

1.5 The Sine Law

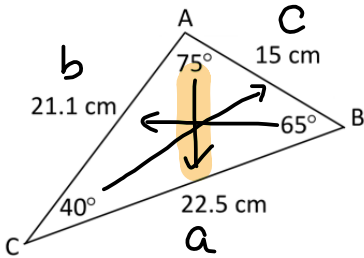
Learning Goals: I am learning to...

- Determine the measure of an unknown angle and/or side using the Sine Law
- Understand and explain when to use the Sine Law



The three primary trigonometric ratios (SOH CAH TOA) can only be used for right angle triangles, however, the Sine Law can be used for any type of triangle to find an unknown side or angle.

Investigation: Given the triangle below with all angles and side lengths known, find each ratio in the table below. Round each ratio to **4 decimal places**.



$\frac{\sin A}{a} = \frac{\sin 75^\circ}{22.5}$ $= 0.0429$	$\frac{\sin B}{b} = \frac{\sin 65^\circ}{21.1}$ $= 0.0430$	$\frac{\sin C}{c} = \frac{\sin 40^\circ}{15}$ $= 0.0429$
$\frac{a}{\sin A} = \frac{22.5}{\sin 75^\circ}$ $= 23.2937$	$\frac{b}{\sin B} = \frac{21.1}{\sin 65^\circ}$ $= 23.2813$	$\frac{c}{\sin C} = \frac{15}{\sin 40^\circ}$ $= 23.3359$

What do you notice about the ratios in the table above?

The ones with sine in the numerator are almost identical
The ones with sine in the denominator are almost equal

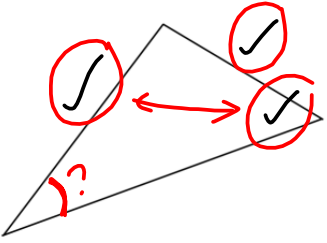
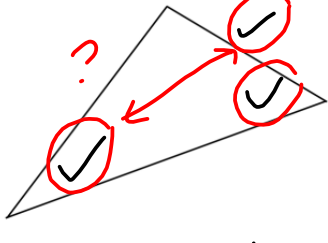
Sine Law:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
OR
 $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

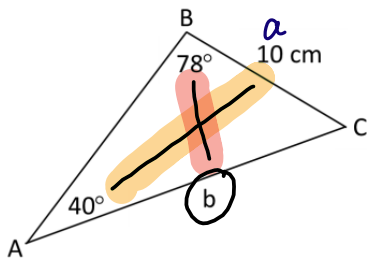
- Note: An angle is labeled with a capital letter while a side is labeled with a lower case letter. Angles and sides are labeled opposite each other.
- The equation has three parts, but only two parts are used at the same time
- To find an unknown, you need one complete pair (side and opposite angle) plus one extra side or angle.

The Sine Law can be used for any type of triangle, but it depends on what you are given as to whether you can use it or not. The Sine Law works with pairs made up of an angle and its opposite side. We must know at least one full pair (side and opposite angle) in order to use the Sine Law. It is always important to check before applying the Sine Law.

MAP4C1 Unit 1: Trigonometry

Finding an Unknown Angle If we know 2 sides and 1 angle	Finding an Unknown Side If we know 2 angles and 1 side
 <p>We must know at least 1 full pair plus one extra!</p>	

Example 1: Use the Sine Law to find the unknown side length, b.



$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

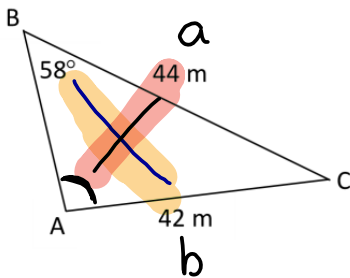
$$\frac{b}{\sin 78^\circ} = \frac{10}{\sin 40^\circ}$$

$$\frac{b(\sin 40^\circ)}{\sin 40^\circ} = \frac{10(\sin 78^\circ)}{\sin 40^\circ}$$

$$\boxed{b = 15.2 \text{ cm}}$$

*Hint: Put your unknown in the numerator to make solving quicker.

Example 2: Use the Sine Law to find the unknown angle, A.



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

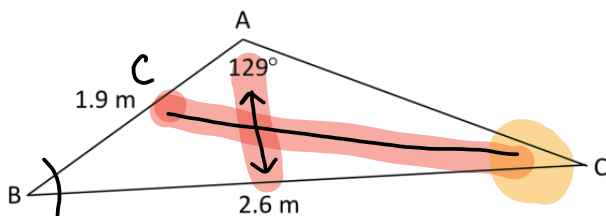
$$\frac{\sin A}{44} = \frac{\sin 58^\circ}{42}$$

$$\frac{42(\sin A)}{42} = \frac{44(\sin 58^\circ)}{42}$$

$$\sin A = \frac{44(\sin 58^\circ)}{42}$$

$$\sin A = 0.8884 \rightarrow \boxed{A = 62.7^\circ}$$

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Example 3: Determine the measure of angle B.

② find angle B:

$$B = 180^\circ - 129^\circ - 34.6^\circ$$

$$\boxed{B = 16.4^\circ}$$

① find angle C.

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin C}{1.9} = \frac{\sin 129^\circ}{2.6}$$

$$\frac{2.6(\sin C)}{2.6} = \frac{1.9(\sin 129^\circ)}{2.6}$$

$$\sin C = 0.5679$$

$$C = \sin^{-1}(0.5679)$$

$$\boxed{C = 34.6^\circ}$$