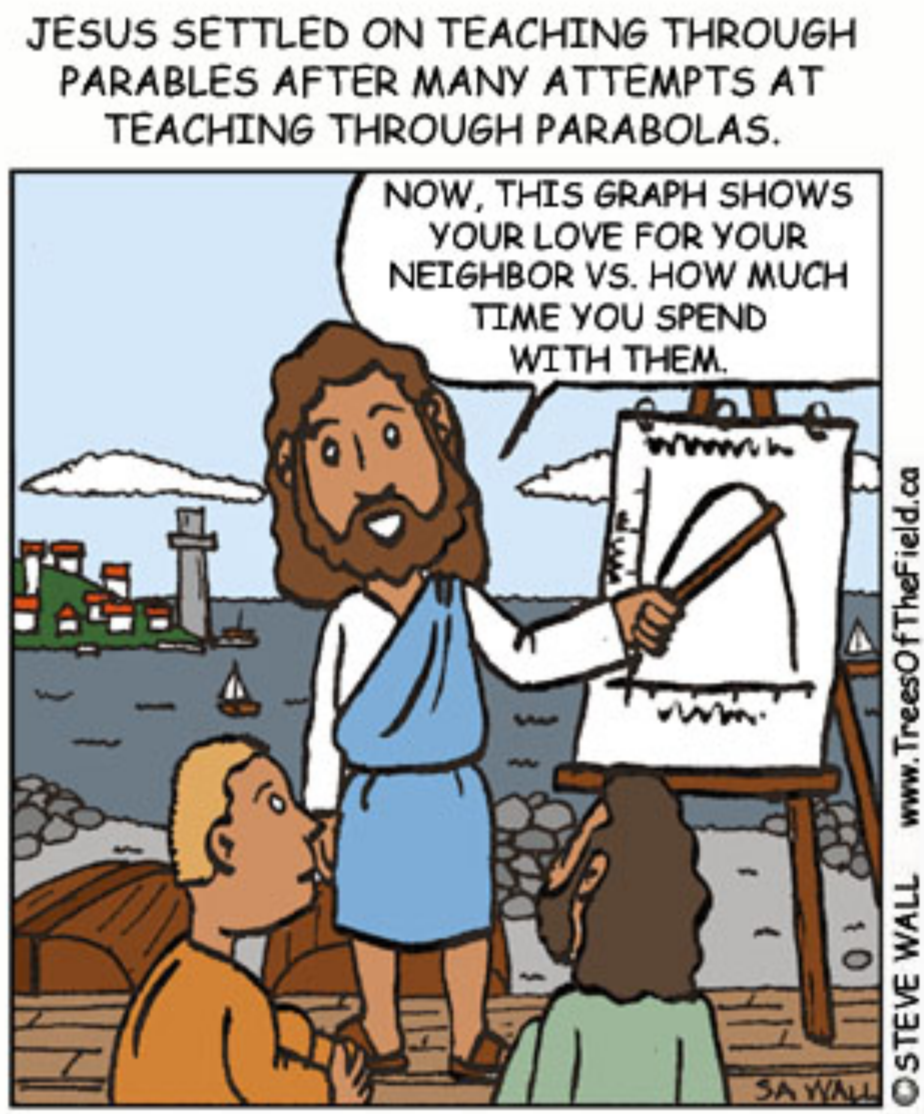


MCF3MI

Unit 2: Quadratic Functions – Forms and Transformations

UNIT 2A: QUADRATIC FUNCTIONS: TRANSFORMATIONS – ESSENTIAL LEARNINGS

You will have at least three opportunities to demonstrate the essential learnings in this course. The first opportunity will occur during the unit with formative quizzes and homework checks. Your second opportunity will be on the unit test. Your third opportunity will be on the exam. You need to demonstrate all essential learnings to earn this credit.

Please take the opportunity to keep track of the essential learnings you've demonstrated during this unit using the checklist below.

Essential Learnings: Connecting Graphs and Equations of Quadratic Functions & Solving Problems with Quadratic Functions	***Homework***	***Associated Lessons***
☐ Distinguish between quadratic and linear functions, with an emphasis on characteristics of quadratic functions	*pg. 28 – 30* #1 – 4	Lesson 2.1
☐ Describe *a*, *h*, and *k* in quadratics in vertex form in terms of their transformations of the parent function	*pg. 38 – 39* #1, 2, 4, 5, 6, 7	Lesson 2.2
	pg. 45 – 46 #1 – 3, 5, 8, 9, 11	Lesson 2.3
	worksheet	Lesson 2.4
☐ Sketch graphs in vertex form, including all proper labels	*pg. 51 – 52* #1 – 6, 8	Lesson 2.5
Unit Review:	*pg. 54 – 55* #7 – 11 *pg. 56 – 57* #2 – 5, 9, 10, 12, 13	

Analysing Quadratic Functions

A. Recall

Linear equations ($y = mx + b$) produce straight ***lines***.

Quadratic equations ($y = ax^2 + bx + c,\ where\ \ a,\ b,\ c \in R\ \ and\ \ a \neq 0$) produce ***parabolas***.

Both linear and quadratic equations represent **relations** that are also **functions**.

B. First and Second Differences – *A Review*

Ex. 1: Complete the table of values for this **linear** function.

x	$y = 3x + 1$	**First Differences**
−2	$y =$	
−1	$y =$	
0	$y =$	
1	$y =$	
2	$y =$	

What is true about the 1st differences?

Thus, for linear functions, the 1st differences are ____________________.

Ex. 2: Complete the table of values for this **quadratic** function.

x	$y = 3x^2 - 2x + 1$	**First Differences**	**Second Differences**
−2	$y =$		
−1	$y =$		
0	$y =$		
1	$y =$		
2	$y =$		

What is true about the 2nd differences?

Thus, for quadratic functions, the 2nd differences are __________________.

So far, we know how to distinguish between ***linear*** and ***quadratic*** functions by examining their 1st and 2nd differences. What about just looking at their equations? The word ***degree*** is used to describe the type of equation you have. The degree of a *polynomial* with a single variable is the value of the highest exponent in the equation. Here are some examples…

Equation	***Degree***	***Function Type***	***Constant Differences***	***Sketch***
$y=3x+1$	1	linear		
$y=3x^2-2x+1$	2	quadratic		
$y=-5x+4$				
$y=-3x^2+1$				
$y=x^3-2x^2+1$ *bonus!				
$y=x^4-5x^3-31$ *bonus!				

C. Terminology for Working with Quadratics

- The ***direction of opening*** for the quadratic equation can be determined by inspecting the ***a*** value:
  - if ***a > 0*** (+) the parabola opens up
  - if ***a < 0*** (–) the parabola opens down.
- The ***vertex*** of a parabola is the maximum or minimum point that the parabola reaches. If the parabola opens down, the vertex is a *maximum*; if it opens up, the vertex is a *minimum*.
  - If a graph or table of values has been created, the vertex can be found by inspection.
  - If the equation is given in vertex form… $y = a(x-h)^2 + k$, the vertex is always at the point (h, k) .
  - If the equation is given in standard form… $y = ax^2 + bx + c$, the *x*-value of the vertex is $h = \frac{-b}{2a}$.
- The ***axis of symmetry*** is the vertical mirror-line that divides the parabola in half through its vertex. The equation of the axis of symmetry is the x-value of the vertex (*i.e.* x = *h*)

