1. Compute the following limit: $\displaystyle\lim_{x\to -4}\frac{x^2+7x+12}{x^2-16}$

2. Compute the following limit: $\displaystyle\lim_{x\to -3}\frac{x^2+4x+3}{x^2+3x}$

3. Compute the following limit: $\displaystyle\lim_{x\to -3}\frac{x^2+5x+6}{x^2+7x+12}$

4. Compute the following limit: $\displaystyle\lim_{x\to 2}\frac{x^2-6x+8}{x^2-7x+10}$

5. Compute the following limit: $\displaystyle\lim_{x\to 4}\frac{x^2-6x+8}{x^2-2x-8}$

6. Compute the following limit: $\displaystyle\lim_{x\to -2}\frac{x^2+x-2}{x^2-3x-10}$

7. Compute the following limit: $\displaystyle\lim_{x\to -5}\frac{x^2-25}{x^2+x-20}$

8. Compute the following limit: $\displaystyle\lim_{x\to -3}\frac{x^2-9}{x^2+6x+9}$

9. Compute the following limit: $\displaystyle\lim_{x\to 2}\frac{x^2+2x-8}{x^2-4x+4}$

10. Compute the following limit: $\displaystyle\lim_{x\to 1}\frac{x^2-2x+1}{x^2-1}$

Answers: 1. $\frac{1}{8}$ 2. $\frac{2}{3}$ 3. -1 4. $\frac{2}{3}$ 5. $\frac{1}{3}$ 6. $\frac{3}{7}$ 7. $\frac{10}{9}$ 8. DNE 9. DNE 10. 0

Solutions:

1. $\lim_{x\to -4} \dfrac{x^2+7x+12}{x^2-16}$ ▶ Factorize:

$= \lim_{x\to -4} \dfrac{(x+4)(x+3)}{(x+4)(x-4)}$ ▶ Simplify the common factor:

$= \lim_{x\to -4} \dfrac{x+3}{x-4}$ ▶ Use substitution to compute the limit:

$= \dfrac{-4+3}{-4-4}$ ▶ Simplify:

$= \dfrac{1}{8}$

2. $\lim_{x\to -3} \dfrac{x^2+4x+3}{x^2+3x}$ ▶ Factorize:

$= \lim_{x\to -3} \dfrac{(x+3)(x+1)}{(x+3)(x)}$ ▶ Simplify the common factor:

$= \lim_{x\to -3} \dfrac{x+1}{x}$ ▶ Use substitution to compute the limit:

$= \dfrac{-3+1}{-3}$ ▶ Simplify:

$= \dfrac{2}{3}$

3. $\lim_{x\to -3} \dfrac{x^2+5x+6}{x^2+7x+12}$ ▶ Factorize:

$= \lim_{x\to -3} \dfrac{(x+3)(x+2)}{(x+3)(x+4)}$ ▶ Simplify the common factor:

$= \lim_{x\to -3} \dfrac{x+2}{x+4}$ ▶ Use substitution to compute the limit:

$= \dfrac{-3+2}{-3+4}$ ▶ Simplify:

$= -1$

4. $\lim_{x\to 2} \dfrac{x^2-6x+8}{x^2-7x+10}$ ▶ Factorize:

$= \lim_{x\to 2} \dfrac{(x-2)(x-4)}{(x-2)(x-5)}$ ▶ Simplify the common factor:

$= \lim_{x\to 2} \dfrac{x-4}{x-5}$ ▶ Use substitution to compute the limit:

$= \dfrac{2-4}{2-5}$ ▶ Simplify:

$= \dfrac{2}{3}$

5. $\lim_{x\to 4} \dfrac{x^2-6x+8}{x^2-2x-8}$ ▶ Factorize:

$= \lim_{x \to 4} \frac{(x-4)(x-2)}{(x-4)(x+2)}$ ▶ Simplify the common factor:

$= \lim_{x \to 4} \frac{x-2}{x+2}$ ▶ Use substitution to compute the limit:

$= \frac{4-2}{4+2}$ ▶ Simplify:

$= \frac{1}{3}$

6. $\lim_{x \to -2} \frac{x^2+x-2}{x^2-3x-10}$ ▶ Factorize:

$= \lim_{x \to -2} \frac{(x+2)(x-1)}{(x+2)(x-5)}$ ▶ Simplify the common factor:

$= \lim_{x \to -2} \frac{x-1}{x-5}$ ▶ Use substitution to compute the limit:

$= \frac{-2-1}{-2-5}$ ▶ Simplify:

$= \frac{3}{7}$

7. $\lim_{x \to -5} \frac{x^2-25}{x^2+x-20}$ ▶ Factorize:

$= \lim_{x \to -5} \frac{(x+5)(x-5)}{(x+5)(x-4)}$ ▶ Simplify the common factor:

$= \lim_{x \to -5} \frac{x-5}{x-4}$ ▶ Use substitution to compute the limit:

$= \frac{-5-5}{-5-4}$ ▶ Simplify:

$= \frac{10}{9}$

8. $\lim_{x \to -3} \frac{x^2-9}{x^2+6x+9}$ ▶ Factorize:

$= \lim_{x \to -3} \frac{(x+3)(x-3)}{(x+3)(x+3)}$ ▶ Simplify the common factor:

$= \lim_{x \to -3} \frac{x-3}{x+3}$ ▶ Use substitution to compute the limit:

$= \frac{-3-3}{-3+3}$ ▶ Simplify:

$= DNE$

9. $\lim_{x \to 2} \frac{x^2+2x-8}{x^2-4x+4}$ ▶ Factorize:

$= \lim_{x \to 2} \frac{(x-2)(x+4)}{(x-2)(x-2)}$ ▶ Simplify the common factor:

$= \lim_{x \to 2} \frac{x+4}{x-2}$ ▶ Use substitution to compute the limit:

$= \dfrac{2+4}{2-2}$ ▶ Simplify:

$= DNE$

10. $\displaystyle\lim_{x\to1} \frac{x^2-2x+1}{x^2-1}$ ▶ Factorize:

$= \displaystyle\lim_{x\to1} \frac{(x-1)(x-1)}{(x-1)(x+1)}$ ▶ Simplify the common factor:

$= \displaystyle\lim_{x\to1} \frac{x-1}{x+1}$ ▶ Use substitution to compute the limit:

$= \dfrac{1-1}{1+1}$ ▶ Simplify:

$= 0$

1. Compute the following limit: $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$

2. Compute the following limit: $\lim_{x\to \frac{25}{9}} \frac{\sqrt{x}-\frac{5}{3}}{x-\frac{25}{9}}$

3. Compute the following limit: $\lim_{x\to \frac{25}{4}} \frac{\sqrt{x}-\frac{5}{2}}{x-\frac{25}{4}}$

4. Compute the following limit: $\lim_{x\to \frac{1}{16}} \frac{\sqrt{x}-\frac{1}{4}}{x-\frac{1}{16}}$

5. Compute the following limit: $\lim_{x\to \frac{1}{9}} \frac{\sqrt{x}-\frac{1}{3}}{x-\frac{1}{9}}$

6. Compute the following limit: $\lim_{x\to \frac{4}{9}} \frac{\sqrt{x}-\frac{2}{3}}{x-\frac{4}{9}}$

7. Compute the following limit: $\lim_{x\to \frac{16}{9}} \frac{\sqrt{x}-\frac{4}{3}}{x-\frac{16}{9}}$

8. Compute the following limit: $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

9. Compute the following limit: $\lim_{x\to \frac{9}{4}} \frac{\sqrt{x}-\frac{3}{2}}{x-\frac{9}{4}}$

10. Compute the following limit: $\lim_{x\to \frac{1}{4}} \frac{\sqrt{x}-\frac{1}{2}}{x-\frac{1}{4}}$

Answers: 1. $\frac{1}{2}$ 2. $\frac{3}{10}$ 3. $\frac{1}{5}$ 4. 2 5. $\frac{3}{2}$ 6. $\frac{3}{4}$ 7. $\frac{3}{8}$ 8. $\frac{1}{4}$ 9. $\frac{1}{3}$ 10. 1

Solutions:

1. $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{x-1}$ ▶ Multiply by the conjugate radical:

$= \lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{x-1} \times \dfrac{\sqrt{x}+1}{\sqrt{x}+1}$ ▶ Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{x})^2 = x$

$= \lim\limits_{x\to 1} \dfrac{x-1}{(x-1)(\sqrt{x}+1)}$ ▶ Simplify the common factor:

$= \lim\limits_{x\to 1} \dfrac{1}{\sqrt{x}+1}$ ▶ Use substituion to compute the limit:

$= \dfrac{1}{\sqrt{1}+1}$ ▶ Simplify:

$= \dfrac{1}{2}$

2. $\lim\limits_{x\to \frac{25}{9}} \dfrac{\sqrt{x}-\frac{5}{3}}{x-\frac{25}{9}}$ ▶ Multiply by the conjugate radical:

$= \lim\limits_{x\to \frac{25}{9}} \dfrac{\sqrt{x}-\frac{5}{3}}{x-\frac{25}{9}} \times \dfrac{\sqrt{x}+\frac{5}{3}}{\sqrt{x}+\frac{5}{3}}$ ▶ Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{x})^2 = x$

$= \lim\limits_{x\to \frac{25}{9}} \dfrac{x-\frac{25}{9}}{(x-\frac{25}{9})(\sqrt{x}+\frac{5}{3})}$ ▶ Simplify the common factor:

