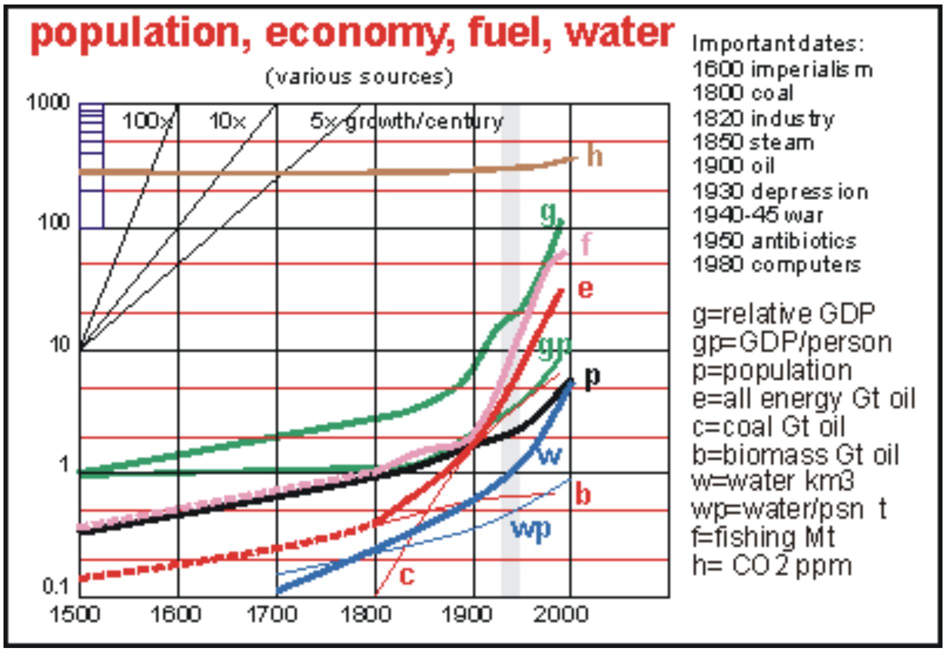
# Chapter 4: Exponential Functions

## § 4.1: Exploring Growth and Decay

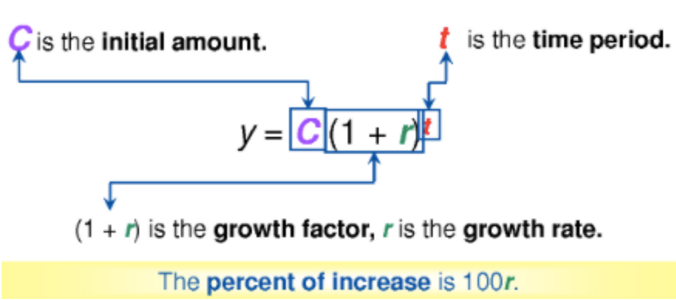
Some real-world situations can be modeled by functions whose first differences follow a multiplicative pattern.



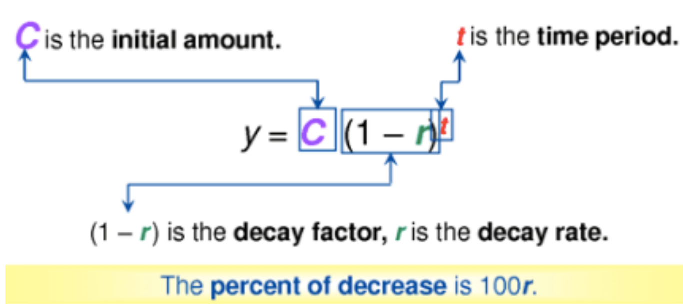

The scatter plots for these situations show _______________ or ____________ nonlinear patterns.

The domain and range of a function should be considered in terms of the situation it is modeling.

