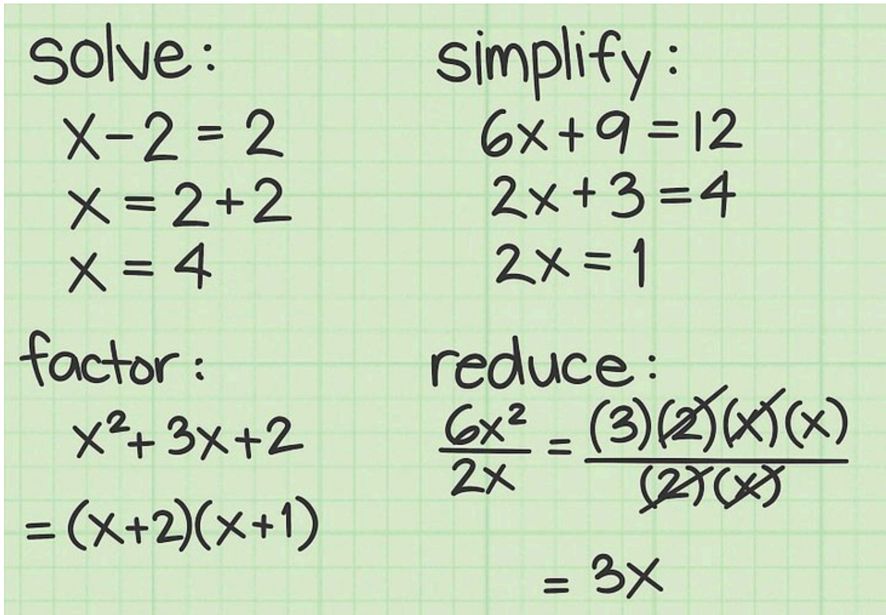
Unit 4: Algebra Basics

V. Nguyen



Exponent Laws

Algebra Basics 1

* Goal: Understand the definition of exponentiation in algebra.

Exponent Rules For $a \neq 0, b \neq 0$	
Product Rule	$a^x \times a^y = a^{x+y}$
Quotient Rule	$a^x \div a^y = a^{x-y}$
Power Rule	$\left(a^x\right)^y = a^{xy}$
Power of a Product Rule	$\left(ab\right)^x = a^x b^x$
Power of a Fraction Rule	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Zero Exponent	$a^0 = 1$
Negative Exponent	$a^{-x} = \frac{1}{a^x}$
Fractional Exponent	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

Exponent Laws

Consider the expression $a^2 \cdot a^3$, using the definition of exponentiation:
$a^2 \cdot a^3 =$

If the bases are *not* the same, this rule *does not* apply. For example, the expression $2^4 \cdot 3^2$ cannot be combined.

Product of Like Powers **Law:**

For any real non-zero values x, y, and a, $\boldsymbol{a^x \cdot a^y = a^{x+y}}$

Now, $\dfrac{a^3}{a^2} =$

Quotient of Like Powers **Law:**

For any real non-zero values x, y, and a, $\dfrac{a^x}{a^y} = a^{x-y}$

Just like the Product Law, the bases **must** be the same.

Exponent Laws

Now, consider $(a^2)^3$

In general, $(a^x)^y$

Power of a Power **Law:**

For any real non-zero values *x*, *y*, and *a*, $\mathbf{(a^x)^y = a^{xy}}$

Exponent Laws

Next, consider $(ab)^3$

In general, $(ab)^x$

<u>Power</u> <u>of a Product</u> **<u>Law:</u>**
For any real, non-zero values a, b, and x, $\boldsymbol{(ab)^x = a^x b^x}$

Finally, consider $\left(\frac{a}{b}\right)^2 =$

In general, $\left(\frac{a}{b}\right)^x =$

<u>Power</u> <u>of a Quotient</u> **<u>Law:</u>**
For any real, non-zero values a, b, and x, $\left(\frac{a}{b}\right)^x =$

Exponent Laws

Example 1:

Simplify the expression (a) x^4x^7 (b) $\dfrac{y^8}{y^6}$ (c) $(z^3)^3$ (d) x^4x^7 (e) $\left(\dfrac{x}{2}\right)^5$

Exponent Laws

Zero Exponent Law: For any real, non-zero value of a, $a^0 = 1$.

Why?

Negative Exponent Law: For any real, non-zero value of a and any real, positive value of x, $a^{-x} = \frac{1}{a^x}$

Why?

Exponent Laws

Example 2: Evaluate $1\ 234\ 567^0$

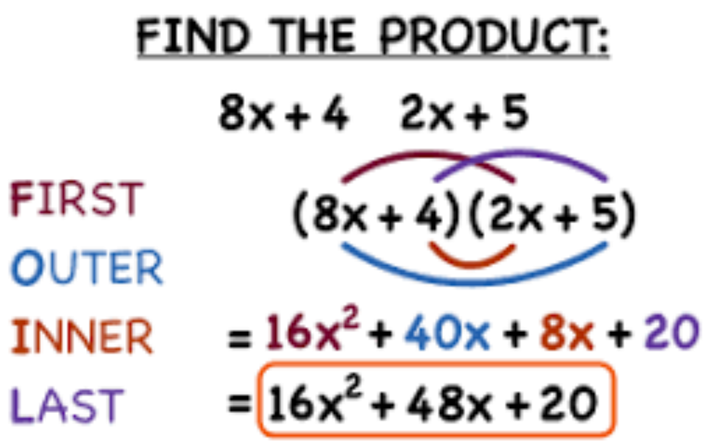
Example 3: Express x^{-4} using positive exponents.