$= \lim\limits_{x\to \frac{25}{9}} \dfrac{1}{\sqrt{x}+\frac{5}{3}}$ ▶ Use substituion to compute the limit:

$= \dfrac{1}{\sqrt{\frac{25}{9}}+\frac{5}{3}}$ ▶ Simplify:

$= \dfrac{3}{10}$

3. $\lim\limits_{x\to \frac{25}{4}} \dfrac{\sqrt{x}-\frac{5}{2}}{x-\frac{25}{4}}$ ▶ Multiply by the conjugate radical:

$= \lim\limits_{x\to \frac{25}{4}} \dfrac{\sqrt{x}-\frac{5}{2}}{x-\frac{25}{4}} \times \dfrac{\sqrt{x}+\frac{5}{2}}{\sqrt{x}+\frac{5}{2}}$ ▶ Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{x})^2 = x$

$= \lim\limits_{x\to \frac{25}{4}} \dfrac{x-\frac{25}{4}}{(x-\frac{25}{4})(\sqrt{x}+\frac{5}{2})}$ ▶ Simplify the common factor:

$= \lim\limits_{x\to \frac{25}{4}} \dfrac{1}{\sqrt{x}+\frac{5}{2}}$ ▶ Use substituion to compute the limit:

$= \dfrac{1}{\sqrt{\frac{25}{4}}+\frac{5}{2}}$ ▶ Simplify:

$= \dfrac{1}{5}$

4. $\lim\limits_{x\to \frac{1}{16}} \dfrac{\sqrt{x}-\frac{1}{4}}{x-\frac{1}{16}}$ ▶ Multiply by the conjugate radical:

$= \lim_{x \to \frac{1}{16}} \frac{\sqrt{x} - \frac{1}{4}}{x - \frac{1}{16}} \times \frac{\sqrt{x} + \frac{1}{4}}{\sqrt{x} + \frac{1}{4}}$ ▶ Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{x})^2 = x$

$= \lim_{x \to \frac{1}{16}} \frac{x - \frac{1}{16}}{(x - \frac{1}{16})(\sqrt{x} + \frac{1}{4})}$ ▶ Simplify the common factor:

$= \lim_{x \to \frac{1}{16}} \frac{1}{\sqrt{x} + \frac{1}{4}}$ ▶ Use substituion to compute the limit:

$= \frac{1}{\sqrt{\frac{1}{16}} + \frac{1}{4}}$ ▶ Simplify:

$= 2$

5. $\lim_{x \to \frac{1}{9}} \frac{\sqrt{x} - \frac{1}{3}}{x - \frac{1}{9}}$ ▶ Multiply by the conjugate radical:

$= \lim_{x \to \frac{1}{9}} \frac{\sqrt{x} - \frac{1}{3}}{x - \frac{1}{9}} \times \frac{\sqrt{x} + \frac{1}{3}}{\sqrt{x} + \frac{1}{3}}$ ▶ Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{x})^2 = x$

$= \lim_{x \to \frac{1}{9}} \frac{x - \frac{1}{9}}{(x - \frac{1}{9})(\sqrt{x} + \frac{1}{3})}$ ▶ Simplify the common factor:

$= \lim_{x \to \frac{1}{9}} \frac{1}{\sqrt{x} + \frac{1}{3}}$ ▶ Use substituion to compute the limit:

$= \frac{1}{\sqrt{\frac{1}{9}} + \frac{1}{3}}$ ▶ Simplify:

$= \frac{3}{2}$

6. $\lim_{x \to \frac{4}{9}} \frac{\sqrt{x} - \frac{2}{3}}{x - \frac{4}{9}}$ ▶ Multiply by the conjugate radical:

$= \lim_{x \to \frac{4}{9}} \frac{\sqrt{x} - \frac{2}{3}}{x - \frac{4}{9}} \times \frac{\sqrt{x} + \frac{2}{3}}{\sqrt{x} + \frac{2}{3}}$ ▶ Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{x})^2 = x$

$= \lim_{x \to \frac{4}{9}} \frac{x - \frac{4}{9}}{(x - \frac{4}{9})(\sqrt{x} + \frac{2}{3})}$ ▶ Simplify the common factor:

$= \lim_{x \to \frac{4}{9}} \frac{1}{\sqrt{x} + \frac{2}{3}}$ ▶ Use substituion to compute the limit:

$= \frac{1}{\sqrt{\frac{4}{9}} + \frac{2}{3}}$ ▶ Simplify:

$= \frac{3}{4}$

7. $\lim_{x \to \frac{16}{9}} \frac{\sqrt{x} - \frac{4}{3}}{x - \frac{16}{9}}$ ▶ Multiply by the conjugate radical:

$= \lim_{x \to \frac{16}{9}} \frac{\sqrt{x} - \frac{4}{3}}{x - \frac{16}{9}} \times \frac{\sqrt{x} + \frac{4}{3}}{\sqrt{x} + \frac{4}{3}}$ ▶ Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{x})^2 = x$

$$= \lim_{x\to\frac{16}{9}} \frac{x-\frac{16}{9}}{(x-\frac{16}{9})(\sqrt{x}+\frac{4}{3})}$$ ► Simplify the common factor:

$$= \lim_{x\to\frac{16}{9}} \frac{1}{\sqrt{x}+\frac{4}{3}}$$ ► Use substituion to compute the limit:

$$= \frac{1}{\sqrt{\frac{16}{9}}+\frac{4}{3}}$$ ► Simplify:

$$= \frac{3}{8}$$

8. $\displaystyle\lim_{x\to4} \frac{\sqrt{x}-2}{x-4}$ ► Multiply by the conjugate radical:

$$= \lim_{x\to4} \frac{\sqrt{x}-2}{x-4} \times \frac{\sqrt{x}+2}{\sqrt{x}+2}$$ ► Use: $(a-b)(a+b) = a^2-b^2$ and $(\sqrt{x})^2 = x$

$$= \lim_{x\to4} \frac{x-4}{(x-4)(\sqrt{x}+2)}$$ ► Simplify the common factor:

$$= \lim_{x\to4} \frac{1}{\sqrt{x}+2}$$ ► Use substituion to compute the limit:

$$= \frac{1}{\sqrt{4}+2}$$ ► Simplify:

$$= \frac{1}{4}$$

9. $\displaystyle\lim_{x\to\frac{9}{4}} \frac{\sqrt{x}-\frac{3}{2}}{x-\frac{9}{4}}$ ► Multiply by the conjugate radical:

$$= \lim_{x\to\frac{9}{4}} \frac{\sqrt{x}-\frac{3}{2}}{x-\frac{9}{4}} \times \frac{\sqrt{x}+\frac{3}{2}}{\sqrt{x}+\frac{3}{2}}$$ ► Use: $(a-b)(a+b) = a^2-b^2$ and $(\sqrt{x})^2 = x$

$$= \lim_{x\to\frac{9}{4}} \frac{x-\frac{9}{4}}{(x-\frac{9}{4})(\sqrt{x}+\frac{3}{2})}$$ ► Simplify the common factor:

$$= \lim_{x\to\frac{9}{4}} \frac{1}{\sqrt{x}+\frac{3}{2}}$$ ► Use substituion to compute the limit:

$$= \frac{1}{\sqrt{\frac{9}{4}}+\frac{3}{2}}$$ ► Simplify:

$$= \frac{1}{3}$$

10. $\displaystyle\lim_{x\to\frac{1}{4}} \frac{\sqrt{x}-\frac{1}{2}}{x-\frac{1}{4}}$ ► Multiply by the conjugate radical:

$$= \lim_{x\to\frac{1}{4}} \frac{\sqrt{x}-\frac{1}{2}}{x-\frac{1}{4}} \times \frac{\sqrt{x}+\frac{1}{2}}{\sqrt{x}+\frac{1}{2}}$$ ► Use: $(a-b)(a+b) = a^2-b^2$ and $(\sqrt{x})^2 = x$

$$= \lim_{x\to\frac{1}{4}} \frac{x-\frac{1}{4}}{(x-\frac{1}{4})(\sqrt{x}+\frac{1}{2})}$$ ► Simplify the common factor:

$$= \lim_{x\to\frac{1}{4}} \frac{1}{\sqrt{x}+\frac{1}{2}}$$ ► Use substituion to compute the limit:

$$= \frac{1}{\sqrt{\frac{1}{4}} + \frac{1}{2}}$$ ▶ Simplify:

$$= 1$$

1. Compute the following limit: $\lim_{x\to -1} \frac{x^3+1}{x^2-1}$

2. Compute the following limit: $\lim_{x\to 2} \frac{x^3-8}{x^2-4}$

3. Compute the following limit: $\lim_{x\to 4} \frac{x^3-64}{x^2-16}$

4. Compute the following limit: $\lim_{x\to -4} \frac{x^3+64}{x^2-16}$

5. Compute the following limit: $\lim_{x\to -3} \frac{x^3+27}{x^2-9}$

6. Compute the following limit: $\lim_{x\to 1} \frac{x^3-1}{x^2-1}$

7. Compute the following limit: $\lim_{x\to -2} \frac{x^3+8}{x^2-4}$

8. Compute the following limit: $\lim_{x\to -5} \frac{x^3+125}{x^2-25}$

9. Compute the following limit: $\lim_{x\to 3} \frac{x^3-27}{x^2-9}$

10. Compute the following limit: $\lim_{x\to 5} \frac{x^3-125}{x^2-25}$

Answers: 1. $\frac{-3}{2}$ 2. 3 3. 6 4. -6 5. $\frac{-9}{2}$ 6. $\frac{3}{2}$ 7. -3 8. $\frac{-15}{2}$ 9. $\frac{9}{2}$ 10. $\frac{15}{2}$

