MAP4C1 Unit 2: Geometry

2.6 2D Optimization

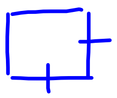
Learning Goals: I am learning to…
- ☐ Investigate the properties of 2D optimization and determine the dimensions of a shape that would produce the maximum area or minimum perimeter

In lesson 2.5 we investigated how we could optimize the area and then perimeter of an animal pen. We defined **optimization** as the process of finding the most efficient use of materials for a given situation.

Recall:

1. To **optimize area** (max), given a specific perimeter, a square has the maximum area.
2. To **optimize perimeter** (min), given a specific area, a square has the minimum perimeter.

Example 1: What dimensions produce an optimal area of a rectangle, with a perimeter of 60 cm? What is the area?

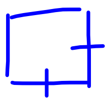


$\frac{60}{4} = 15$

dimensions 15 x 15cm.

$A = 15(15)$
$= 225\text{ cm}^2$

Example 2: What dimensions produce an optimal perimeter of a rectangle, with an area of 81 cm^2? What is the perimeter?



$\sqrt{81} = 9$

dimensions 9cm x 9cm

$P = 4(9)$
$= 36\text{cm}$

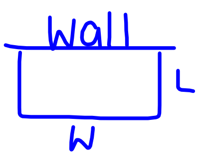
2D Optimization with Restrictions

Sometimes when trying to optimize you may come across a situation where it may not be possible to form a square because of certain restrictions, such as:

- The length and width of the rectangle need to be whole numbers
- One or more sides are enclosed by natural boundaries (i.e. wall, fence, house etc.)

Example 3: A rectangular garden is to be fenced using the wall of a house as one on the sides of the garden. The garden must have an area of 40 m^2. Determine the minimum perimeter and dimensions of the garden if:

a) The dimensions must be whole numbers



$A = 40\text{m}^2$

L	W	P
1	40	42
2	20	24
4	10	18
5	8	18
8	5	21

Dimensions could be 5m x 8m or 4m x 10m

b) The dimensions can be decimals (1dp)

check between 4 & 5

L	W	P
4.2	9.52	17.92
4.4	9.09	17.89
4.5	8.89	17.89
4.6	8.70	17.9

min Perimeter is approx. 17.89m

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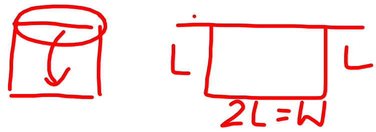
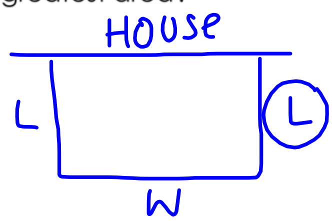
When there are restrictions such as the above situation, a square cannot be formed to form the optimal shape. In this situation,

Total Length = Total Width

2L = W



Example 4: Carlo has 28 m of fencing to build a pen for his dog. He plans to build the pen along one wall of his house to save on materials. What dimensions of the pen will give the greatest area?



$P = 28m$

$P = 2L + W$

but $W = 2L$

$P = 2L + 2L$

$P = 4L$

$\frac{28}{4} = \frac{\cancel{4}L}{\cancel{4}}$

$7 = L$

$W = 2L$
$= 2(7)$
$= 14$

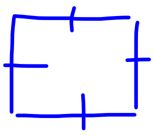
dimensions are $7m \times 14m$

$Area = LW$
$= 7(14)$
$= 98$

$\therefore$ The optimal area is $98m^2$

Example 5: A farmer wants to create a fenced exercise yard for her horses. She has 900 m of flexible fencing and wants to create the maximum area possible for yard. She can't decide if she should make a rectangular fence or a circular shaped fence. Determine which enclosure will produce the greatest area, the rectangle or the circle.

option ①: rectangle. (square)



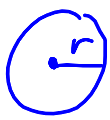
$P = 900m$

$\frac{900}{4} = 225m$

$A = 225(225)$
$= 50625m^2$

$\therefore$ The circular enclosure will produce the largest area.

option ②: circular



$C = 900m$

$C = 2\pi r$ or $C = \pi d$

$\frac{900}{2\pi} = \frac{\cancel{2\pi}r}{\cancel{2\pi}}$

$143.2 = r$

$A = \pi r^2$
$= \pi(143.2)^2$
$= 64422.3m^2$