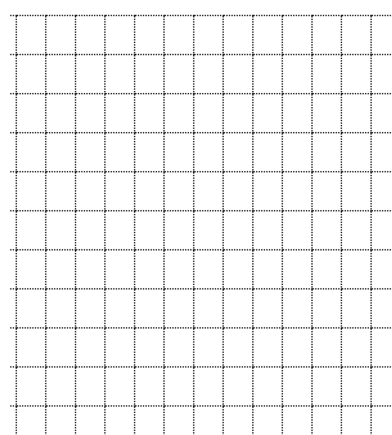
Ex. 3: Graph the table of values provided:

- calculate the 1st and 2nd differences
- what type of function is it? ____________
- state the direction of opening __________
- state the vertex ____________________
- is the vertex a maximum or a minimum?
- give the equation of the axis of symmetry

  ____
- find the range of this function

  R = ____________________________

Time (s)	Height (m)
0	0
1	35
2	60
3	75
4	80
5	75
6	60
7	35
8	0



D. Domain and Range – *Revisited*!

Linear functions	***Quadratic*** functions
The ***Domain*** is ALWAYS… $D = \{ x \in R \}$ The ***Range*** is ALWAYS … $R = \{ y \in R \}$ There are however, two exceptions… **vertical** and **horizontal** lines. Example 1: Vertical Line x = 5 *D* = ____________________ *R* = ____________________ Example 2: Horizontal Line y = – 2 *D* = ____________________ *R* = ____________________	The ***Domain*** is ALWAYS… $D = \{ x \in R \}$ The ***Range*** however … 1) If the equation is in **vertex form**, that is $y = a(x - h)^2 + k$, then use the vertex and the direction of opening to determine the ***Range***. 2) If the equation is in standard form, that is $y = ax^2 + bx + c$, then find the vertex (h, k) using $h = \frac{-b}{2a}$ and proceed as above.

Ex. 4: Use the rules from the table above to state the **domain** and **range** for each function below.

a) $y = 3x - 4$

D = ____________________

R = ____________________

b) $y = -2(x-4)^2 + 1$

D = ____________________

R = ____________________

c) $P = 0.5t^2 + 10t + 200$

D = ____________________

R = ____________________

If you're having troubles with first and second differences, read the example on page 27 of the text before beginning the homework.

HW: p. 28-30 #1-4

A

For help with questions 1 and 2, refer to Example 2.

1. Determine if each equation represents a linear or quadratic function.

a) $y = 2x + 1$

b) $f(x) = x^2 + 9$

c) $h(t) = -4.9t^2 + 19.6t + 2$

d) $3x + 4y = 12$

e) $y = 1$

f) $g(x) = 2(x - 3)^2$

g) $y = \frac{2}{3}x - 5$

h) $f(x) = -13 - 0.9x^2$

2. For each set of data, identify the relation as linear, quadratic, or neither. Calculate the first differences and second differences, if necessary.

a)

x	y
0	1
1	3
2	6
3	10
4	15
5	21

b)

Year	Population (millions)
1999	25.0
2000	26.5
2001	28.0
2002	29.5
2003	31.0
2004	32.5

c)

T	H
0	0
1	40
2	60
3	60
4	40
5	0

d)

Time (h)	Bacteria (billions)
0	1
1	2
2	4
3	8
4	16
5	32

Connect and Apply

B

3. The height h, in metres, of a ball t seconds after being thrown from a certain height is modelled by the function shown.
$h(t) = -4.9(t - 3)^2 + 60$

a) Create a table of values for $t = 0, 1, 2, \ldots, 6$.

b) Calculate the first and second differences.

c) Explain how your results in part b) show that the function is quadratic.

d) Give another reason why you know that the function is quadratic.

4. Refer to question 3.

a) Use the table of values to draw a graph for the function.

b) Identify the axis of symmetry, direction of opening, the coordinates of the vertex, and the domain and range.

1.3 Analyse Quadratic Functions, pages 23 – 30

1. a) linear **b)** quadratic
c) quadratic **d)** linear
e) linear **f)** quadratic
g) linear **h)** quadratic

2. a) quadratic **b)** linear
c) quadratic **d)** neither linear nor quadratic

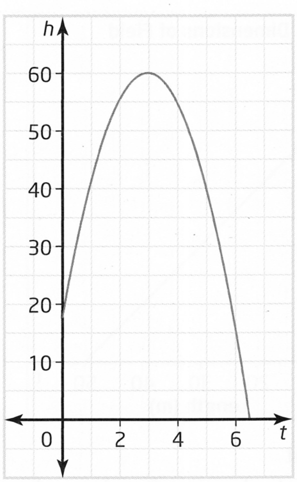
3. a), b)

t	*h*	First Differences	Second Differences
0	15.9		
		24.5	
1	40.4		9.8
		14.7	
2	55.1		9.8
		4.9	
3	60.0		9.8
		−4.9	
4	55.1		9.8
		−14.7	
5	40.4		9.8
		−24.5	
6	15.9		

c) The second differences are constant, so the function is quadratic.

d) Answers may vary. For example: The function is quadratic because the power of the independent variable, t, is 2.

4. a) Diagrams may vary. For example:



b) Axis of symmetry: $t = 3$
Direction of opening: downward
Vertex: (3, 60)
Domain: $\{t \in \mathbf{R} \mid 0 \leq t \leq 6.5\}$
Range: $\{h \in \mathbf{R} \mid 0 \leq h \leq 60\}$

Transformations I: Stretches and Reflections

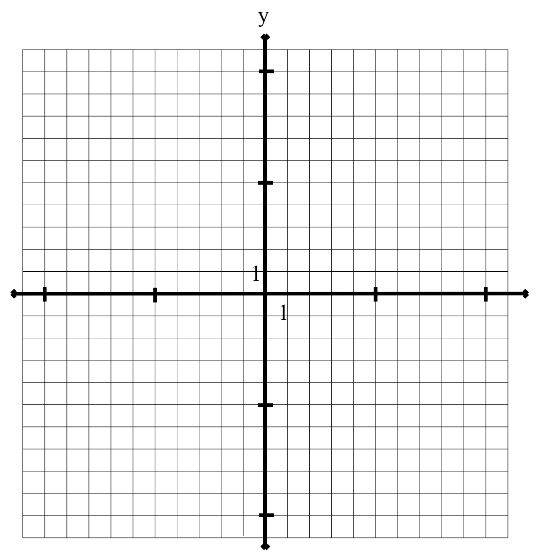
A. What is a transformation?

A **transformation** is a change made to a function or relation such that its graph is shifted or changed in shape. For a quadratic function, we use the base (or "parent") function $\boldsymbol{y = x^2}$ and transform its shape using the parameters ***a, h*** & ***k*** according to the vertex form $\boldsymbol{y = a(x - h)^2 + k}$.

B. Stretches and Reflections: Comparing the graphs of $y = x^2$ and $y = ax^2$

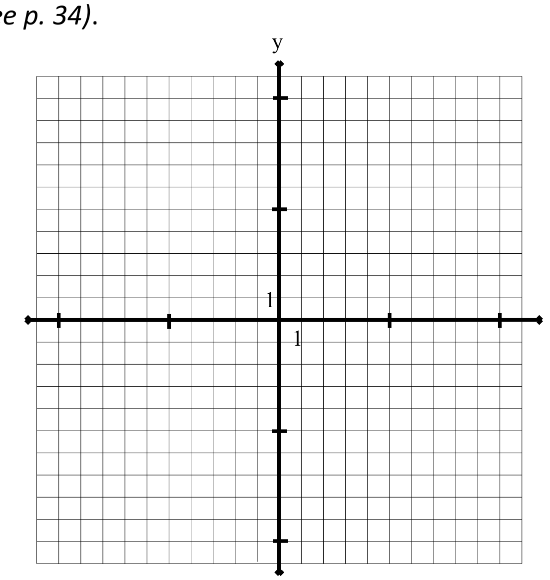
Turn to the bottom of page 33 in your text. Read pages 33-37 while copying the relevant information into this note. Don't forget to read the information in the margins – it's important too!