Solutions:

1. $\lim_{x\to -1} \frac{x^3+1}{x^2-1}$ ► Use: $(a+b)^3=(a+b)(a^2-ab+b^2)$ or $(a-b)^3=(a-b)(a^2+ab+b^2)$

$= \lim_{x\to -1} \frac{(x+1)(x^2-x+1)}{(x+1)(x-1)}$ ► Simplify the common factor:

$= \lim_{x\to -1} \frac{x^2-x+1}{x-1}$ ► Use substitution to compute the limit:

$= \frac{1-1(-1)+1}{-1-1}$ ► Simplify:

$= \frac{-3}{2}$

2. $\lim_{x\to 2} \frac{x^3-8}{x^2-4}$ ► Use: $(a+b)^3=(a+b)(a^2-ab+b^2)$ or $(a-b)^3=(a-b)(a^2+ab+b^2)$

$= \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}$ ► Simplify the common factor:

$= \lim_{x\to 2} \frac{x^2+2x+4}{x+2}$ ► Use substitution to compute the limit:

$= \frac{4+2(2)+4}{2+2}$ ► Simplify:

$= 3$

3. $\lim_{x\to 4} \frac{x^3-64}{x^2-16}$ ► Use: $(a+b)^3=(a+b)(a^2-ab+b^2)$ or $(a-b)^3=(a-b)(a^2+ab+b^2)$

$= \lim_{x\to 4} \frac{(x-4)(x^2+4x+16)}{(x-4)(x+4)}$ ► Simplify the common factor:

$= \lim_{x\to 4} \frac{x^2+4x+16}{x+4}$ ► Use substitution to compute the limit:

$= \frac{16+4(4)+16}{4+4}$ ► Simplify:

$= 6$

4. $\lim_{x\to -4} \frac{x^3+64}{x^2-16}$ ► Use: $(a+b)^3=(a+b)(a^2-ab+b^2)$ or $(a-b)^3=(a-b)(a^2+ab+b^2)$

$= \lim_{x\to -4} \frac{(x+4)(x^2-4x+16)}{(x+4)(x-4)}$ ► Simplify the common factor:

$= \lim_{x\to -4} \frac{x^2-4x+16}{x-4}$ ► Use substitution to compute the limit:

$= \frac{16-4(-4)+16}{-4-4}$ ► Simplify:

$= -6$

5. $\lim_{x\to -3} \frac{x^3+27}{x^2-9}$ ► Use: $(a+b)^3=(a+b)(a^2-ab+b^2)$ or $(a-b)^3=(a-b)(a^2+ab+b^2)$

$= \lim_{x\to -3} \dfrac{(x+3)(x^2-3x+9)}{(x+3)(x-3)}$ ▶ Simplify the common factor:

$= \lim_{x\to -3} \dfrac{x^2-3x+9}{x-3}$ ▶ Use substitution to compute the limit:

$= \dfrac{9-3(-3)+9}{-3-3}$ ▶ Simplify:

$= \dfrac{-9}{2}$

6. $\lim_{x\to 1} \dfrac{x^3-1}{x^2-1}$ ▶ Use: $(a+b)^3 = (a+b)(a^2-ab+b^2)$ or $(a-b)^3 = (a-b)(a^2+ab+b^2)$

$= \lim_{x\to 1} \dfrac{(x-1)(x^2+x+1)}{(x-1)(x+1)}$ ▶ Simplify the common factor:

$= \lim_{x\to 1} \dfrac{x^2+x+1}{x+1}$ ▶ Use substitution to compute the limit:

$= \dfrac{1+1(1)+1}{1+1}$ ▶ Simplify:

$= \dfrac{3}{2}$

7. $\lim_{x\to -2} \dfrac{x^3+8}{x^2-4}$ ▶ Use: $(a+b)^3 = (a+b)(a^2-ab+b^2)$ or $(a-b)^3 = (a-b)(a^2+ab+b^2)$

$= \lim_{x\to -2} \dfrac{(x+2)(x^2-2x+4)}{(x+2)(x-2)}$ ▶ Simplify the common factor:

$= \lim_{x\to -2} \dfrac{x^2-2x+4}{x-2}$ ▶ Use substitution to compute the limit:

$= \dfrac{4-2(-2)+4}{-2-2}$ ▶ Simplify:

$= -3$

8. $\lim_{x\to -5} \dfrac{x^3+125}{x^2-25}$ ▶ Use: $(a+b)^3 = (a+b)(a^2-ab+b^2)$ or $(a-b)^3 = (a-b)(a^2+ab+b^2)$

$= \lim_{x\to -5} \dfrac{(x+5)(x^2-5x+25)}{(x+5)(x-5)}$ ▶ Simplify the common factor:

$= \lim_{x\to -5} \dfrac{x^2-5x+25}{x-5}$ ▶ Use substitution to compute the limit:

$= \dfrac{25-5(-5)+25}{-5-5}$ ▶ Simplify:

$= \dfrac{-15}{2}$

9. $\lim_{x\to 3} \dfrac{x^3-27}{x^2-9}$ ▶ Use: $(a+b)^3 = (a+b)(a^2-ab+b^2)$ or $(a-b)^3 = (a-b)(a^2+ab+b^2)$

$= \lim_{x\to 3} \dfrac{(x-3)(x^2+3x+9)}{(x-3)(x+3)}$ ▶ Simplify the common factor:

$= \lim_{x\to 3} \dfrac{x^2+3x+9}{x+3}$ ▶ Use substitution to compute the limit:

$= \dfrac{9+3(3)+9}{3+3}$ ► Simplify:

$= \dfrac{9}{2}$

10. $\lim_{x\to 5} \dfrac{x^3-125}{x^2-25}$ ► Use: $(a+b)^3 = (a+b)(a^2-ab+b^2)$ or $(a-b)^3 = (a-b)(a^2+ab+b^2)$

$= \lim_{x\to 5} \dfrac{(x-5)(x^2+5x+25)}{(x-5)(x+5)}$ ► Simplify the common factor:

$= \lim_{x\to 5} \dfrac{x^2+5x+25}{x+5}$ ► Use substitution to compute the limit:

$= \dfrac{25+5(5)+25}{5+5}$ ► Simplify:

$= \dfrac{15}{2}$

1. Compute the following limit: $\lim_{x\to 4} \frac{\sqrt{x-1}-\sqrt{3}}{x-4}$

2. Compute the following limit: $\lim_{x\to -2} \frac{\sqrt{x+9}-\sqrt{7}}{x+2}$

3. Compute the following limit: $\lim_{x\to -2} \frac{\sqrt{x+7}-\sqrt{5}}{x+2}$

4. Compute the following limit: $\lim_{x\to 4} \frac{\sqrt{x-3}-\sqrt{1}}{x-4}$

5. Compute the following limit: $\lim_{x\to -5} \frac{\sqrt{x+7}-\sqrt{2}}{x+5}$

6. Compute the following limit: $\lim_{x\to 4} \frac{\sqrt{x+7}-\sqrt{11}}{x-4}$

7. Compute the following limit: $\lim_{x\to -3} \frac{\sqrt{x+13}-\sqrt{10}}{x+3}$

8. Compute the following limit: $\lim_{x\to -3} \frac{\sqrt{x+8}-\sqrt{5}}{x+3}$

9. Compute the following limit: $\lim_{x\to 0} \frac{\sqrt{x+15}-\sqrt{15}}{x-0}$

10. Compute the following limit: $\lim_{x\to -1} \frac{\sqrt{x+9}-\sqrt{8}}{x+1}$

Answers:
1. $\frac{1}{2\sqrt{3}}$ 2. $\frac{1}{2\sqrt{7}}$ 3. $\frac{1}{2\sqrt{5}}$ 4. $\frac{1}{2\sqrt{1}}$ 5. $\frac{1}{2\sqrt{2}}$ 6. $\frac{1}{2\sqrt{11}}$ 7. $\frac{1}{2\sqrt{10}}$ 8. $\frac{1}{2\sqrt{5}}$ 9. $\frac{1}{2\sqrt{15}}$
10. $\frac{1}{2\sqrt{8}}$

Solutions:

1. $\lim\limits_{x\to 4}\dfrac{\sqrt{x-1}-\sqrt{3}}{x-4}$ ► Multiply by the conjugate radical:

$= \lim\limits_{x\to 4}\dfrac{\sqrt{x-1}-\sqrt{3}}{x-4}\times\dfrac{\sqrt{x-1}+\sqrt{3}}{\sqrt{x-1}+\sqrt{3}}$ ► Use: $(a-b)(a+b)=a^2-b^2$ and $(\sqrt{a})^2=a$

$= \lim\limits_{x\to 4}\dfrac{(x-1)-3}{(x-4)(\sqrt{x-1}+\sqrt{3})}$ ► Remove brackets and simplify:

$= \lim\limits_{x\to 4}\dfrac{x-4}{(x-4)(\sqrt{x-1}+\sqrt{3})}$ ► Simplify the common factor:

