MAP4C1 Unit 2: Geometry

2.7 3D Optimization

Learning Goals: I am learning to…
- ☐ Investigate the properties of 3D optimization and determine the dimensions of an object that would produce the maximum volume and minimum surface area

Optimization is the process of finding the most efficient use of available materials with given constraints. (i.e. finding the maximum or minimum for a specific variable)

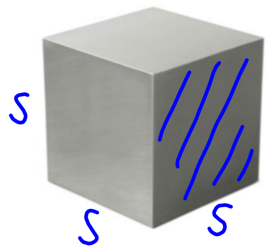
For 2D optimization, we concluded that to optimize both perimeter and area, a **square** produces the optimal dimensions. What about 3D objects? How could we extend this?

1. For a rectangular prism, with a given surface area, a cube has the **maximum volume.**
2. For a rectangular prism, with a given volume, a cube has the **minimum surface area.**

Volume:

$V = s^3$

reverse: $\sqrt[3]{}$



Surface Area:

$SA = 6s^2$

Example 1: What dimensions produce a minimum surface area of a rectangular prism with a volume of 1000 cm^3

minimum surface area → cube

Calculator: $\sqrt[3]{}$ $\sqrt[x]{}$

$V = 1000\text{ cm}^3$

$V = s^3$

$1000 = s^3$

$\sqrt[3]{1000} = s$

$s = 10$

∴ The dimensions are 10cm × 10cm × 10cm

Example 2: What dimensions of a rectangular prism will produce a maximum volume if the surface area is 486 cm^2

max volume → cube

$SA = 486\text{ cm}^2$

$SA = 6s^2$

$\frac{486}{6} = \frac{\cancel{6}s^2}{\cancel{6}}$

$81 = s^2$

$\sqrt{81} = s$

$9 = s$

∴ The dimensions are 9cm × 9cm × 9cm

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3D Optimization with Restrictions

Similar to 2D optimization, there may certain situations where it may not be possible to form a cube because of certain restrictions, such as:

- The dimensions need to be whole numbers or multiples of other numbers
- Sometimes more surfaces are missing or are blocked by other barriers

Example 3: Jeff is designing a glass candle holder. It will be a square-based rectangular prism with an outer surface area of, 225 cm^2 and no top. Determine the **maximum volume** of the candle holder to the nearest cm^3. What are the dimensions of the candle holder?

Base Length (cm)	Height (cm)	Volume (m^3)	Surface Area (cm^2)
1			225
2			225
3			225
4			225
5			225
6			225
7			225
8			225
9			225
10			225