Solving Exponential Equations

Date:

An ***exponential equation*** is an equation in which the variable is in the exponent rather than it the base.

The "BATS" Method:

If two powers are equal to each other and their **B**ases **A**re **T**he **S**ame, then the exponents must also be equal.	For example: if $3^x = 3^7$ $\therefore\ x = 7$	In general: if $a^x = a^y$ then $x = y$ (as long as: $a \neq 1, 0, -1$)

Example 1a: $3^x = 81$ **Example 1b:** $(-2)^x = -128$	**Example 2** $6^{m+2} = 36$	**Example 3** $2^{7-x} = \frac{1}{2}$
Example 4 $7^{6-x} = 49^{x-3}$	**Example 5** $32^{2t} = 8^{2t+2}$	**Example 6** $10^{2x+3} = 0.001$
Example 7a $\left(\frac{1}{4}\right)^{x-2} = \left(\frac{1}{8}\right)^{x+1}$	**Example 7b** (alternate method) $\left(\frac{1}{4}\right)^{x-2} = \left(\frac{1}{8}\right)^{x+1}$	**Example 9** $7\left(6^{2x+3}\right) = 1512$

p. 23 # 2, 4, 5, 6, 7

Solving Exponential Equations (Part 2)

The key to solving exponential equations is to be able to *quickly* convert a power term to an equivalent power term with a different, and more suitable, base. In some exponential equations you may have to convert more than one term. Recognizing the proper steps is the problem that practice can help solve.

Example 1:

$8^{2x-1} = 16^{x-1}$

Example 2

$3^{x+1} - 2 \cdot 3^{x-2} = 25$

Example 3

$2(4^{v+1}) = 1$

Example 4

Solve and Check

$9^{x} - 4 \cdot 3^{x} + 3 = 0$

Example 5

$0.3^{3x-2} = 1$

Example 6

$3^{x+2} - 3^{x} = 216$

Example 7

$2^{x^2-7x+10} = 1$

Homework P. 24 # 8 – 11 Handout: Solving Exponential Equations

Solving Exponential Equations II

Date:

The key to solving exponential equations is to be able to *quickly* convert a power term to an equivalent power term with a different and more suitable base. In some exponential equations you may have to convert more than one term.

1. Express each of the following as powers, given the required base.

a) 16^{x-1}, base 2
b) 125^{3x}, base 5
c) $\frac{1}{8^x}$, base 2
d) $\left(\frac{9}{16}\right)^{3x-1}$, base $\frac{3}{4}$

e) $\left(\frac{9}{16}\right)^{3x-1}$, base $\frac{4}{3}$
f) 128^{x+3}, base 4
g) 16^{3-2x}, base 4
h) 16^{3-2x}, base $\frac{1}{8}$

2. Solve each of the following equations.

a) $2^{7-x} = \frac{1}{2}$
b) $2^{x-2} = 4^{x+2}$
c) $\left(\frac{1}{4}\right)^{x-2} = \left(\frac{1}{8}\right)^{x+1}$

d) $9^{2x+1} = 81(27^x)$
e) $9^{x+2} = \left(\frac{1}{27}\right)^{x+2}$
f) $4^x + 4^{x+1} = 80$

g) $3^x - 3^{x-1} = 54$
h) $16^k = 64^{2k-1}$
i) $2 = 6(3^{4k-2})$

j) $5 = 25^{\frac{x}{2}}$
k) $16^{3+x} = 32^{1-2x}$
l) $3(5^{x^2+3x}) = \frac{12}{100}$

m) $4^{2x} - 20(4^x) + 64 = 0$
n) $(3^{x-3})^x = \frac{1}{9}$
o) $(3^{2x+4})(27^{4-x}) = \left(\frac{1}{81}\right)^{x-2}$

p) $5^{2x} - 4(5^x) - 5 = 0$
q) $5^{2x^2+2} = 125^x(25^x)^x$
r) $2^{2x} - 2^{x+2} - 32 = 0$

Answers

1. a) 2^{4x-4} b) 5^{9x} c) 2^{-3x} d) $\left(\frac{3}{4}\right)^{6x-2}$

e) $\left(\frac{4}{3}\right)^{2-6x}$ f) $4^{\frac{7}{2}x+\frac{21}{2}}$ g) 4^{6-4x} h) $\left(\frac{1}{8}\right)^{\frac{8}{3}x-4}$

2. a) $x=8$ b) $x=-6$ c) $x=-7$ d) $x=2$ e) $x=-2$ f) $x=2$

g) $x=4$ h) $x=\frac{3}{4}$ i) $k=\frac{1}{4}$ j) $x=1$ k) $x=\frac{-1}{2}$ l) $x=-1,-2$

m) $x=1,2$ n) $x=1,2$ o) $x=\frac{-8}{3}$ p) $x=1$ q) $x=\frac{2}{3}$ r) $x=3$

A Quick Exponent Review

Date: ____________

Although many exponent law questions can be evaluated on the calculator, it is important that you are able to THINK your way to an answer.

1. Evaluate or simplify as required.

a) $\left(\frac{8}{27}\right)^{\frac{1}{3}}$ b) -7^0 c) $(25x^2)^{\frac{1}{2}}$

d) $\frac{9^0}{16^{-\frac{3}{4}}}$ e) $\frac{1}{81^{-\frac{3}{4}}}$ f) $(-8)^{-\frac{2}{3}}$

g) $(16x^{12}y^8)^{\frac{3}{4}}$ h) $\frac{1}{2^{-3}}+\frac{1}{2^{-2}}$ i) $\frac{1}{2^{-3}+2^{-2}}$

2. Express each first number as a power of the second number.

a) 32, 2 b) 64, 4 c) 1, 4

d) –8, -2 e) 81, 3 f) 32, 4

3. Simplify each of the following.

a) $(3^x)(3^x)(3^x)$ b) $3^x+3^x+3^x$ c) $8(2^x)$

Answers:

1a) $\frac{2}{3}$ b) -1 c) $5x$ d) 8 e) 27 f) $\frac{1}{4}$ g) $8x^9y^6$ h) 12 i) $\frac{8}{3}$

2a) 5 b) 3 c) 0 d) 3 e) 4 f) $\frac{5}{2}$

3a) 3^{3x} b) 3^{1+x} c) 2^{3+x}

Applications of Exponential Equations

Date:

Exponential Decay

Let A represent the amount (mass) of radioactive substance at time instant t.
Let A_0 represent the initial amount (mass) of that radioactive substance.
Let h represent the half-life of that substance i.e. the time it takes for half of the substance to decay.

Then $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$

Ex. Radon has a half-life of 25 days. How much time does it take for a sample of 200 g to decay to 12.5 g?

Population Growth

Let P represent the population at time instant t.
Let P_0 represent the initial population.
Let d represent the doubling time of that population i.e. the time it takes for the population to double.

Then $P = P_0\left(2\right)^{\frac{t}{d}}$

Ex. The number of insects in a colony doubles every month. If there are 250 insects now, how long will it take for the colony to grow to 8000 insects?