

1.7 Applications of the Sine and Cosine Law

Learning Goals: I am learning to...

- Solve application based problems involving the Sine and Cosine Law
- Apply a variety of trigonometry solving techniques to real-world problems
- Identify when to use different trigonometry skills



<p>Primary Trig Ratios: </p> <p>$\sin \theta = \frac{opp}{hyp}$ (SOH)</p> <p>$\cos \theta = \frac{adj}{hyp}$ (CAH)</p> <p>$\tan \theta = \frac{opp}{adj}$ (TOA)</p>	<p>Pythagorean Theorem:</p> <p style="text-align: center;">$a^2 + b^2 = c^2$</p> <p>*c is always the longest side (hypotenuse)</p>	<p>Sum of Interior Angles:</p> <p style="text-align: center;">$\angle A + \angle B + \angle C = 180^\circ$</p>
<p>Angle of Elevation (Inclination): The angle formed by the horizontal and the line of sight to an object above the horizontal.</p> <p>Angle of Depression: The angle between the horizontal line and the line of sight to an object below the horizontal.</p> <div style="text-align: center; margin-top: 10px;"> </div>		<p>Sine Law:</p> $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
		<p>Cosine Law:</p> $c^2 = a^2 + b^2 - 2ab \cos C$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

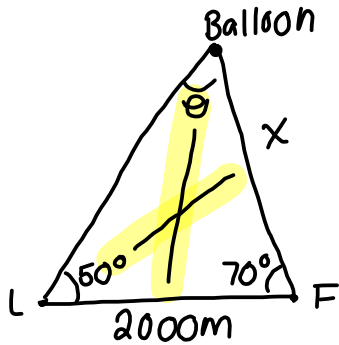
Trigonometric application problems can be solved by applying a variety of different skills and problem solving techniques.

1. Sketch a well-labeled diagram to model the situation (if appropriate).
2. Apply the appropriate trig strategy or strategies to solve.
 - Pythagorean Theorem $a^2 + b^2 = c^2$
 - Sum of angles is equal to 180°
 - Primary trig ratios – (SOH CAH TOA)
 - Sine or cosine law
3. Conclude the problem and explain what the solution means, including units!

MAP4C1 Unit 1: Trigonometry

Example 1: Lynn and Fred standing 2000 m apart, spotted a hot air balloon at angles of elevation of 50° and 70° respectively. The hot air balloon is located between them. What is the distance from Fred directly to the hot air balloon? Show all your work.

Law used: Sine



let x rep. the distance from Fred to the balloon.

$$\textcircled{1} \theta = 180^\circ - 50^\circ - 70^\circ = 60^\circ$$

$$\textcircled{2} \frac{x}{\sin 50^\circ} = \frac{2000}{\sin 60^\circ}$$

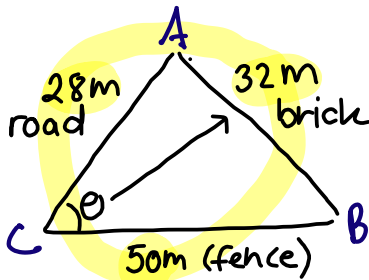
$$\frac{x(\sin 60^\circ)}{\sin 60^\circ} = \frac{2000(\sin 50^\circ)}{\sin 60^\circ}$$

$$x = 1769.1 \text{ m}$$

∴ The distance between Fred and the balloon is 1769.1m

Example 2: A triangular piece of land is surrounded by 32 m of brick wall, 50 m of fencing, and 28 m of road frontage. What angle does the fence make with the road? Show all your work!

Law used: Cosine



let θ rep. the angle between the fence and the road.

$$\cos c = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos \theta = \frac{28^2 + 50^2 - 32^2}{2(28)(50)}$$

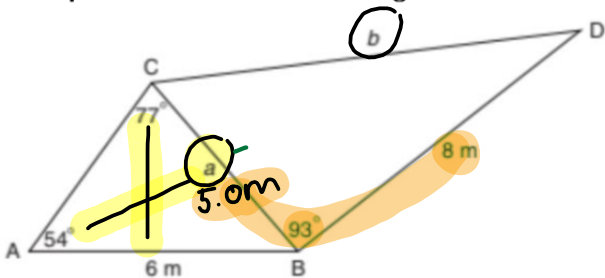
$$\cos \theta = \frac{2260}{2800}$$

$$\theta = \cos^{-1}\left(\frac{2260}{2800}\right)$$

$$\theta = 36.2^\circ$$

∴ The angle of the road with the fence is 36.2°.

Example 3: Determine side lengths, a and b. Round to one decimal place.



① Side a:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 54^\circ} = \frac{6}{\sin 77^\circ}$$

$$\frac{a(\sin 77^\circ)}{\sin 77^\circ} = \frac{6(\sin 54^\circ)}{\sin 77^\circ}$$

$$a = 5.0 \text{ m}$$

② side b:

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$b^2 = 5.0^2 + 8^2 - 2(5.0)(8)\cos 93^\circ$$

$$b^2 = 25 + 64 - 80 \cos 93^\circ$$

$$b^2 = 89 - (80 \cos 93^\circ)$$

$$b^2 = 93.19$$

$$b = 9.7 \text{ m}$$