Exponential Growth Models




Exponential Decay Models




**Homework: p. 216 #2**

# § 4.2: Working With Integer Exponents

An integer base raised to a negative exponent is equivalent to the ________________ of the

same base raised to the __________________ exponent.

$$\boxed{b^{-n} = \frac{1}{b^n}, \qquad \text{where} \quad b \neq 0}$$

A _______________ base raised to a negative exponent is equivalent to the reciprocal of the

same base raised to the opposite exponent.

$$\boxed{\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = \left(\frac{b}{a}\right)^n, \qquad \text{where} \quad a \neq 0, b \neq 0}$$

A number (or expression), other than 0, raised to the power of zero is equal to ____.

$$\boxed{b^0 = 1, \qquad \text{where} \quad b \neq 0}$$

When multiplying powers with the same base, add exponents: $\boxed{b^m \times b^n = b^{m+n}}$

When dividing powers with the same base, subtract exponents: $\boxed{b^m \div b^n = b^{m-n} \quad \text{if} \quad b \neq 0}$

To raise a power to a power, multiply exponents: $\boxed{(b^m)^n = b^{mn}}$

When simplifying numerical expressions involving powers, present answer as an <u>integer</u>, a

<u>fraction</u>, or a <u>decimal</u> ALSO with <u>positive exponents</u>.

<u>Example 1:</u>

a) $\left(-\frac{1}{5}\right)^{-3}$

b) $\frac{5^{-3}}{5^{-1}}$

<u>Example 2:</u>

a) $3^3(3^{-2})^{-3}$

b) $2^2(2^{-2}(2^2))$

**In Class Assignment: p. 216 #1, 2, 3; Homework: p. 217 #4, 6, 8, 9, 11, 15**

# § 4.3: Working with Rational Exponents

A number raised to a rational exponent is equivalent to a ________________________.

The rational exponent _____ indicates the $n$th root of the base. If $n > 1$ and $n \in N$

(n belongs to the set of Natural numbers), then ____________, where __________.

If the numerator of a rational exponent is not 1, and if $m$ and $n$ are positive integers, then

___________________________, where ____________.

The product of power rule:

The quotient of powers rule:

Some roots of negative numbers do not have real solutions. For example, -16 does not have a real-number square root. Odd roots can have negative bases, but even ones cannot.

**Example 1: Write in radical form, then evaluate.**

a) $49^{\frac{1}{2}}$

b) $(-125)^{\frac{1}{3}}$

c) $-16^{0.25}$

**Example 2: Write in exponent form, then evaluate.**

a) $\sqrt[9]{512}$

b) $\sqrt[5]{\frac{-32}{243}}$

c) $\sqrt[4]{(\frac{16}{81})^{-1}}$

**In Class Assignment: p. 229 #1bf, 2bd, 3; Homework: p. 230 #4bd, 5ac, 6df**

# § 4.4: Simplifying Algebraic Expressions Involving Exponents

Algebraic expressions involving powers containing integer and rational exponents can be simplified with the use of the exponent rules in the same way numerical expressions can be simplified. When evaluating an algebraic expression by substitution, _______________ prior to substituting. The answer will be the same if substitution is done prior to simplifying, but the number of calculations will be ____________________.

Algebraic expressions involving radicals can often be simplified by changing the expression in to the exponential form and applying the rules for exponents.

**Example 1: Simplify, express answer with positive exponents.**

a) $x^4(x^3)$ b) $\dfrac{m^5}{m^{-3}}$ c) $-\left(k^6\right)^{-1}$ d) $\dfrac{\left(b^{-7}\right)^2}{b\left(b^{-5}\right)b^9}$

**Example 2: Evaluate, express answers in rational form with positive exponents.**

a) $(16x^6y^4)^{\frac{1}{2}}$ for $x = 2, y = 1$

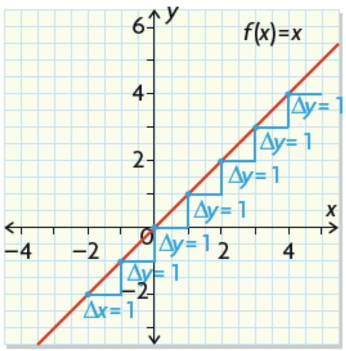
b) $\left(\dfrac{(25a^4)^{-1}}{(7a^{-2}b)^2}\right)^{\frac{1}{2}}$ for $a = 11, b = 10$

