

### 1.4 Sine, Cosine and Tangent of Obtuse Angles

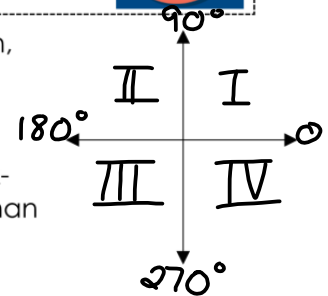
Learning Goals: I am learning to...

- Identify the sign of the three primary trig ratios in quadrant I and II
- Determine the properties of supplementary angles given the trig ratio



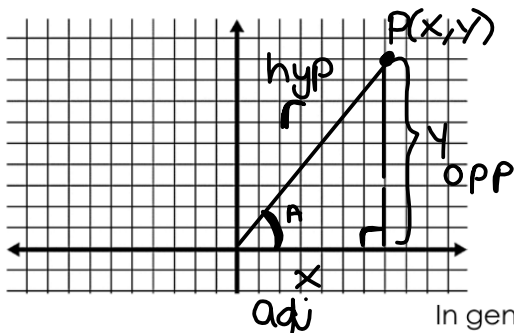
In lesson 1.3, we concluded that by knowing what quadrant an angle is in, we can determine the sign of the trigonometric ratio.

Recall: The Cartesian plane is divided into four quadrants, numbered in a counter-clockwise direction using Roman numerals. Starting at 0° on the x-axis, any oblique triangle can be defined, since the angles must be less than 180°.



#### Defining Trigonometric Ratios

Any trigonometric ratio can be defined given a point,  $P(x, y)$  on a Cartesian plane, by making a right triangle with the x-axis. The hypotenuse can also be found by using Pythagorean Theorem.



General case: In triangle PBA,

$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r}$$

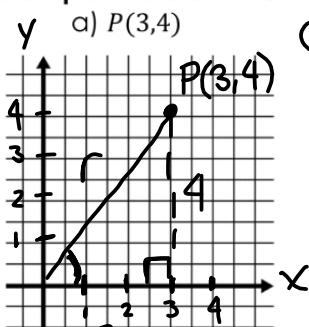
$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

In general, for point,  $P(x, y)$ .

Trigonometric Ratio	Quadrant I	Quadrant II
Sine	+	+
Cosine	+	-
Tangent	+	-

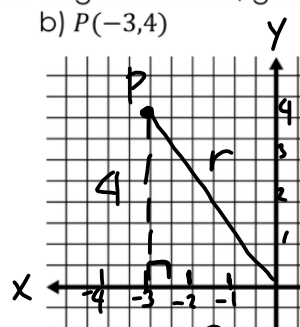
(acute)      (obtuse)

Example 1: Determine the trigonometric ratios for angles A and B, given point P.



$$\begin{aligned} \textcircled{1} \quad x^2 + y^2 &= r^2 \\ 3^2 + 4^2 &= r^2 \\ 9 + 16 &= r^2 \\ 25 &= r^2 \\ 5 &= r \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \sin A &= \frac{4}{5} \\ \cos A &= \frac{3}{5} \\ \tan A &= \frac{4}{3} \end{aligned}$$



$$\begin{aligned} \textcircled{1} \quad x^2 + y^2 &= r^2 \\ (-3)^2 + 4^2 &= r^2 \\ 9 + 16 &= r^2 \\ 25 &= r^2 \\ 5 &= r \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \sin A &= \frac{4}{5} \\ \cos A &= -\frac{3}{5} \\ \tan A &= -\frac{4}{3} \end{aligned}$$

MAP4C1 Unit 1: Trigonometry

**Properties of Supplementary Angles**

By knowing what quadrant an angle lies in, we can derive a general rule for the properties of an obtuse angle given the acute angle A.

- $\sin A = \sin (180^\circ - A)$
- $\cos A = -\cos (180^\circ - A)$
- $\tan A = -\tan (180^\circ - A)$



**Example 2:** Determine the measure of each supplementary obtuse angle.

a) The sine of acute  $\angle P$  is 0.65

Method 1	Method 2
Determine the measure of the acute angle first $\sin P = 0.65$ $P = \sin^{-1}(0.65)$ $P = 40.5^\circ$ obtuse: $180^\circ - 40.5^\circ$ $= \underline{\underline{139.5^\circ}}$	Determine the measure of the obtuse angle based on the supplementary angle property $\sin P = \sin (180^\circ - P)$ $0.65 = \sin (180^\circ - P)$ $\sin^{-1}(0.65) = 180^\circ - P$ $40.5^\circ = 180^\circ - P$ $P = 180^\circ - 40.5^\circ$ $P = 139.5^\circ$

b) The cosine of acute  $\angle R$  is 0.22

$$\begin{aligned} \cos R &= 0.22 \\ R &= \cos^{-1}(0.22) \\ R &= 77.3^\circ \\ &\text{(acute)} \end{aligned}$$

$$\begin{aligned} \text{obtuse: } 180^\circ - 77.3^\circ \\ = 102.7^\circ \end{aligned}$$

c) The tangent of acute  $\angle S$  is 0.44

$$\begin{aligned} \tan S &= 0.44 \\ S &= \tan^{-1}(0.44) \\ S &= 23.7^\circ \end{aligned}$$

$$\begin{aligned} \text{Obtuse: } 180^\circ - 23.7^\circ \\ = 156.3^\circ \end{aligned}$$