I. Graph and label $y = x^2$ and $y = 3x^2$ on the same set of axes *(see p. 33)*.



- Every point on $y = 3x^2$ is ______ times as far from the x-axis as the corresponding point on the graph of $y = x^2$.
- The parabola appears ____________ than the parabola for $y = x^2$ as the value of a increases from 1.
- The graph of $y = ax^2$, where $\boldsymbol{a > 1}$ is called a **<u>vertical expansion</u>** of the graph $y = x^2$ by a factor of _____.
- For each x-value, the y-value of $y = ax^2$ is multiplied by _____.
- Point (x, y) on $y = x^2$ is transformed to point (x, $2y$) on $y = 2x^2$.

II. Graph and label $y = x^2$, $y = \frac{1}{2}x^2$ and $y = -x^2$ on the same set of axes *(see p. 34)*.



- Every point on $y = \frac{1}{2}x^2$ is __________ as far from the x-axis as the corresponding point on the graph of $y = x^2$.
- The graph of $y = ax^2$, where $\boldsymbol{0 < a < 1}$ is called a **<u>vertical compression</u>** of the graph $y = x^2$ by a factor of _____.
- The graph for $y = -x^2$ *is a* ______________ *image* of the graph of $y = x^2$. The parabola opens _____________________ instead of upward.
- The graph of $y = ax^2$, where $\boldsymbol{a < 0}$, involves a ________________ of the graph $y = x^2$ in the ____-axis.

∴ The point (x, y) on $y = x^2$ is transformed to the point (x, ay) on $y = ax^2$.

*If **a > 1**, the graph is **vertically expanded** by a factor of **a**.*
*If **0 < a < 1**, the graph is **vertically compressed** by a factor of **a**.*
*If **a < 0**, the graph is **reflected** across the x-axis.*

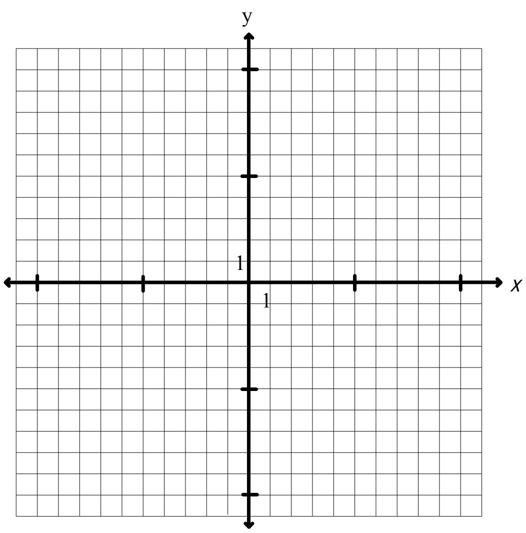
Example 1 **Graph a Vertical Stretch** *(see p. 34-35)*

For each function, describe in words the transformation relative to $y = x^2$ and then sketch the graph.

a) $y = 2x^2$

Transformations:

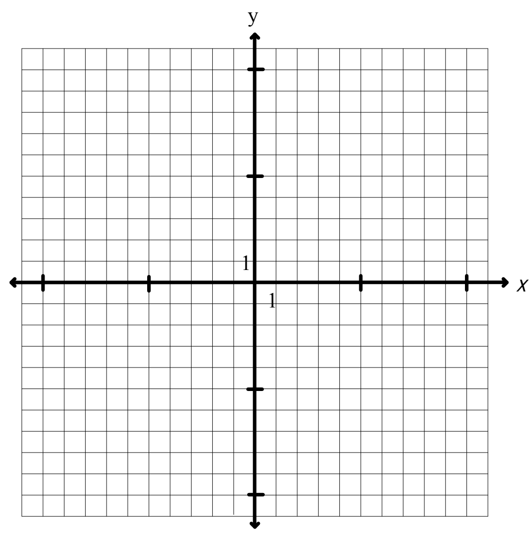
Graph:



b) $y = -3x^2$

Transformations:

Graph:



Example 2 **Find the Stretch Factor** *(see p. 36)*

The point (3, 12) is on the graph of the function $y = ax^2$. Use **algebra** to find the value of a, the stretch factor.

HW: p. 37 "Communicate Your Understanding" C1 and C2; p. 38-39 #1, 2, 4, 5, 6, 7

Communicate Your Understanding

C1 A parabola has its vertex at the origin and passes through the point (4, 4). Explain how you know that this parabola represents a vertical compression of the graph of $y = x^2$ by a factor of 0.25.

C2 You are asked to graph the function $y = -0.5x^2$. Three methods are presented. Do all of them work? If so, which one will you use? Explain.

a) Compress the graph of $y = x^2$ vertically by a factor of 0.5. Then, reflect the resulting graph in the x-axis.

b) Reflect the graph of $y = x^2$ in the x-axis. Then, compress the resulting graph vertically by a factor of 0.5.

c) Use points on the graph of $y = x^2$ that are at the corners of grid squares, such as (1, 1) and (2, 4). Multiply each y-coordinate by -0.5. Plot these points and join them with a smooth curve.

A

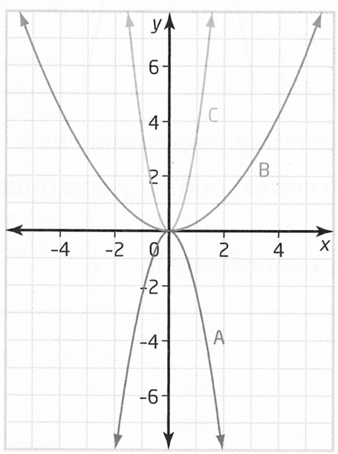
For help with questions 1 to 3, refer to Example 1.

1. Match each equation to the corresponding graph.

a) $y = 0.25x^2$

b) $y = 3x^2$

c) $y = -2x^2$



2. Graph these three functions on the same set of axes. Label the graphs.

a) $f(x) = x^2$

b) $g(x) = 4x^2$

c) $h(x) = -0.5x^2$

4. Write an equation for the graph that results from each transformation.

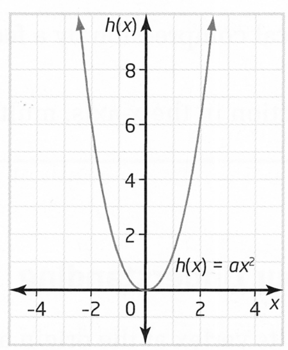
a) The graph of $f(x) = x^2$ is stretched vertically by a factor of 10.

b) The graph of $r(x) = x^2$ is compressed vertically by a factor of 0.25.

c) The graph of $t(x) = x^2$ is stretched vertically by a factor of 5 and reflected in the x-axis.

5. The point (5, 10) is on the graph of the function $g(x) = ax^2$. Find the value of a.

6. The graph of the function $h(x) = ax^2$ is shown. What is the value of a?



Connect and Apply

B

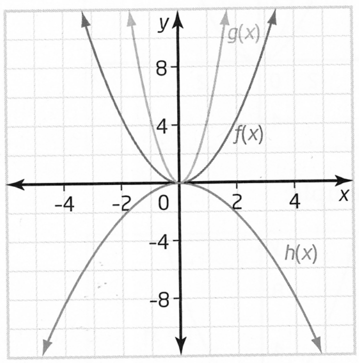
7. a) How do you use the points on the graph of $y = x^2$ to find the corresponding points on the graph of $y = 4x^2$?

b) Draw the graphs of $y = x^2$ and $y = 4x^2$ on the same set of axes.