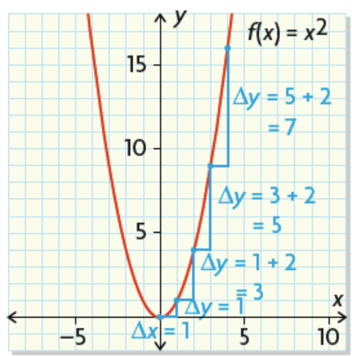
**ICA: p. 235 #1bde, 2ace, 3; Homework: pp. 236-237 #4af, 5bd, 6ace, 7bc**

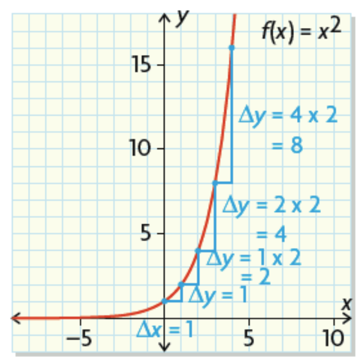
# § 4.5: Exploring Properties of Exponential Functions

Linear, quadratic, and exponential functions have unique first-difference patterns that allow them to be recognized.

| Linear | Quadratic | Exponential |
|---|---|---|
| Linear functions have constant first differences. | Quadratic functions have first differences that are related by addition. Their second differences are constant. | Exponential functions have first differences that are related by multiplication. Their second finite differences are not constant. |




The exponential function $f(x) = b^x$ is

- An increasing function representing growth when ____________________
- A decreasing function representing decay when ______________________

It has the following characteristics:

- If $b > 0$, then the function is defined, its domain is $\{x \in \mathbb{R}\}$ and range is $\{y \in \mathbb{R} |\ y \geq 0\}$.
- If $b > 1$, then the greater the value of $b$, the faster the growth.
- If $0 < b < 1$, then the lesser the value of $b$, the faster the decay.
- The function has the $x$-axis, $y = 0$, as horizontal *asymptote.*
- The function has a $y$-intercept of 1.

**Example: Use differences to identify the type of function.**

| *x* | *y* | *FD* | *SD* | *Ratio* |
|---|---|---|---|---|
| -5 | 32 | | | |
| -4 | 16 | | | |
| -3 | 8 | | | |
| -2 | 4 | | | |
| -1 | 2 | | | |
| 0 | 1 | | | |

**Homework: pp. 243 #1, 2**

# § 4.7 – Applications Involving Exponential Functions

The exponential function $f(x) = ab^x$ and its graph can be used as a model to solve problems involving exponential growth and decay. Note that:

- $f(x)$ is the final amount or number
- $a$ is the initial amount or number
- for exponential growth, $b = 1 +$ growth rate;
- for exponential decay, $b = 1 -$ decay rate
- $x$ is the number of growth or decay periods

When solving questions involving exponential function:

- If the ***growth rate*** (as a percent) is given, then the base of the power in the equation can be obtained by _______________ the rate, as a decimal, to 1.
    - If the growth rate is 8%, then it involves multiplying by 1.08 repeatedly.

- If the ***decay rate*** (as a percent) is given, then the base of the power in the equation can be

  obtained by _______________ the rate, as a decimal, to 1.

  - If the decay rate is <u>8%,</u> then it involves multiplying by <u>0.92</u> repeatedly.

**<u>Example 1: Solve for exponential equation.</u>**

a) $A = 250(1.05)^{10}$

b) $50 = N_0(1.25)^{1.25}$

**<u>Example 2:</u>**

The growth in population of a small town since 1996 is given by the function

$$P(n) = 1250(1.03)^n$$

a) What is the initial population? Growth rate?

b) Determine the population in the year 2007.

c) In which year does the population reach 2000 people?

**In Class Assignment: p. 261 #1bd, 2; Homework: pp. 262 #4, 5, 6, 15**