Example 4: Simplify $\dfrac{x^5y^3}{x^2y^7}$ using positive exponents.

Distributive Law

Algebra Basics 2

- Goal: Understand and use the distributive law when multiplying two binomials.



Products of Two Binomials

Example 1: Expand and simplify $(x + 2)(x + 5)$

Example 2: Expand and simplify $(2x - 1)(3x - 6)$

Products of Two Binomials

Example 3: Expand and simplify $(7 - 2x)(7 + 2x)$

Example 4: Expand and simplify $(4r + 5s)(2r - 6s)$

Distributive Law Extension

Example 5: Expand and simplify $(x - 2)(x^2 + 3x - 5)$

Example 6: Expand and simplify $(x^2 + 5x - 1)(2x^2 - 4x + 3)$

Perfect Squares

$(ax + c)^2 =$

Example 7: Expand and simplify $(2x + 7)^2$

Differences of Squares

$(ax + c)(ax - c) =$

Example 8: Expand and simplify $(3x + 9)(3x - 9)$

Factoring Polynomials

Algebra Basics 3

* <u>Goal:</u> Make use of distributive law to common factor polynomials.

Summary:

$$
\begin{aligned}
6x + 4 &= 2 \cdot 3x + 2 \cdot 2 \\
&= 2(3x + 2)
\end{aligned}
$$

Check by distributing:

$$2(3x + 2) = 6x + 4$$

Common Factoring

Recall that the Greatest Common Factor (GCF) of two or more numbers is the greatest value that divides evenly into those numbers. For example:

GCF(6,9) =

GCF(x^3,x^5) =

Example 9: Factor $8x - 12$

Example 10: Factor $28xy - 7x$

Common Factoring

Example 11: Factor $3x^2 - 15x + 21$

Example 12: Factor $5x^3 - 2x^2 + 3x$

Common Factoring

Example 13: Factor $15x^4 + 25x^3 - 30x$

Example 14: Factor $16x^3y^2 + 24x^5y^2$

Common Factoring

Example 15: Factor $2x^2 + 5x - 3$

Factoring by Grouping

Recap: GCF of $12x^4 + 14x^3 - 20x^2$ is ____

Factoring by Grouping

Example 1: Factor $3x(x - 5) + 2(x - 5)$

Example 2: Factor $5x^2(2x + 7) + (2x + 7)$

Factoring by Grouping

Consider the cubic expression $x^3 + 2x^2 + 8x + 16$

Example 3: Factor $x^3 - 2x^2 + 6x + 9$

Factoring by Grouping

Example 4: Factor $xy - 4y - 3x + 12$

Example 5: Factor $2a^2 + 21b - 6ab - 7a$.

Summary: *Factor Polynomials*

What to do	How to do it
Factor: **common factors**:	$ax + ay = a(x + y)$
Distribute: *left / right*	$ax + bx = (a + b)x$
Perfect square trinomial.	$a^2x^2 \pm 2abx + b^2$
binomial squared	$(ax \pm b)^2$
Difference of Squares:	$A^2 - B^2 = (A + B)(A - B)$
Conjugate Pairs:	$a^2x^2 - b^2y^2 = (ax+by)(ax - by)$

Factoring Simple Trinomials

Recap: Expand and simplify $(x + 2)(x + 5)$

Recall that a quadratic expression has the form $ax^2 + bx + c$

If $a = 1$, then the expression will have the *simple* trinomial form.

Simple trinomials usually result from multiplying two binomials, each of the form $x + k$.

For example, $(x + 4)(x - 7) = x^2 - 3x - 28$.

Factoring Simple Trinomials

Example 1: Factor $x^2 + 16x + 15$.

Example 2: Factor $x^2 + 11x + 24$.

Factoring Simple Trinomials

Example 3: Factor $x^2 - 7x + 10$.

Example 4: Factor $x^2 + x - 12$.

Factoring Simple Trinomials

Example 5: Factor $x^2 - 7x - 18$.

Example 6: Factor $x^2 + 5x + 2$.

Factoring Complex Trinomials

When factoring complex trinomial, we must determine two values such with a sum of ***b*** and a product of ***ac***. To make things work, we **decompose** the middle term into separate two terms.

Example 1: Factor $4x^2 - 8x - 21$

Factor $4x^2 - 8x - 21$

Factoring Complex Trinomials

Example 2: Factor $6x^2 + 19x - 7$

Factor $6x^2 + 19x - 7$

Factoring Complex Trinomials

Example 3: Factor $25x^2 - 20x + 4$

Factor $25x^2 - 20x + 4$

Factoring Complex Trinomials

Example 4: Factor $4 - 20x + 25x^2$

Factor $4 - 20x + 25x^2$

Multi-Stage Factoring

Recap: Factor $1 + 10x + 25x^2$

Factor $1 + 10x + 25x^2$

Multi-Stage Factoring

Example 1: Fully factor $2x^2 - 4x - 30$

Example 2: Fully factor $20x^3 - 60x^2 + 45x$

Multi-Stage Factoring

Example 3: Factor $3x^4 - 15x^2 + 18$

Example 4: Factor $12x^2 - 27$

Multi-Stage Factoring

<u>Example 5:</u> Factor $16x^4 - 1$

<u>Example 6*:</u> Factor $x^3 - 8$

*You will not be tested on this