1.4 Stretches of Functions, pages 31 - 39

1. a) B **b)** C **c)** A

2. a) to **c)** Diagrams may vary. For example:



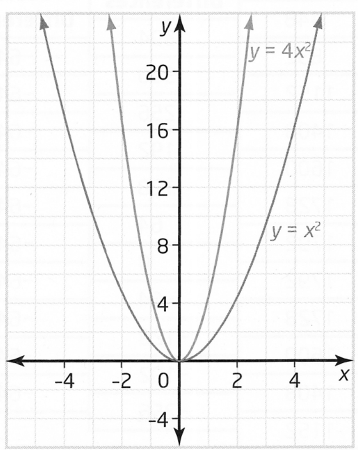
4. a) $y = 10x^2$ **b)** $y = 0.25x^2$ **c)** $y = -5x^2$

5. 0.4

6. 1.5

7. a) Answers may vary. For example: Multiply all the y-coordinates of the points on the graph of $y = x^2$ by 4 to get the corresponding points on the graph of $y = x^2$.

b) Diagrams may vary. For example:



Transformations II: Translations of Functions

Recall – Stretches and Reflections: Comparing the graphs of $f(x) = x^2$ and $f(x) = ax^2$

A **stretch** is a change made to a function or relation such that its graph is **expanded** or **compressed** relative to its distance from the *x*-axis. A **reflection** in the *x*-axis occurs when the *a*-value is negative.

The point (x, y) on $y = x^2$ is transformed to the point (x, *ay*) on $y = ax^2$:

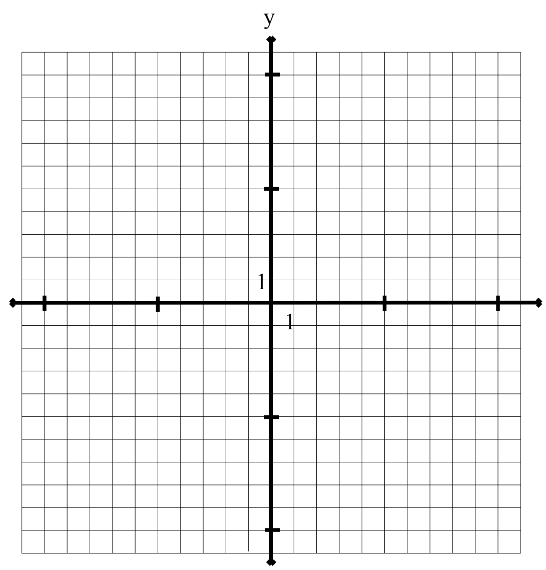
*If **$a > 1$**, the graph is **vertically expanded** by a factor of **a**.*
*If **$0 < a < 1$**, the graph is **vertically compressed** by a factor of **a**.*
*If **$a < 0$**, the graph is **reflected** across the x-axis.*

A. Translations of Functions

A **translation** is a *slide* or *shift* that moves the graph of a base function left or right (horizontal translation) and up or down (vertical translation). The **shape** of a graph is **not changed** by a translation (*i.e.* it does not become wider or narrower as a result of a translation).

B. Horizontal Translations: Comparing the graphs of $f(x) = x^2$ and $f(x) = (x - h)^2$

Ex. 1: Graph $y = x^2$, $y = (x - 5)^2$ and $y = (x + 7)^2$ on the same set of axes.



i) $y = x^2$

Vertex : _________

x	*y*
0	0

D = _________________

R = __________________

ii) $y = (x - 5)^2$

Vertex : __________

x	*y*

D = _________________

R = _________________

iii) $y = (x + 7)^2$

Vertex : __________

x	*y*

D = _________________

R = _________________

Summary of Horizontal Translations with $f(x) = x^2$ and $f(x) = (x - h)^2$:

Compared to the graph of $f(x) = x^2$:

- The graph of $f(x) = (x - h)^2$ has its vertex at ____________ , so it is a ___________________________ translation of ______ units to the ________________.
- The graph of $f(x) = (x + h)^2$ has its vertex at _____________, so it is a __________________________ translation of ______ units to the ________________.
- Thus, when $h < 0$, the graph moves _______________. When $h > 0$, the graph moves ____________________.

C. Vertical Translations: Comparing the graphs of $f(x) = x^2$ and $f(x) = x^2 + k$

Ex. 1: Graph $y = x^2$, $y = x^2 + 5$ and $y = x^2 - 10$ on the same set of axes.

i) $y = x^2$

Vertex : ________

x	y

R = __________________

ii) $y = x^2 + 5$

Vertex : _________

x	y

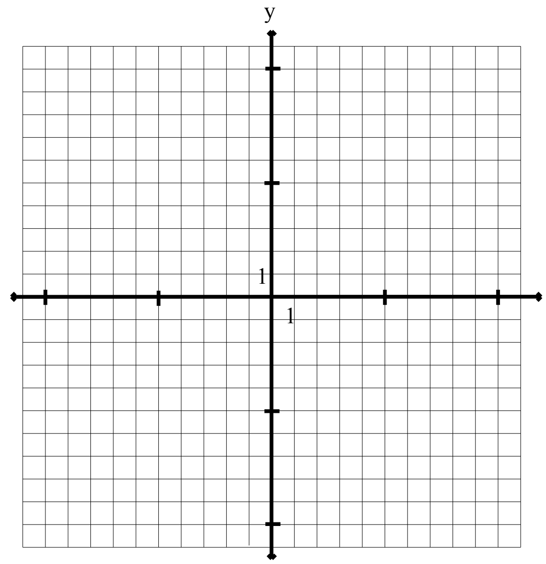
R = _________________

iii) $y = x^2 - 10$

Vertex : _________

x	y

R = _________________



Summary of Vertical Translations with $f(x) = x^2$ and $f(x) = x^2 + k$:

Compared to the graph of $f(x) = x^2$:

- The graph of $f(x) = x^2 + k$ is a ______________________ translation of ______ units ________.
- The graph of $f(x) = x^2 - k$ is a ______________________ translation of ______ units _____________.
- Thus, when $k > 0$, the graph moves _____________. When $k < 0$, the graph moves ____________________.