$= \lim\limits_{x\to 4}\dfrac{1}{\sqrt{x-1}+\sqrt{3}}$ ► Use substitution to compute the limit:

$= \dfrac{1}{\sqrt{4-1}+\sqrt{3}}$ ► Simplify:

$= \dfrac{1}{2\sqrt{3}}$

2. $\lim\limits_{x\to -2}\dfrac{\sqrt{x+9}-\sqrt{7}}{x+2}$ ► Multiply by the conjugate radical:

$= \lim\limits_{x\to -2}\dfrac{\sqrt{x+9}-\sqrt{7}}{x+2}\times\dfrac{\sqrt{x+9}+\sqrt{7}}{\sqrt{x+9}+\sqrt{7}}$ ► Use: $(a-b)(a+b)=a^2-b^2$ and $(\sqrt{a})^2=a$

$= \lim\limits_{x\to -2}\dfrac{(x+9)-7}{(x+2)(\sqrt{x+9}+\sqrt{7})}$ ► Remove brackets and simplify:

$= \lim\limits_{x\to -2}\dfrac{x+2}{(x+2)(\sqrt{x+9}+\sqrt{7})}$ ► Simplify the common factor:

$= \lim\limits_{x\to -2}\dfrac{1}{\sqrt{x+9}+\sqrt{7}}$ ► Use substitution to compute the limit:

$= \dfrac{1}{\sqrt{-2+9}+\sqrt{7}}$ ► Simplify:

$= \dfrac{1}{2\sqrt{7}}$

3. $\lim\limits_{x\to -2}\dfrac{\sqrt{x+7}-\sqrt{5}}{x+2}$ ► Multiply by the conjugate radical:

$= \lim\limits_{x\to -2}\dfrac{\sqrt{x+7}-\sqrt{5}}{x+2}\times\dfrac{\sqrt{x+7}+\sqrt{5}}{\sqrt{x+7}+\sqrt{5}}$ ► Use: $(a-b)(a+b)=a^2-b^2$ and $(\sqrt{a})^2=a$

$= \lim\limits_{x\to -2}\dfrac{(x+7)-5}{(x+2)(\sqrt{x+7}+\sqrt{5})}$ ► Remove brackets and simplify:

$= \lim\limits_{x\to -2}\dfrac{x+2}{(x+2)(\sqrt{x+7}+\sqrt{5})}$ ► Simplify the common factor:

$= \lim\limits_{x\to -2}\dfrac{1}{\sqrt{x+7}+\sqrt{5}}$ ► Use substitution to compute the limit:

$= \dfrac{1}{\sqrt{-2+7}+\sqrt{5}}$ ► Simplify:

$= \dfrac{1}{2\sqrt{5}}$

4. $\displaystyle\lim_{x\to 4} \frac{\sqrt{x-3}-\sqrt{1}}{x-4}$ ▶ Multiply by the conjugate radical:

$= \displaystyle\lim_{x\to 4} \frac{\sqrt{x-3}-\sqrt{1}}{x-4} \times \frac{\sqrt{x-3}+\sqrt{1}}{\sqrt{x-3}+\sqrt{1}}$ ▶ Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{a})^2 = a$

$= \displaystyle\lim_{x\to 4} \frac{(x-3)-1}{(x-4)(\sqrt{x-3}+\sqrt{1})}$ ▶ Remove brackets and simplify:

$= \displaystyle\lim_{x\to 4} \frac{x-4}{(x-4)(\sqrt{x-3}+\sqrt{1})}$ ▶ Simplify the common factor:

$= \displaystyle\lim_{x\to 4} \frac{1}{\sqrt{x-3}+\sqrt{1}}$ ▶ Use substitution to compute the limit:

$= \dfrac{1}{\sqrt{4-3}+\sqrt{1}}$ ▶ Simplify:

$= \dfrac{1}{2\sqrt{1}}$

5. $\displaystyle\lim_{x\to -5} \frac{\sqrt{x+7}-\sqrt{2}}{x+5}$ ▶ Multiply by the conjugate radical:

$= \displaystyle\lim_{x\to -5} \frac{\sqrt{x+7}-\sqrt{2}}{x+5} \times \frac{\sqrt{x+7}+\sqrt{2}}{\sqrt{x+7}+\sqrt{2}}$ ▶ Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{a})^2 = a$

$= \displaystyle\lim_{x\to -5} \frac{(x+7)-2}{(x+5)(\sqrt{x+7}+\sqrt{2})}$ ▶ Remove brackets and simplify:

$= \displaystyle\lim_{x\to -5} \frac{x+5}{(x+5)(\sqrt{x+7}+\sqrt{2})}$ ▶ Simplify the common factor:

$= \displaystyle\lim_{x\to -5} \frac{1}{\sqrt{x+7}+\sqrt{2}}$ ▶ Use substitution to compute the limit:

$= \dfrac{1}{\sqrt{-5+7}+\sqrt{2}}$ ▶ Simplify:

$= \dfrac{1}{2\sqrt{2}}$

6. $\displaystyle\lim_{x\to 4} \frac{\sqrt{x+7}-\sqrt{11}}{x-4}$ ▶ Multiply by the conjugate radical:

$= \displaystyle\lim_{x\to 4} \frac{\sqrt{x+7}-\sqrt{11}}{x-4} \times \frac{\sqrt{x+7}+\sqrt{11}}{\sqrt{x+7}+\sqrt{11}}$ ▶ Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{a})^2 = a$

$= \displaystyle\lim_{x\to 4} \frac{(x+7)-11}{(x-4)(\sqrt{x+7}+\sqrt{11})}$ ▶ Remove brackets and simplify:

$= \displaystyle\lim_{x\to 4} \frac{x-4}{(x-4)(\sqrt{x+7}+\sqrt{11})}$ ▶ Simplify the common factor:

$= \displaystyle\lim_{x\to 4} \frac{1}{\sqrt{x+7}+\sqrt{11}}$ ▶ Use substitution to compute the limit:

$= \dfrac{1}{\sqrt{4+7}+\sqrt{11}}$ ▶ Simplify:

$= \dfrac{1}{2\sqrt{11}}$

7. $\lim\limits_{x\to -3} \dfrac{\sqrt{x+13}-\sqrt{10}}{x+3}$ ► Multiply by the conjugate radical:

$= \lim\limits_{x\to -3} \dfrac{\sqrt{x+13}-\sqrt{10}}{x+3} \times \dfrac{\sqrt{x+13}+\sqrt{10}}{\sqrt{x+13}+\sqrt{10}}$ ► Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{a})^2 = a$

$= \lim\limits_{x\to -3} \dfrac{(x+13)-10}{(x+3)(\sqrt{x+13}+\sqrt{10})}$ ► Remove brackets and simplify:

$= \lim\limits_{x\to -3} \dfrac{x+3}{(x+3)(\sqrt{x+13}+\sqrt{10})}$ ► Simplify the common factor:

$= \lim\limits_{x\to -3} \dfrac{1}{\sqrt{x+13}+\sqrt{10}}$ ► Use substitution to compute the limit:

$= \dfrac{1}{\sqrt{-3+13}+\sqrt{10}}$ ► Simplify:

$= \dfrac{1}{2\sqrt{10}}$

8. $\lim\limits_{x\to -3} \dfrac{\sqrt{x+8}-\sqrt{5}}{x+3}$ ► Multiply by the conjugate radical:

$= \lim\limits_{x\to -3} \dfrac{\sqrt{x+8}-\sqrt{5}}{x+3} \times \dfrac{\sqrt{x+8}+\sqrt{5}}{\sqrt{x+8}+\sqrt{5}}$ ► Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{a})^2 = a$

$= \lim\limits_{x\to -3} \dfrac{(x+8)-5}{(x+3)(\sqrt{x+8}+\sqrt{5})}$ ► Remove brackets and simplify:

$= \lim\limits_{x\to -3} \dfrac{x+3}{(x+3)(\sqrt{x+8}+\sqrt{5})}$ ► Simplify the common factor:

$= \lim\limits_{x\to -3} \dfrac{1}{\sqrt{x+8}+\sqrt{5}}$ ► Use substitution to compute the limit:

$= \dfrac{1}{\sqrt{-3+8}+\sqrt{5}}$ ► Simplify:

$= \dfrac{1}{2\sqrt{5}}$

9. $\lim\limits_{x\to 0} \dfrac{\sqrt{x+15}-\sqrt{15}}{x-0}$ ► Multiply by the conjugate radical:

$= \lim\limits_{x\to 0} \dfrac{\sqrt{x+15}-\sqrt{15}}{x-0} \times \dfrac{\sqrt{x+15}+\sqrt{15}}{\sqrt{x+15}+\sqrt{15}}$ ► Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{a})^2 = a$

$= \lim\limits_{x\to 0} \dfrac{(x+15)-15}{(x-0)(\sqrt{x+15}+\sqrt{15})}$ ► Remove brackets and simplify:

$= \lim\limits_{x\to 0} \dfrac{x-0}{(x-0)(\sqrt{x+15}+\sqrt{15})}$ ► Simplify the common factor:

$= \lim\limits_{x\to 0} \dfrac{1}{\sqrt{x+15}+\sqrt{15}}$ ► Use substitution to compute the limit:

$= \dfrac{1}{\sqrt{0+15}+\sqrt{15}}$ ► Simplify:

$= \dfrac{1}{2\sqrt{15}}$

10. $\lim_{x \to -1} \dfrac{\sqrt{x+9} - \sqrt{8}}{x+1}$ ► Multiply by the conjugate radical:

$= \lim_{x \to -1} \dfrac{\sqrt{x+9} - \sqrt{8}}{x+1} \times \dfrac{\sqrt{x+9} + \sqrt{8}}{\sqrt{x+9} + \sqrt{8}}$ ► Use: $(a-b)(a+b) = a^2 - b^2$ and $(\sqrt{a})^2 = a$

$= \lim_{x \to -1} \dfrac{(x+9) - 8}{(x+1)(\sqrt{x+9} + \sqrt{8})}$ ► Remove brackets and simplify:

$= \lim_{x \to -1} \dfrac{x+1}{(x+1)(\sqrt{x+9} + \sqrt{8})}$ ► Simplify the common factor:

$= \lim_{x \to -1} \dfrac{1}{\sqrt{x+9} + \sqrt{8}}$ ► Use substitution to compute the limit:

$= \dfrac{1}{\sqrt{-1+9} + \sqrt{8}}$ ► Simplify:

$= \dfrac{1}{2\sqrt{8}}$

1. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \sqrt{3x-2} & \text{if } x \leqslant 1 \\ x^2+1 & \text{if } x > 1 \end{cases}$$

Analize the continuity of this function at $x = 1$.

2. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \frac{3}{x+2} - 1 & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$$

Analize the continuity of this function at $x = 1$.

3. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \sqrt{-2x^2+3x+8} & \text{if } x \leqslant 1 \\ \frac{-9x}{x+2} + 6 & \text{if } x > 1 \end{cases}$$

Analize the continuity of this function at $x = 1$.

4. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \frac{-18x}{x+3} + 10 & \text{if } x \leqslant 3 \\ \frac{x}{x-2} - 1 & \text{if } x > 3 \end{cases}$$

Analize the continuity of this function at $x = 3$.

5. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \sqrt{x^2-2x-15} & \text{if } x < -3 \\ 0 & \text{if } x = -3 \\ x^2-10 & \text{if } x > -3 \end{cases}$$

Analize the continuity of this function at $x = -3$.

6. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \frac{-6x}{x} + 7 & \text{if } x < 2 \\ 2 & \text{if } x = 2 \\ -3x+8 & \text{if } x > 2 \end{cases}$$

Analize the continuity of this function at $x = 2$.

7. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} x^2 - x - 8 & \text{if } x < -2 \\ -2x^2 - x + 4 & \text{if } x \geqslant -2 \end{cases}$$

Analize the continuity of this function at $x = -2$.

8. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} 2x & \text{if } x < 1 \\ \sqrt{x^2 + 3} & \text{if } x \geqslant 1 \end{cases}$$

Analize the continuity of this function at $x = 1$.

9. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} -2x^2 + 2x + 13 & \text{if } x \leqslant 3 \\ \sqrt{2x^2 - 3x - 9} & \text{if } x > 3 \end{cases}$$

Analize the continuity of this function at $x = 3$.

10. Consider the following piece-wise defined function:

$$f(x) = \begin{cases} \frac{6}{x-3} + 3 & \text{if } x \leqslant 1 \\ -3x^2 - 2x + 4 & \text{if } x > 1 \end{cases}$$

Analize the continuity of this function at $x = 1$.

Answers: 1. discontinuous 2. discontinuous 3. continuous 4. discontinuous 5. discontinuous 6. discontinuous 7. continuous 8. continuous 9. discontinuous 10. discontinuous

Solutions:

1.
$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \sqrt{3x-2} = \sqrt{3(1)-2} = 1$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 + 1 = (1)^2 + 1 = 2$

$f(1) = 1$

So $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x) \qquad \lim_{x \to 1^-} f(x) = f(1) \qquad \lim_{x \to 1^+} f(x) \neq f(1)$

Therefore, the function is discontinuous.

2.
$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{3}{x+2} - 1 = \frac{3}{(1)+2} - 1 = 0$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \sqrt{x} = \sqrt{(1)} = 1$

$f(1) = 1$

So $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x) \qquad \lim_{x \to 1^-} f(x) \neq f(1) \qquad \lim_{x \to 1^+} f(x) = f(1)$

Therefore, the function is discontinuous.

3.
$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \sqrt{-2x^2 + 3x + 8} = \sqrt{-2(1)^2 + 3(1) + 8} = 3$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{-9x}{x+2} + 6 = \frac{-9(1)}{(1)+2} + 6 = 3$

$f(1) = 3$

So $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \qquad \lim_{x \to 1^-} f(x) = f(1) \qquad \lim_{x \to 1^+} f(x) = f(1)$

Therefore, the function is continuous.

4.
$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{-18x}{x+3} + 10 = \frac{-18(3)}{(3)+3} + 10 = 1$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \frac{x}{x-2} - 1 = \frac{(3)}{(3)-2} - 1 = 2$

$f(3) = 1$

So $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x) \qquad \lim_{x \to 3^-} f(x) = f(3) \qquad \lim_{x \to 3^+} f(x) \neq f(3)$

Therefore, the function is discontinuous.

5.
$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \sqrt{x^2 - 2x - 15} = \sqrt{(-3)^2 - 2(-3) - 15} = 0$

$\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} x^2 - 10 = (-3)^2 - 10 = -1$

$f(-3) = 0$

So $\lim_{x \to -3^-} f(x) \neq \lim_{x \to -3^+} f(x) \qquad \lim_{x \to -3^-} f(x) = f(-3) \qquad \lim_{x \to -3^+} f(x) \neq f(-3)$

Therefore, the function is discontinuous.

6.
$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{-6x}{x} + 7 = \frac{-6(2)}{(2)} + 7 = 1$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} -3x + 8 = -3(2) + 8 = 2$

$f(2) = 2$

So $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x) \qquad \lim_{x\to 2^-} f(x) \neq f(2) \qquad \lim_{x\to 2^+} f(x) = f(2)$

Therefore, the function is discontinuous.

7.
$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} x^2 - x - 8 = (-2)^2 - (-2) - 8 = -2$

$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} -2x^2 - x + 4 = -2(-2)^2 - (-2) + 4 = -2$

$f(-2) = -2$

So $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x) \qquad \lim_{x\to -2^-} f(x) = f(-2) \qquad \lim_{x\to -2^+} f(x) = f(-2)$

Therefore, the function is continuous.

8.
$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2x = 2(1) = 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \sqrt{x^2 + 3} = \sqrt{(1)^2 + 3} = 2$

$f(1) = 2$

So $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) \qquad \lim_{x\to 1^-} f(x) = f(1) \qquad \lim_{x\to 1^+} f(x) = f(1)$

Therefore, the function is continuous.

9.
$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} -2x^2 + 2x + 13 = -2(3)^2 + 2(3) + 13 = 1$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \sqrt{2x^2 - 3x - 9} = \sqrt{2(3)^2 - 3(3) - 9} = 0$

$f(3) = 1$

So $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x) \qquad \lim_{x\to 3^-} f(x) = f(3) \qquad \lim_{x\to 3^+} f(x) \neq f(3)$

Therefore, the function is discontinuous.

10.
$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{6}{x-3} + 3 = \frac{6}{(1)-3} + 3 = 0$

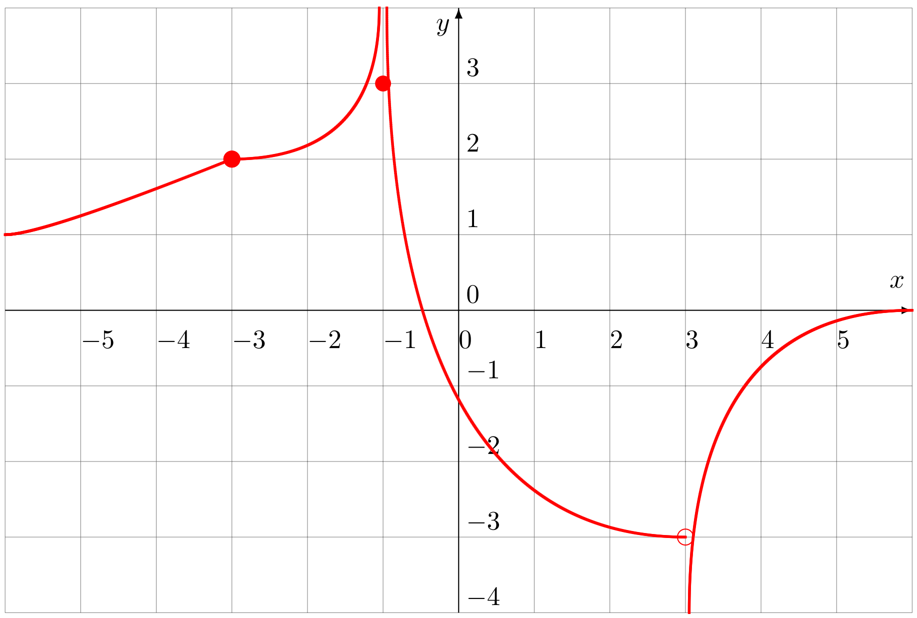
$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} -3x^2 - 2x + 4 = -3(1)^2 - 2(1) + 4 = -1$

$f(1) = 0$

So $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x) \qquad \lim_{x\to 1^-} f(x) = f(1) \qquad \lim_{x\to 1^+} f(x) \neq f(1)$

Therefore, the function is discontinuous.