D. Practice Problems

1. Describe how the graphs of the following functions can be obtained from the graph of the function $y = f(x)$.

   a. $y = x^2 + 8$ ____

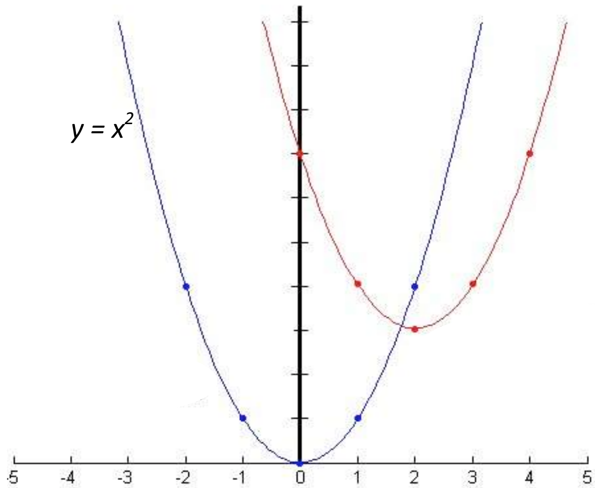
   b. $y = (x - 4)^2$ ____

   c. $y = (x - 2)^2 - 5$ ____

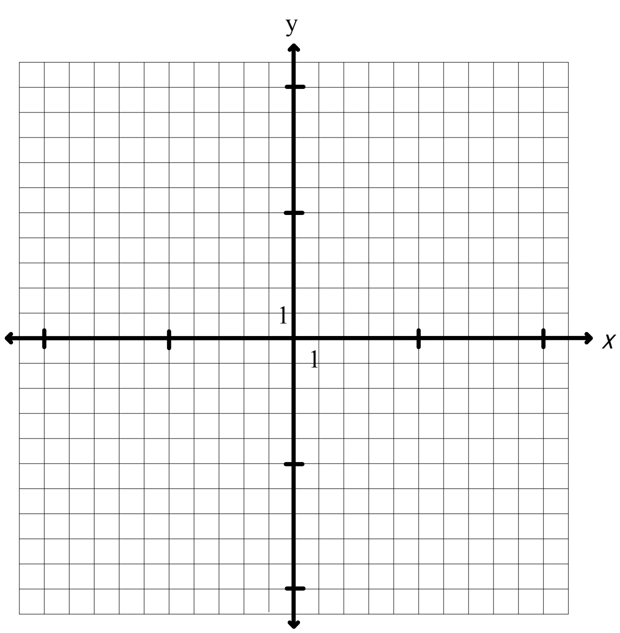
   d. $y = (x + 1)^2 - 12$ ____

2. The graph of $y = x^2$ is translated 3 units to the left and 5 units upward. Write the equation of the transformed function and state its domain and range.

3. The unlabelled parabola is a translation of the function $y = x^2$. Write the equation of the translated graph and state its domain and range.



4. Two points on the graph of the function $f(x) = (x - h)^2 + k$ are (7, 4) and (1, 4). Plot these points on a coordinate grid. Describe how you might find the coordinates of the vertex.

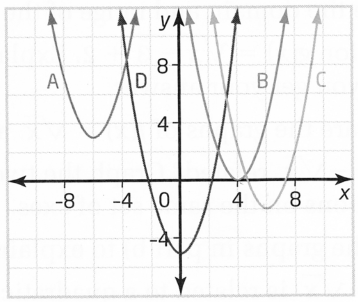


HW: p. 45-46 #1-3, 5, 8, *9, *11

A

For help with questions 1 and 3, refer to the Example.

1. Match each equation to the corresponding graph.
   a) $f(x) = (x - 4)^2$
   b) $g(x) = (x - 6)^2 - 2$
   c) $h(x) = (x + 6)^2 + 3$
   d) $k(x) = x^2 - 5$



2. Write an equation for the graph resulting from each transformation.
   a) The graph of $f(x) = x^2$ is translated 3 units to the right.
   b) The graph of $g(x) = x^2$ is translated 3 units up.
   c) The graph of $p(x) = x^2$ is translated 4 units to the left.
   d) The graph of $q(x) = x^2$ is translated 4 units down.

3. Write the coordinates of the vertex in each graph. Then, sketch the graph.
   a) $f(x) = x^2 - 4$
   b) $g(x) = (x + 3)^2$
   c) $h(x) = (x - 1)^2 + 6$
   d) $k(x) = (x + 5)^2 - 8$

5. The x-intercepts of a parabola that opens upward are -6 and -2.
   a) What important feature of the parabola do you know from the x-intercepts?
   b) Find the equation of the parabola in the form $p(x) = (x - h)^2 + k$.

8. The graph of $f(x) = x^2$ is translated 2 units to the left and 4 units down.
   a) Write an equation for the graph resulting from the transformation.
   b) Sketch the graph.
   c) What are the coordinates of the vertex?
   d) What are the x-intercepts?

9. Relative to the graph of $f(x) = x^2$, the graph of $g(x) = x^2 + 2$ represents a translation of 2 units up, which is toward the positive direction. In Section 1.4, the graph of $h(x) = 2x^2$ represents a vertical stretch by a factor of 2, which is also toward the positive direction. However, the graph of $f(x) = (x + 2)^2$ represents a translation of 2 units to the left, which is toward the negative direction. How can you explain this apparent inconsistency?

11. The domain of the function $f(x) = \sqrt{x}$ is $\{x \in \mathbf{R} \mid x \geq 0\}$ and the range is $\{y \in \mathbf{R} \mid y \geq 0\}$.
    a) Write the domain and range of the function $g(x) = \sqrt{x - 3} + 2$. Explain how you get your answer.
    b) How are the graphs of $f(x) = \sqrt{x}$ and $f(x) = -\sqrt{x}$ related? Graph the two equations on the same set of axes.
    c) Use the graphs in part b) to explain how $f(x) = \sqrt{x}$ is related to a quadratic relation.

1.5 Translations of Functions, pages 40 - 46

1. a) B **b)** C **c)** A **d)** D

2. a) $f(x) = (x - 3)^2$ **b)** $g(x) = x^2 + 3$

c) $p(x) = (x + 4)^2$ **d)** $q(x) = x^2 - 4$

3. Diagrams may vary.

a) (0, −4) **b)** (−3, 0) **c)** (1, 6) **d)** (−5, −8)

5. a) The axis of symmetry of the parabola is between $x = -6$ and $x = -2$, which is $x = -4$. The x-coordinate of the vertex of the parabola is −4.

b) $p(x) = (x + 4)^2 - 4$

8. a) $f(x) = (x + 2)^2 - 4$

b) Sketches may vary.

c) (−2, −4)

d) −4 and 0

9. Answers may vary. For example:
Rearrange the equations to see the numbers corresponding to the transformation in each case.
$g(x) = x^2 + 2$: $y - 2 = x^2$; The graph is shifted up by 2 units.
$h(x) = 2x^2$; $\frac{y}{2} = x^2$; The graph is 2 times as tall.
$f(x) = (x + 2)^2$: $y = (x + 2)^2$; The graph is shifted 2 units to the left.
In each case, the transformation is represented by the opposite of these operations: -2, $\frac{1}{2}$, and $+2$.

10. Answers may vary. For example:
$x^2 + y^2 = 16$ and $(x - 2)^2 + y^2 = 16$

11. a) Domain: $\{x \in \mathbf{R} \mid x \geq 3\}$;
Range: $\{y \in \mathbf{R} \mid y \geq 2\}$; Explanations may vary.

Date: _______________

Analysing the roles of *a*, *h* & *k* in $y = a(x-h)^2 + k$

In a class discussion complete the following graphic organizer to summarize the roles of *a*, *h* and *k* in the vertex form of a quadratic function.

$$y = a(x-h)^2 + k$$

Role of *a*:

Direction of Opening:

- When ***a*** is positive, the parabola opens ____________________.
- When ***a*** is negative, the parabola opens____________________.

Shape:

- If ***a* > 1 or *a* < -1**, then the graph of $y = a(x-h)^2 + k$ has an opening ________________________ than $y = 1(x-h)^2 + k$.
- If ***a*** is between **-1** and **1**, then the graph of $y = a(x-h)^2 + k$ has an opening ________________________ than $y = 1(x-h)^2 + k$.

Role of *h*:

Properties:

- If $h > 0$, then the graph of $y = a(x-h)^2 + k$ is translated horizontally *h* units to the ________________.
- If $h < 0$, then the graph of $y = a(x-h)^2 + k$ is translated horizontally *h* units to the ________________ .

Relation to the Vertex:

- The value of h is the _____ - coordinate of the vertex.

Role of *k*:

Properties:

- If $k > 0$, then the graph of $y = a(x-h)^2 + k$ is translated vertically *k* units _____________.
- If $k < 0$, then the graph of $y = a(x-h)^2 + k$ is translated vertically *k* units ____________.

Relation to the Vertex:

The value of *k* is the _____ - coordinate of the vertex.

x-Intercepts:

- If $k = 0$, then the graph has ____________ x-intercept.
- If $k > 0$, then the graph has ____________ x-intercepts.
- If $k < 0$, then the graph has ____________ x-intercepts.

Example: $y = -2(x-3)^2 + 5$

State:

a) Direction of opening: ___________________

b) Expansion or Compression: ______________________

c) Transformations:

d) Coordinates of the vertex: _______________

e) Number of x-intercepts: _______________

f) Domain and Range:

Demonstrating understanding of the roles of *a*, *h* & *k* in $y = a(x-h)^2 + k$

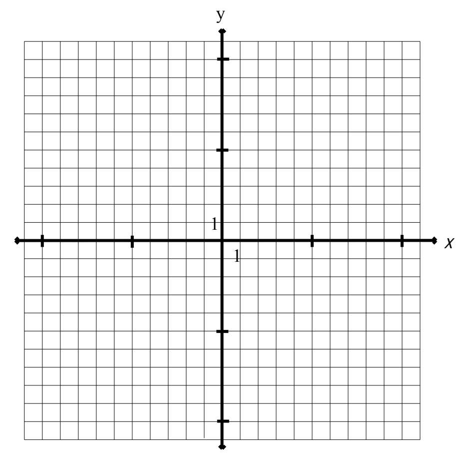
Complete the following table:

Equation	Value of *a*	Value of *h*	Value of *k*	Vertex (*h*, *k*)	# of x-intercepts	Transformations starting from $y = x^2$	Domain & Range
$y = 3(x-2)^2 + 1$	$a = 3$	$h = 2$	$k = 1$	(2, 1)	None	• Vertical expansion by a factor of 3 • No reflection across the *x*-axis • Horizontal translation 2 units right • Vertical translation 1 unit up	$D = \{x \mid x \in R\}$ $R = \{y \mid y \in R, y \geq 1\}$
$y = -2(x-3)^2 + 3$							
$y = \frac{1}{2}(x+1)^2 + 5$							
$y = 0.3(x+2)^2 + 15$							
$y = -\frac{2}{3}(x-4)^2 - 8$							
$y = 2x^2 + 9$							
$y = -3(x+5)^2$							

Sketching Graphs Using Transformations

A. What Are the Roles of *a, h,* and *k* in the Quadratic Function $f(x) = a(x-h)^2 + k$?

$$f(x) = a(x-h)^2 + k$$



B. Order of Operations

When describing transformations on a base function, ORDER MATTERS!

Graph $y = -x^2 + 3$ in two different orders:

a) Reflect $y = x^2$ in the x-axis ***first*** before translating it 3 units up.

b) Translate $y = x^2$ up 3 units ***first*** before reflecting it in the x-axis.

What do you notice? ____

∴ *You must describe combinations of transformations in the following order:*

1. Reflections and Stretches (since both involve *multiplication* of the *y*-value, they must be done first)
2. Translations (just like in BEDMAS, *addition* and *subtraction* on the *x*- and *y*-values must be done last)

****You can't go wrong if you describe the transformations as they occur from left to right across the equation!***

Reflections on the Function $y = f(x)$:

Reflection	**Mathematical Form**	**Effect**
Vertical	$y = -f(x)$	Compared to y = *f*(x), the graph of y = – *f*(x) is a vertical reflection across the x-axis. The point (x, y) on *y* = *f*(x) becomes the point (x, – y) on *y* = – *f*(x).

Stretches on the Function $y = f(x)$:

Stretch	**Mathematical Form**	**Effect**
Vertical	$y = a\,f(x)$	If ***a* > 1**, the graph is **vertically expanded** by a factor of ***a***. If **0 < *a* < 1**, the graph is **vertically compressed** by a factor of ***a***. The point (x, y) on *y* = *f*(x) becomes the point (x, *a*y) on *y* = *af*(x).

Translations on the Function $y = f(x)$:

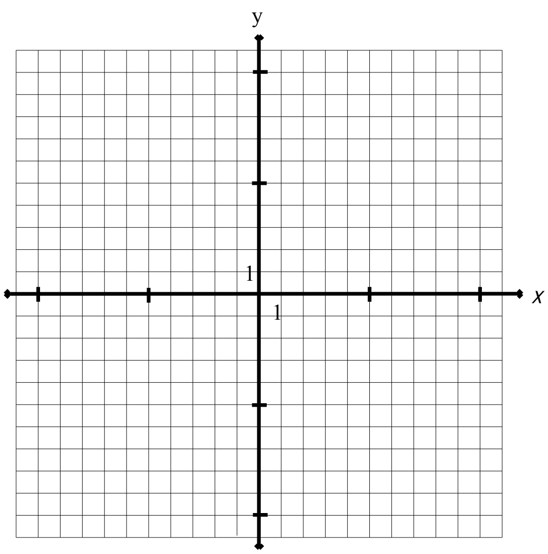
Translation	**Mathematical Form**	**Effect**
Horizontal	$y = f(x - h)$	Compared to the graph of *y* = *f*(x), the graph of *y* = *f*(*x* – h) is a horizontal translation of h units. When **h > 0** the graph is **horizontally translated to the RIGHT h units**. When **h < 0** the graph is **horizontally translated to the LEFT h units**. The point (x, y) on *y* = *f*(x) becomes the point (x + h, *y*) on *y* = *f*(*x* – h).
Vertical	$y = f(x) + k$	Compared to the graph of *y* = *f*(x), the graph of *y* = *f*(x) + k is a vertical translation of k units. When **k > 0** the graph is **vertically translated up k units**. When **k < 0** the graph is **vertically translated down k units**. The point (x, y) on *y* = *f*(x) becomes the point (x, *y* + k) on *y* = *f*(x) + k.

C. Using the Vertex Form to Graph Transformations

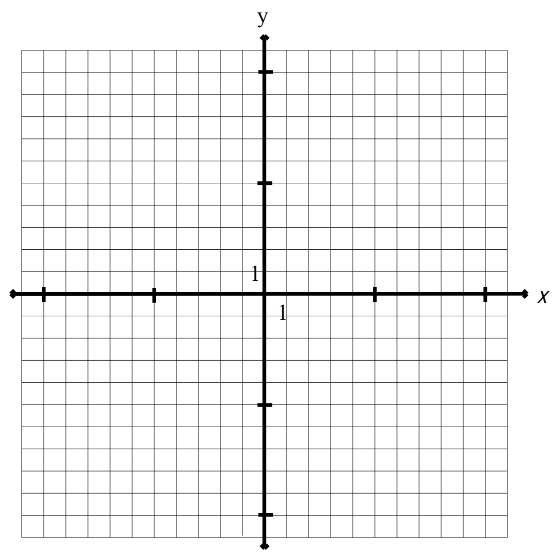
The vertex method is by far the fastest for graphing transformations. You use the vertex (*h, k*), and plot points on either side of it: **go over "*x*" units** and **up/down "$x^2(a)$" units** (go *up* if *a* is positive and *down* if *a* is negative). Plot your first pair of points 1 over and 1^2(a) up/down; then, plot your next pair of points 2 over and 2^2(a) up/down. Join them with a smooth parabolic curve and you're done!

a) Graph the following quadratic functions using the vertex and "$x^2(a)$".
b) Label the vertex and two other points.
c) Write the equation of the axis of symmetry.
d) Describe in words the transformations relative to the graph of $f(x) = x^2$.
e) Write the domain and range of the function.