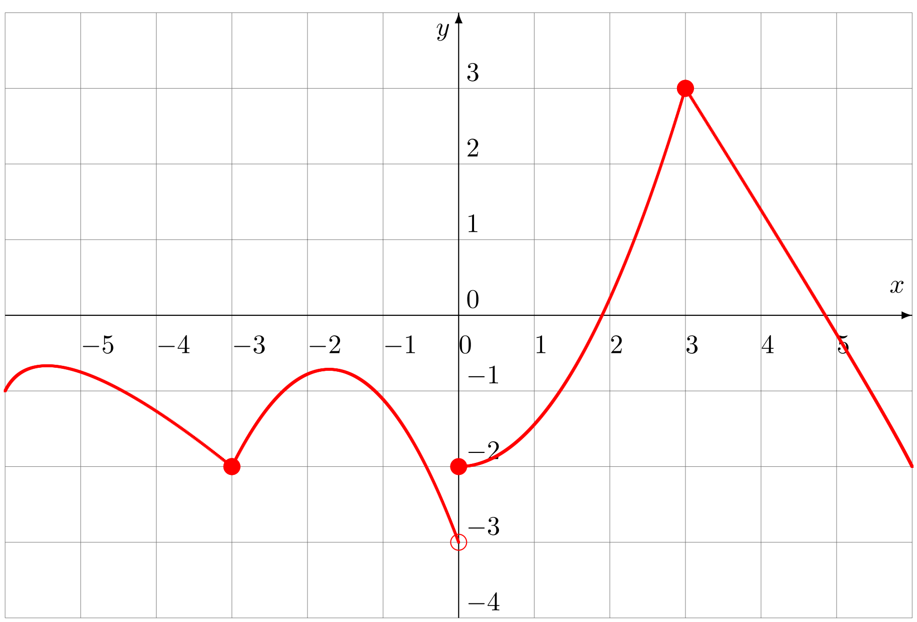
1. Consider the following function defined by its graph:



Analyze the discontinuity of this function (continuous or discontinuous) and the type of discontinuity (removable, jump or infinite discontinuity) at the following numbers:

a) $x = -3$ *b*) $x = -1$ *c*) $x = 3$

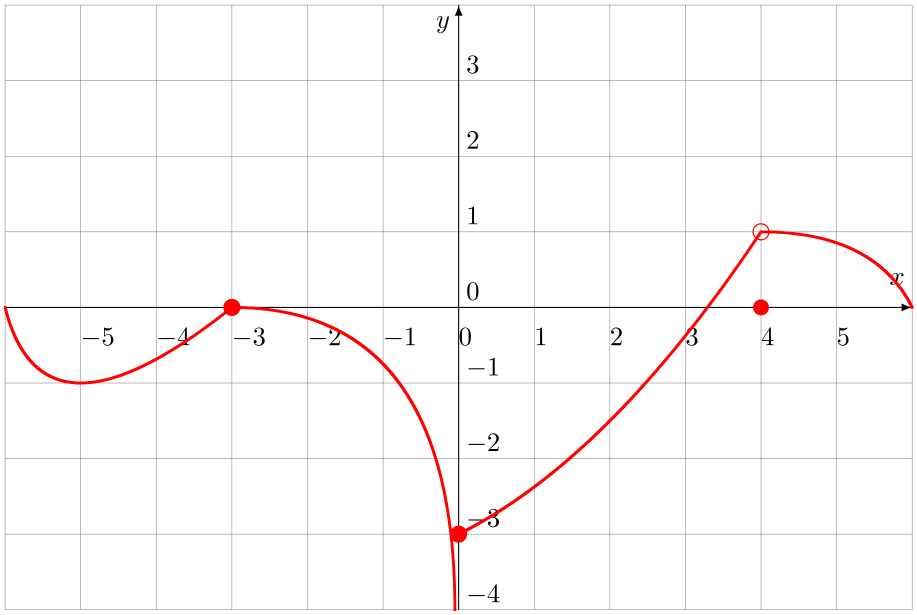
2. Consider the following function defined by its graph:



Analyze the discontinuity of this function (continuous or discontinuous) and the type of discontinuity (removable, jump or infinite discontinuity) at the following numbers:

a) $x = -3$ *b*) $x = 0$ *c*) $x = 3$

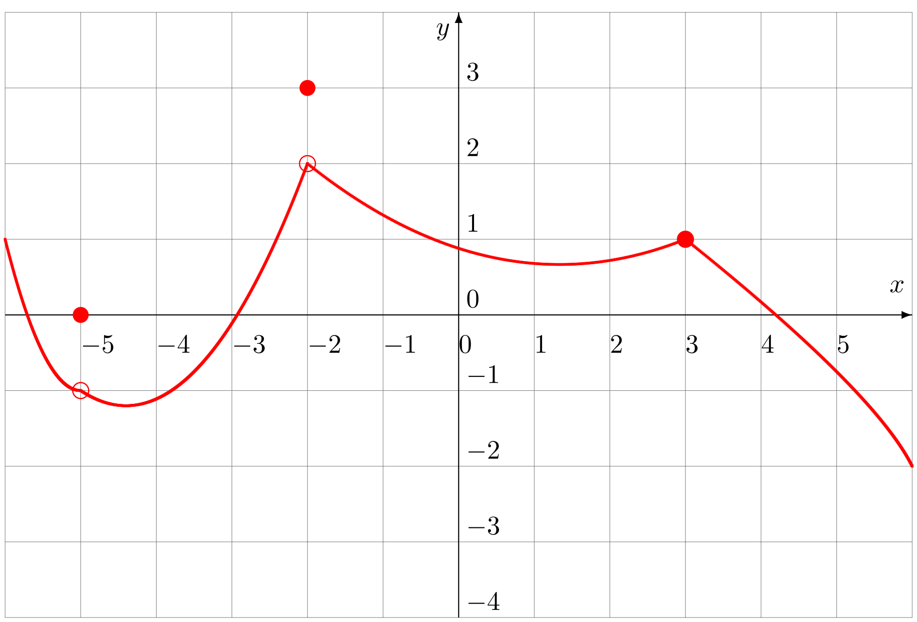
3. Consider the following function defined by its graph:



Analyze the discontinuity of this function (continuous or discontinuous) and the type of discontinuity (removable, jump or infinite discontinuity) at the following numbers:

a) $x = -3$ *b*) $x = 0$ *c*) $x = 4$

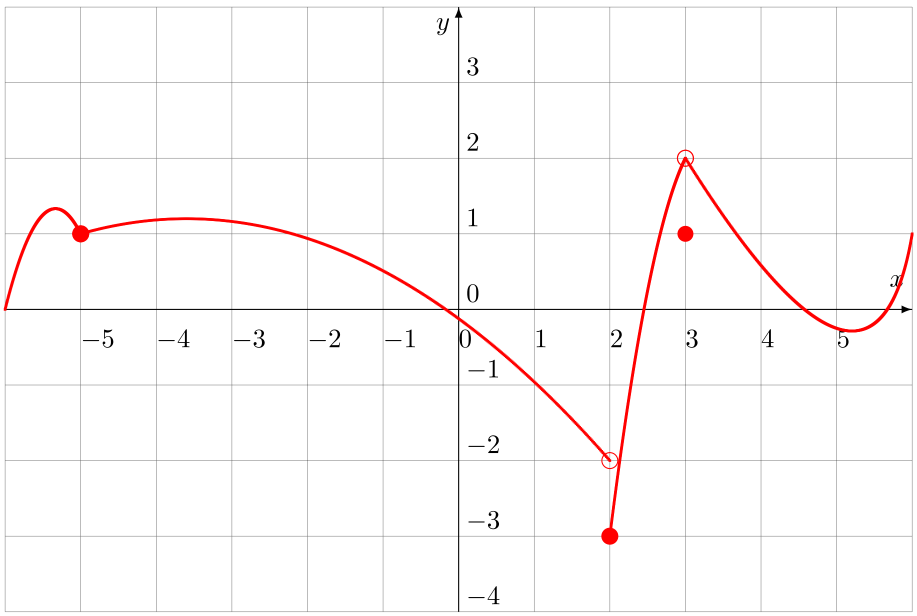
4. Consider the following function defined by its graph:



Analyze the discontinuity of this function (continuous or discontinuous) and the type of discontinuity (removable, jump or infinite discontinuity) at the following numbers:

a) $x = -5$ *b*) $x = -2$ *c*) $x = 3$

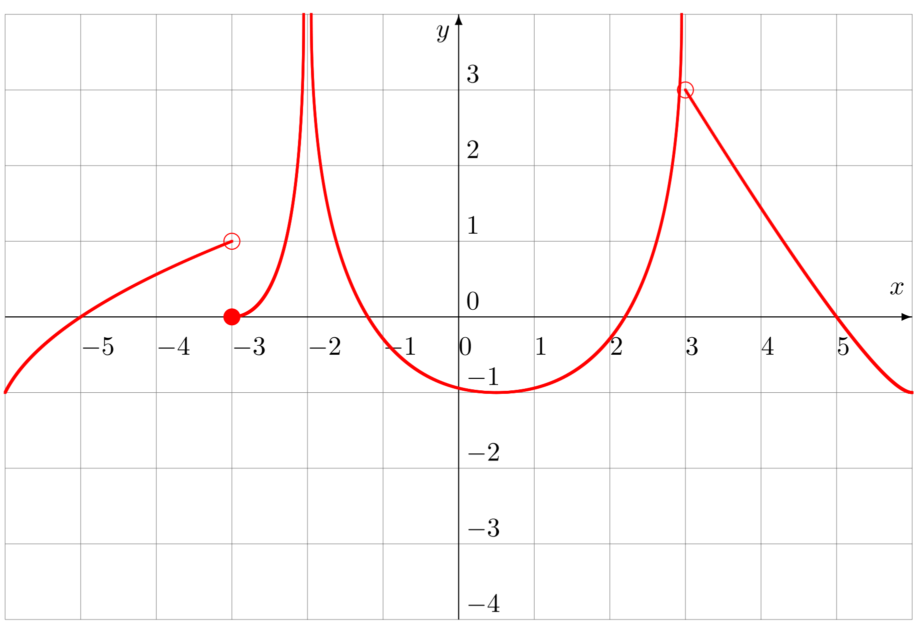
5. Consider the following function defined by its graph:



Analyze the discontinuity of this function (continuous or discontinuous) and the type of discontinuity (removable, jump or infinite discontinuity) at the following numbers:

a) $x = -5$ *b*) $x = 2$ *c*) $x = 3$

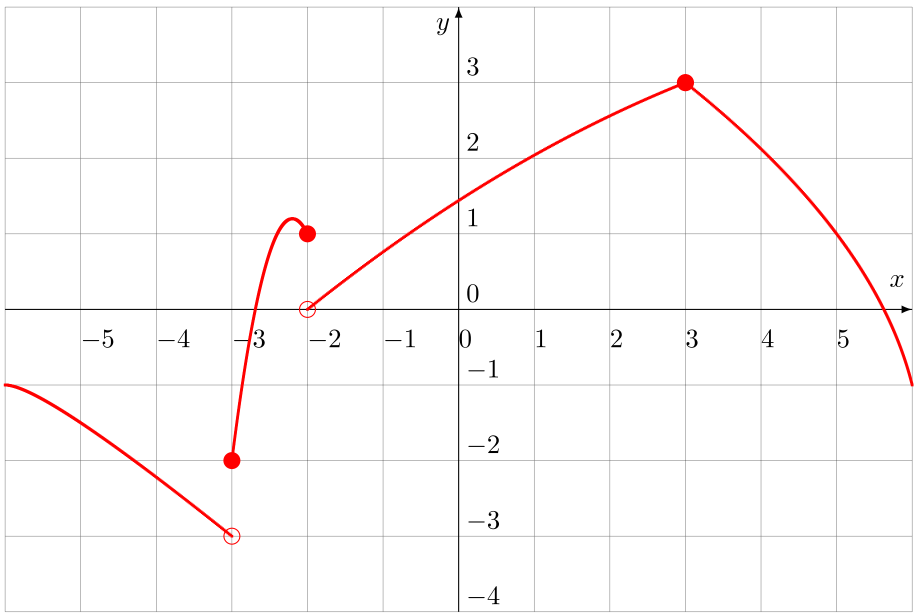
6. Consider the following function defined by its graph:



Analyze the discontinuity of this function (continuous or discontinuous) and the type of discontinuity (removable, jump or infinite discontinuity) at the following numbers:

a) $x = -3$ *b*) $x = -2$ *c*) $x = 3$

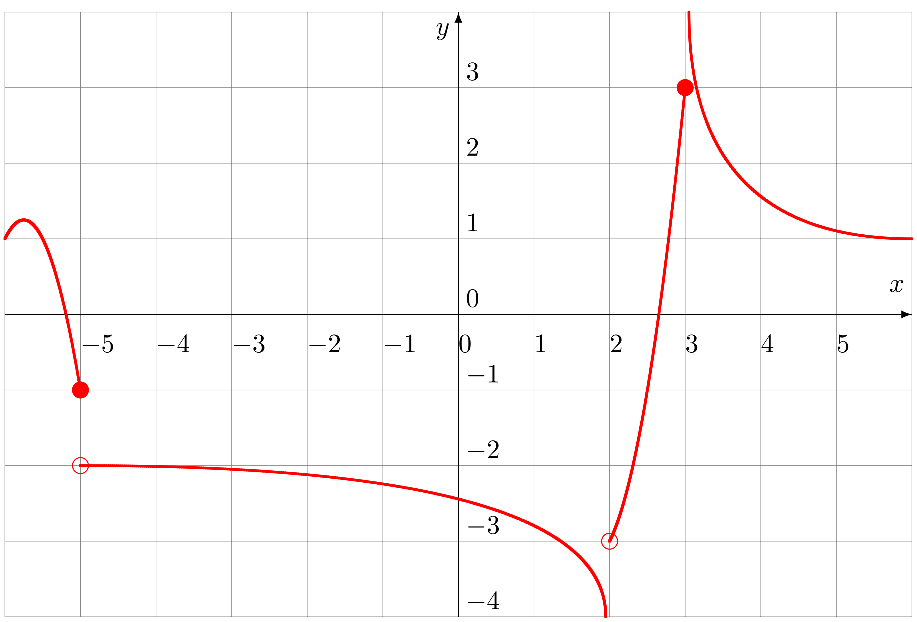
7. Consider the following function defined by its graph:



Analyze the discontinuity of this function (continuous or discontinuous) and the type of discontinuity (removable, jump or infinite discontinuity) at the following numbers:

a) $x = -3$ *b*) $x = -2$ *c*) $x = 3$

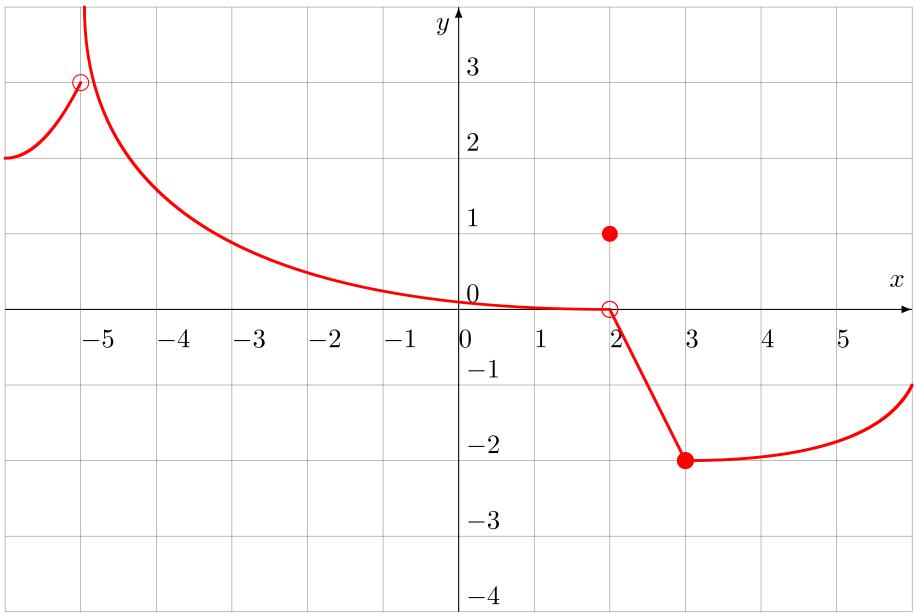
8. Consider the following function defined by its graph:



Analyze the discontinuity of this function (continuous or discontinuous) and the type of discontinuity (removable, jump or infinite discontinuity) at the following numbers:

a) $x = -5$ *b*) $x = 2$ *c*) $x = 3$

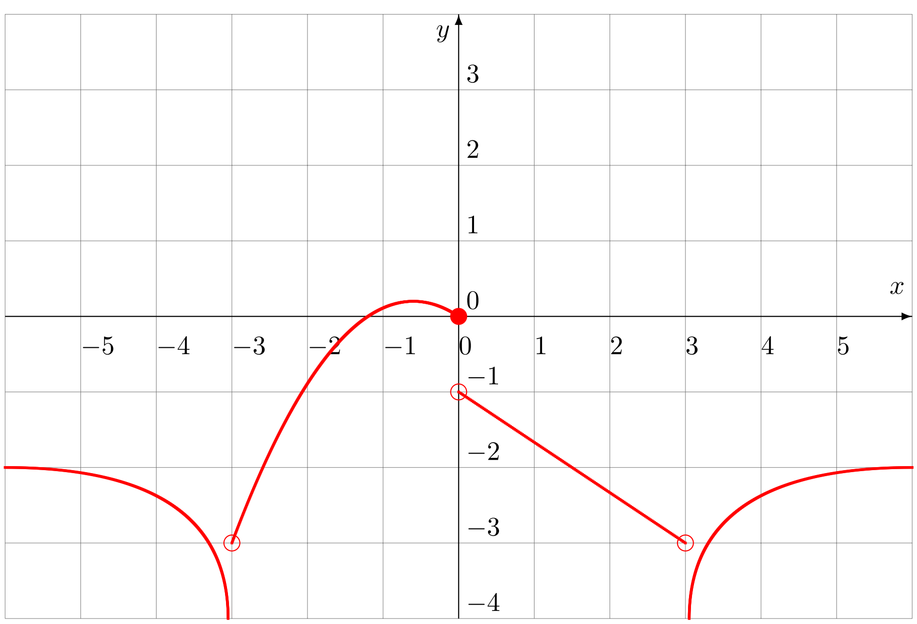
9. Consider the following function defined by its graph:



Analyze the discontinuity of this function (continuous or discontinuous) and the type of discontinuity (removable, jump or infinite discontinuity) at the following numbers:

a) $x = -5$ *b*) $x = 2$ *c*) $x = 3$

10. Consider the following function defined by its graph:



Analyze the discontinuity of this function (continuous or discontinuous) and the type of discontinuity (removable, jump or infinite discontinuity) at the following numbers:

a) $x = -3$ *b*) $x = 0$ *c*) $x = 3$

Answers: 1. a) continuous b) discontinuous (infinite discontinuity) c) discontinuous (infinite discontinuity)
2. a) continuous b) discontinuous (jump discontinuity) c) continuous
3. a) continuous b) discontinuous (infinite discontinuity) c) discontinuous (removable discontinuity)
4. a) discontinuous (removable discontinuity) b) discontinuous (removable discontinuity) c) continuous
5. a) continuous b) discontinuous (jump discontinuity) c) discontinuous (removable discontinuity)
6. a) discontinuous (jump discontinuity) b) discontinuous (infinite discontinuity) c) discontinuous (infinite discontinuity)
7. a) discontinuous (jump discontinuity) b) discontinuous (jump discontinuity) c) continuous
8. a) discontinuous (jump discontinuity) b) discontinuous (infinite discontinuity) c) discontinuous (infinite discontinuity)
9. a) discontinuous (infinite discontinuity) b) discontinuous (removable discontinuity) c) continuous
10. a) discontinuous (infinite discontinuity) b) discontinuous (jump discontinuity) c) discontinuous (infinite discontinuity)

Solutions:

1.

a) $\lim_{x \to -3^-} f(x) = 2$ $\lim_{x \to -3^+} f(x) = 2$ $f(-3) = 2$

Therefore the function is continuous.

b) $\lim_{x \to -1^-} f(x) = \infty$ $\lim_{x \to -1^+} f(x) = \infty$ $f(-1) = 3$

Therefore the function is discontinuous. There is an infinite discontinuity.

c) $\lim_{x \to 3^-} f(x) = -3$ $\lim_{x \to 3^+} f(x) = -\infty$ $f(3) = DNE$

Therefore the function is discontinuous. There is an infinite discontinuity.

2.

a) $\lim_{x \to -3^-} f(x) = -2$ $\lim_{x \to -3^+} f(x) = -2$ $f(-3) = -2$

Therefore the function is continuous.

b) $\lim_{x \to 0^-} f(x) = -3$ $\lim_{x \to 0^+} f(x) = -2$ $f(0) = -2$ $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

Therefore the function is discontinuous. There is a jump discontinuity.

c) $\lim_{x \to 3^-} f(x) = 3$ $\lim_{x \to 3^+} f(x) = 3$ $f(3) = 3$

Therefore the function is continuous.

3.

a) $\lim_{x \to -3^-} f(x) = 0$ $\lim_{x \to -3^+} f(x) = 0$ $f(-3) = 0$

Therefore the function is continuous.

b) $\lim_{x \to 0^-} f(x) = -\infty$ $\lim_{x \to 0^+} f(x) = -3$ $f(0) = -3$

Therefore the function is discontinuous. There is an infinite discontinuity.

c) $\lim_{x \to 4^-} f(x) = 1$ $\lim_{x \to 4^+} f(x) = 1$ $f(4) = 0$ $\lim_{x \to 4^-} f(x) \neq f(4)$ $\lim_{x \to 4^+} f(x) \neq f(4)$

Therefore the function is discontinuous. There is a removable discontinuity.

4.

a) $\lim_{x \to -5^-} f(x) = -1$ $\lim_{x \to -5^+} f(x) = -1$ $f(-5) = 0$ $\lim_{x \to -5^-} f(x) \neq f(-5)$ $\lim_{x \to -5^+} f(x) \neq f(-5)$

Therefore the function is discontinuous. There is a removable discontinuity.

b) $\lim_{x \to -2^-} f(x) = 2$ $\lim_{x \to -2^+} f(x) = 2$ $f(-2) = 3$ $\lim_{x \to -2^-} f(x) \neq f(-2)$ $\lim_{x \to -2^+} f(x) \neq f(-2)$

Therefore the function is discontinuous. There is a removable discontinuity.

c) $\lim_{x \to 3^-} f(x) = 1$ $\lim_{x \to 3^+} f(x) = 1$ $f(3) = 1$

Therefore the function is continuous.

5.

a) $\lim_{x \to -5^-} f(x) = 1$ $\lim_{x \to -5^+} f(x) = 1$ $f(-5) = 1$

Therefore the function is continuous.

b) $\lim_{x \to 2^-} f(x) = -2$ $\lim_{x \to 2^+} f(x) = -3$ $f(2) = -3$ $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$

Therefore the function is discontinuous. There is a jump discontinuity.

c) $\lim_{x \to 3^-} f(x) = 2$ $\lim_{x \to 3^+} f(x) = 2$ $f(3) = 1$ $\lim_{x \to 3^-} f(x) \neq f(3)$ $\lim_{x \to 3^+} f(x) \neq f(3)$

Therefore the function is discontinuous. There is a removable discontinuity.

6.

a) $\lim_{x \to -3^-} f(x) = 1$ $\lim_{x \to -3^+} f(x) = 0$ $f(-3) = 0$ $\lim_{x \to -3^-} f(x) \neq \lim_{x \to -3^+} f(x)$

Therefore the function is discontinuous. There is a jump discontinuity.

b) $\lim_{x \to -2^-} f(x) = \infty$ $\lim_{x \to -2^+} f(x) = \infty$ $f(-2) = DNE$

Therefore the function is discontinuous. There is an infinite discontinuity.

c) $\lim_{x \to 3^-} f(x) = \infty$ $\lim_{x \to 3^+} f(x) = 3$ $f(3) = DNE$

Therefore the function is discontinuous. There is an infinite discontinuity.

7.
a) $\lim\limits_{x\to -3^-} f(x) = -3$ $\quad \lim\limits_{x\to -3^+} f(x) = -2$ $\quad f(-3) = -2$ $\quad \lim\limits_{x\to -3^-} f(x) \neq \lim\limits_{x\to -3^+} f(x)$
Therefore the function is discontinuous. There is a jump discontinuity.
b) $\lim\limits_{x\to -2^-} f(x) = 1$ $\quad \lim\limits_{x\to -2^+} f(x) = 0$ $\quad f(-2) = 1$ $\quad \lim\limits_{x\to -2^-} f(x) \neq \lim\limits_{x\to -2^+} f(x)$
Therefore the function is discontinuous. There is a jump discontinuity.
c) $\lim\limits_{x\to 3^-} f(x) = 3$ $\quad \lim\limits_{x\to 3^+} f(x) = 3$ $\quad f(3) = 3$
Therefore the function is continuous.

8.
a) $\lim\limits_{x\to -5^-} f(x) = -1$ $\quad \lim\limits_{x\to -5^+} f(x) = -2$ $\quad f(-5) = -1$ $\quad \lim\limits_{x\to -5^-} f(x) \neq \lim\limits_{x\to -5^+} f(x)$
Therefore the function is discontinuous. There is a jump discontinuity.
b) $\lim\limits_{x\to 2^-} f(x) = -\infty$ $\quad \lim\limits_{x\to 2^+} f(x) = -3$ $\quad f(2) = DNE$
Therefore the function is discontinuous. There is an infinite discontinuity.
c) $\lim\limits_{x\to 3^-} f(x) = 3$ $\quad \lim\limits_{x\to 3^+} f(x) = \infty$ $\quad f(3) = 3$
Therefore the function is discontinuous. There is an infinite discontinuity.

9.
a) $\lim\limits_{x\to -5^-} f(x) = 3$ $\quad \lim\limits_{x\to -5^+} f(x) = \infty$ $\quad f(-5) = DNE$
Therefore the function is discontinuous. There is an infinite discontinuity.
b) $\lim\limits_{x\to 2^-} f(x) = 0$ $\quad \lim\limits_{x\to 2^+} f(x) = 0$ $\quad f(2) = 1$ $\quad \lim\limits_{x\to 2^-} f(x) \neq f(2)$ $\quad \lim\limits_{x\to 2^+} f(x) \neq f(2)$
Therefore the function is discontinuous. There is a removable discontinuity.
c) $\lim\limits_{x\to 3^-} f(x) = -2$ $\quad \lim\limits_{x\to 3^+} f(x) = -2$ $\quad f(3) = -2$
Therefore the function is continuous.

10.
a) $\lim\limits_{x\to -3^-} f(x) = -\infty$ $\quad \lim\limits_{x\to -3^+} f(x) = -3$ $\quad f(-3) = DNE$
Therefore the function is discontinuous. There is an infinite discontinuity.
b) $\lim\limits_{x\to 0^-} f(x) = 0$ $\quad \lim\limits_{x\to 0^+} f(x) = -1$ $\quad f(0) = 0$ $\quad \lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+} f(x)$
Therefore the function is discontinuous. There is a jump discontinuity.
c) $\lim\limits_{x\to 3^-} f(x) = -3$ $\quad \lim\limits_{x\to 3^+} f(x) = -\infty$ $\quad f(3) = DNE$
Therefore the function is discontinuous. There is an infinite discontinuity.