Ex. 1: $f(x) = 2(x-3)^2 - 2$



Ex. 2: $y = -(x+1)^2 + 5$



HW: p. 51-52 #1-6, 8 (*hint*: for #8a the ball is thrown at $t = 0$)

1. Copy and complete the table.

Equation of Parabola	Coordinates of Vertex	Equation of Axis of Symmetry	Direction of Opening	Range of Function
$f(x) = -5x^2 + 20$				
$f(x) = 9(x - 5)^2$				
$f(x) = 9(x - 5)^2 - 18$				
$f(x) = -2(x + 1)^2 + 32$				
$f(x) = 0.5(x + 1)^2 - 3$				

For help with question 2, refer to the Example.

2. **Use Technology** Describe the graph of each function in terms of transformations on the graph of $y = x^2$. Then, sketch the graph. Clearly label the vertex, the axis of symmetry, and one other point. Check your result with a graphing calculator.
   a) $y = 2(x + 3)^2$
   b) $f(x) = -x^2 + 5$
   c) $g(x) = 4(x + 2)^2 - 8$
   d) $h(x) = -3(x - 1)^2 - 1$

3. The graph of $f(x) = x^2$ has been stretched vertically by a factor of 10 and translated 5 units to the right and 8 units down.
   a) Write the equation of the graph resulting from the transformations.
   b) Sketch the graph of $f(x) = x^2$ and its image after the transformations.

4. Write an equation of a parabola that satisfies each set of conditions.
   a) vertex $(4, -6)$
   congruent in shape to the graph of $y = 3x^2$
   range: $\{y \in \mathbf{R} \mid y \leq -6\}$
   b) vertex $(-2, 0)$
   y-intercept: 4
   c) opens downward
   congruent in shape to the graph of $y = 2x^2$
   x-intercepts: 5 and 9

Connect and Apply

B

5. a) Describe how the graphs of the three functions are related.
   i) $f(x) = (x + 2)^2 - 4$
   ii) $g(x) = 2(x + 2)^2 - 4$
   iii) $h(x) = -(x + 2)^2 - 4$
   b) Sketch the three graphs on the same set of axes to verify your answer in part a).

6. a) Write the coordinates of two points other than the vertex on the graph of $f(x) = 2x^2$.
   b) Explain how these points can help you draw the graph of $g(x) = 2(x - 10)^2 - 32$.
   c) Graph the function $g(x) = 2(x - 10)^2 - 32$. Label the vertex and the axis of symmetry and write the domain and range.

8. The height, in metres, of a ball t seconds after being thrown is modelled by the function $h(t) = -4.9(t - 2)^2 + 45$.
   a) From what height is the ball thrown?
   b) What is the maximum height of the ball and when does this occur?
   c) Write the range of this function.
   d) **Use Technology** Use a graphing calculator to graph the function. Determine how long it takes the ball to land.
   e) Write the domain of the function.

1.

Equation of Parabola	Coordinates of Vertex	Equation of Axis of Symmetry	Direction of Opening	Range of Function
$f(x) = -5x^2 + 20$	(0, 20)	x = 0	downward	$\{y \in R \mid y \leq 20\}$
$f(x) = 9(x - 5)^2$	(5, 0)	x = 5	upward	$\{y \in R \mid y \geq 0\}$
$f(x) = 9(x - 5)^2 - 18$	(5, -18)	x = 5	upward	$\{y \in R \mid y \geq -18\}$
$f(x) = -2(x + 1)^2 + 32$	(-1, 32)	x = -1	downward	$\{y \in R \mid y \leq 32\}$
$f(x) = 0.5(x + 1)^2 - 3$	(-1, -3)	x = -1	upward	$\{y \in R \mid y \geq -3\}$

2. Sketches may vary.

a) Answers may vary. For example: The graph of $y = 2(x + 3)^2$ is a vertical stretch of the graph of $y = x^2$ by a factor of 2 followed by a translation of 3 units to the left.

b) Answers may vary. For example: The graph of $f(x) = -x^2 + 5$ is a reflection of the graph of $y = x^2$ in the x-axis followed by a translation of 5 units up.

c) Answers may vary. For example: The graph of $g(x) = 4(x + 2)^2 - 8$ is a vertical stretch of the graph of $y = x^2$ by a factor of 4 followed by a translation of 2 units to the left and 8 units down.

d) Answers may vary. For example: The graph of $h(x) = -3(x - 1)^2 - 1$ is a vertical stretch of the graph of $y = x^2$ by a factor of 3 followed by a reflection in the x-axis and a translation of 1 unit to the right and 1 unit down.

3. a) $g(x) = 10(x - 5)^2 - 8$

b) Sketches may vary.

4. a) $y = -3(x - 4)^2 - 6$

b) $y = (x + 2)^2$

c) $y = 2(x - 7)^2 - 8$

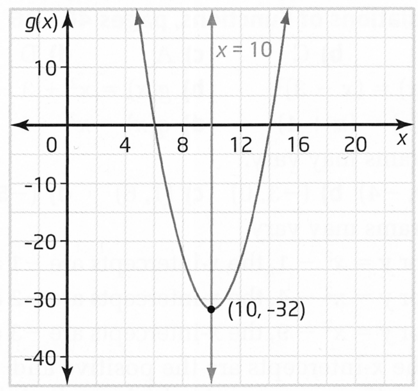
5. a) Answers may vary. For example: All three graphs are parabolas with the vertex at $(-2, -4)$; Graph ii) is a vertical stretch of graph i) by a factor of 2; Graph iii) is a reflection of graph i) in the line $y = -4$.

b) Sketches may vary.

6. a) Answers may vary. For example: The vertex is at (0, 0). Two other points are (1, 2) and (2, 8).

b) Answers may vary. For example: Relative to the graph of $f(x) = 2x^2$, the graph of $f(x) = 2(x - 10)^2 - 32$ is a translation of 10 units to the right and 32 units down. The shape of the parabola is congruent in shape to the parabola for $f(x) = 2x^2$. Add 10 to the x-coordinates and subtract 32 from the y-coordinates of points on the graph of $f(x) = 2x^2$ to get the corresponding points on the graph of $f(x) = 2(x - 10)^2 - 32$.

c) Diagrams may vary. For example:



Domain: $\{x \mid x \in \mathbf{R}\}$;
Range: $\{y \in \mathbf{R} \mid y \geq -32\}$

8. a) 25.4 m

b) 45 m, 2 s after the ball is thrown

c) $\{h \in \mathbf{R} \mid 0 \leq h \leq 45\}$

d) 5 s

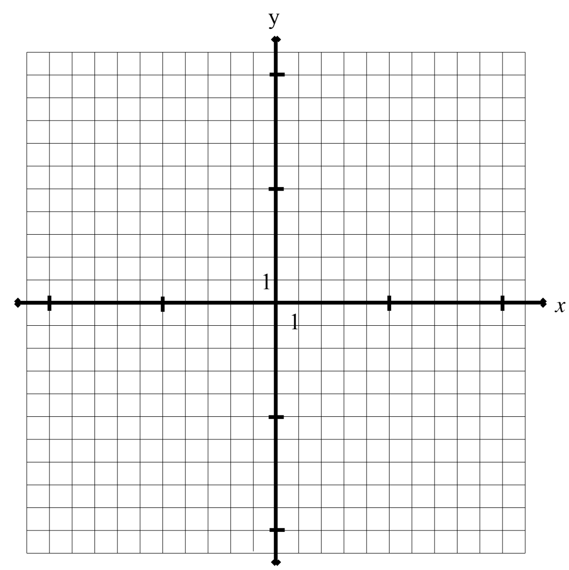
e) $\{t \in \mathbf{R} \mid 0 \leq t\}$

Review for Unit 2 Part A Test

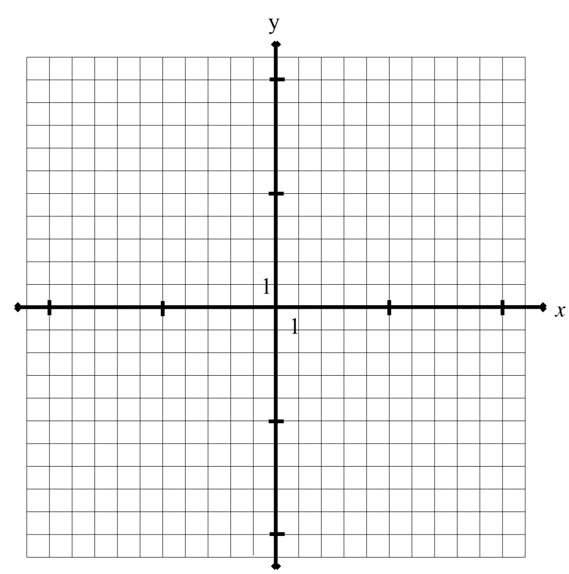
Representative Questions for Review:

1) Explain how you can identify whether a function is linear or quadratic from its table of values.

2) Given the quadratic function $y = 3(x - 1)^2 + 5$:
   a) What is its vertex?
   b) What is its range?
   c) Describe the transformations to the parent function in order
   c) Graph it, with all appropriate labels



3) Given the quadratic function $y = -2x^2 - 3$:
   a) What is its vertex?
   b) What is its range?
   c) Describe the transformations to the parent function in order
   d) Graph it, with all appropriate labels



Textbook review questions:

p. 54-55 #7, 8, 9, 10, 11

p. 56-57 #2, 3, 4, 5, 9, 10, 12, 13

7. Does each relation represent a quadratic function? If not, explain why.

a) $h(t) = -2(t - 3)^2 + 10$

b) $2x + y = 11$

c) $x^2 = 1$

8. For each set of data, identify the relation as linear or quadratic. Calculate the first differences, and second differences, if necessary.

a)

x	y
-2	9
-1	6
0	5
1	6
2	9
3	14

b)

Hours	Wages
1	15
2	19
3	23
4	27
5	31
6	35

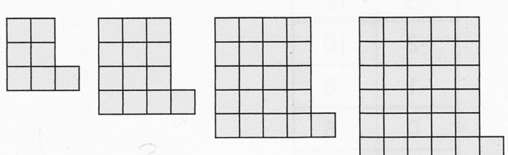
c)

x	y
-3	0
-2	2
-1	4
0	6
1	8
2	10
3	12

d)

Time (s)	Height (m)
0	125
1	120
2	105
3	80
4	45
5	0

9. The diagram shows the first four terms of a pattern of cubes. Create a table of values and determine if the relation between Term Number and Number of Cubes is a quadratic function.



10. For each function, describe the graph in terms of transformations on the graph of $y = x^2$. Then, sketch the graph. Label the vertex, axis of symmetry, and two other points.

a) $f(x) = -x^2 + 9$

b) $g(x) = 2(x - 5)^2$

c) $y = -4(x + 2)^2 + 8$

11. Write an equation for the parabola that satisfies each set of conditions.

a) vertex at $(10, -5)$
congruent in shape to the graph of $y = 3x^2$
with no x-intercepts

b) vertex at $(-3, 0)$
congruent in shape to the graph of $y = 0.5x^2$
range: $\{y \in \mathbf{R} \mid y \leq 0\}$

2. Which set of numbers best represents the range of the parabola given by $f(x) = -5(x - 9)^2 + 12$?

A any real number greater than or equal to 12

B $\{y \in \mathbf{R} \mid y \leq 12\}$

C $\{11, 10, 9, 8, \ldots\}$

D $\{y \in \mathbf{R} \mid y \leq 9\}$

3. Which statement is not true for the parabola given by $h(t) = 3(t - 10)^2 - 50$?

A Its vertex is located at $(10, -50)$.

B It opens upward.

C It passes through the point $(5, 25)$.

D The domain is any number greater than or equal to 10.

4. The point $(-2, -6)$ is on the graph of the function $y = a(x + 1)^2 - 3$. What is the value of a?

A 1

B 3

C -1

D -3

5. The graph of $y = x^2$ is compressed vertically by a factor of 0.5 and translated 2 units down. Which equation represents the resulting graph?

A $y = 0.5x^2 + 2$

B $y = \frac{1}{2}x^2 - 2$

C $y = 2(x + 2)^2$

D $y = 2x^2 - 2$

9. Write an equation for the graph resulting from each transformation.

a) The graph of $f(x) = x^2$ is translated 2 units left.

b) The graph of $h(t) = t^2$ is translated 5 units up.

c) The graph of $A(r) = \pi r^2$ is translated 4 units right.

d) The graph of $f(x) = 2x^2$ is translated 3 units down.

10. Write the coordinates of the vertex in each graph.

a) $f(x) = x^2 + 2$

b) $g(x) = (x - 3)^2 + 1$

c) $h(x) = -(x + 1)^2 - 1$

d) $t(x) = 3(x + 1)^2 - 1$

12. Describe the graph of each function in terms of transformations on the graph of $y = x^2$.

a) $y = x^2 + 7$

b) $y = (x - 1)^2 + 2$

c) $y = 5(x + 1)^2 - 4$

d) $y = -(x - 3)^2 + 5$

13. A parabola is modelled by the function $g(x) = -(x - 4)^2 + 9$.

a) Sketch the parabola. Label the vertex, axis of symmetry, and two other points.

b) Write the domain and range of the function.

7. a) Yes

b) No; The graph of the relation is a straight line.

c) No; The graph of the relation are the two vertical lines represented by $x = 1$ and $x = -1$. Each x value can be mapped onto numerous y-values.

8. a) quadratic **b)** linear

c) linear **d)** quadratic

9.

Term Number	Number of Cubes	First Differences	Second Differences
1	7	6	
2	13	8	2
3	21	10	2
4	31	12	2
5	43	14	2
6	57		

The relation between the term number and the number of cubes is a quadratic function.

10. Sketches may vary.

a) The graph is a result of these transformations on the graph of $y = x^2$: a reflection in the x-axis and a translation of 9 units up.

b) The graph is a result of these transformations on the graph of $y = x^2$: a vertical stretch by a factor of 2 and a translation of 5 units to the right.

c) The graph is a result of these transformations on the graph of $y = x^2$: a vertical stretch by a factor of 4, a reflection in the x-axis, and a translation of 2 units to the left and 8 units up.

11. a) $y = -3(x - 10)^2 - 5$

b) $y = -0.5(x + 3)^2$

Chapter 1 Practice Test, pages 56–57

1. C

2. B

3. D

4. D

5. B

9. a) $f(x) = (x + 2)^2$ **b)** $h(t) = t^2 + 5$

c) $A(r) = \pi(r - 4)^2$ **d)** $f(x) = x^2 - 3$

10. a) $(0, 2)$ **b)** $(3, 1)$

c) $(-1, -1)$ **d)** $(-1, -1)$

12. a) The graph is a result of these transformations on the graph of $y = x^2$: a translation of 7 units up.

b) The graph is a result of these transformations on the graph of $y = x^2$: a translation of 1 unit to the right and 2 units up.

c) The graph is a result of these transformations on the graph of $y = x^2$: a vertical stretch by a factor of 5 and a translation of 1 unit to the left and 4 units down.

d) The graph is a result of these transformations on the graph of $y = x^2$: a reflection in the x-axis and a translation of 3 units to the right and 5 units up.

13. a) Sketches may vary.

b) Domain: $\{x \mid x \in \mathbf{R}\}$;
Range: $\{g(x) \in \mathbf{R} \mid g(x) \leq 9\}$